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Assessment of EGM2008 in Europe using accurate astrogeodetic vertical deflections and omission error estimates from SRTM/DTM2006.0 residual terrain model data

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Abstract: We assess the new EGM2008 Earth gravitational model using a set of 1056 astrogeodetic vertical deflections over parts of continental Europe. Our astrogeodetic vertical deflection data set originates from zenith camera observations performed during 1983-2008. This set, which is completely independent from EGM2008, covers, e.g., Switzerland, Germany, Portugal and Greece, and samples a variety of topography – level terrain, medium elevated and rugged Alpine areas. We describe how EGM2008 is used to compute vertical deflections according to Helmert's (surface) definition. Particular attention is paid to estimating the EGM2008 signal omission error from residual terrain model (RTM) data. The RTM data is obtained from the Shuttle Radar Topography Mission (SRTM) elevation model and the DTM2006.0 high degree spherical harmonic reference surface. The comparisons between the astrogeodetic and EGM2008 vertical deflections show an agreement of about 3 arc seconds (root mean square, RMS). Adding omission error estimates from RTM to EGM2008 significantly reduces the discrepancies from the complete European set of astrogeodetic deflections to 1 arc second (RMS). Depending on the region, the RMS errors vary between 0.4 and 1.5 arc seconds. These values not only reflect EGM2008 commission errors, but also short-scale mass-density anomalies not modelled from the RTM data. Given (1) formally stated EGM2008 commission error estimates of about 0.6-0.8 arc seconds for vertical deflections, and (2) that short-scale mass-density anomalies may affect vertical deflections by about 1 arc second, the agreement between EGM2008 and our astrogeodetic deflection data set is very good. Further focus is placed on the investigation of the high-degree spectral bands of EGM2008. As a general conclusion, EGM2008 – enhanced by RTM data – is capable of predicting Helmert vertical deflections at the 1 arc second accuracy level over Europe.

Keywords: model validation, EGM2008, vertical deflections, residual terrain model (RTM), omission error, commission error

1 Introduction

In 2008, the high-resolution (~10 km) Earth Gravitational Model EGM2008 [*Pavlis et al.* 2008] was released by the US National Geospatial Intelligence Agency (NGA). It is complete to spherical harmonic degree and order 2159, but contains additional spherical harmonic coefficients to degree 2190 and order 2159. EGM2008 is constructed from a combination of GRACE satellite data [*Mayer-Guerr* 2007], topographic data [*Saleh and Pavlis*, 2002; *Pavlis et al.*, 2007], altimetry on sea [e.g., *Anderson et al.*, 2010; *Sandwell and Smith*, 2009] and gravity observations on land areas [e.g., *Pavlis et al.*, 2007, 2008].

An important task is the quality assessment of EGM2008 by means of external validation techniques. EGM2008 has already been evaluated regionally and globally from a range of external data sets, such as height anomalies at GNSS/levelling stations, other gravity field models (global spherical harmonic models or regional geoid/quasigeoid solutions), terrestrial gravity observations and vertical deflections. These efforts are documented through 25 validation reports from different authors in *Newton's Bulletin* [2009].

However, only a couple of these external validations tested EGM2008 with astrogeodetic vertical deflections: *Huang and Veronneau* [2009] used a set of 939 vertical deflections over Canada, and *Claessens et al.* [2009] deployed a set of 1080 vertical deflections over Australia. In addition, the EGM2008 development team used 3561 vertical deflections over the US, and the 1080 vertical deflections over Australia for EGM2008 evaluation [cf. *Pavlis* *et al., 2008*]. Vertical deflections, being first-order horizontal derivatives of the disturbing potential, are particularly powerful for testing the high-frequency components of an Earth Gravitational Model (EGM) [*Jekeli,* 1999].

Sometimes, the assessment of EGMs is difficult because data sets such as gravity anomalies were already used for the computation of the model coefficients and are, consequently, not independent [e.g., *Gruber*, 2009; *Claessens et al.,* 2009]. Furthermore, the assessment of EGMs with gravity field observations always poses the problem of signal omission [e.g., *Torge,* 1981]. This is because any EGM is limited by its spectral resolution (in the case of EGM2008: 5’ (arc minutes), which equates ~10 km in the latitude direction), while terrestrial observations (such as gravity, height anomalies and vertical deflections) contain the full spectral signal power [e.g., *Gruber,* 2009].

Therefore, comparisons among the model and observations not only reflect the errors of the model (the commission error), but also the limited spectral content of the model (the omission error). To the authors’ understanding, none of the evaluation reports published in *Newton’s Bulletin* [2009] made attempts to model the EGM2008 signal omission error from digital elevation models as an auxiliary data source. However, residual terrain model (RTM) data [cf. *Forsberg,* 1984] may be used for modeling some parts of the omission error as shown by *Hirt* [2010]. This allows better validation of EGMs, because the comparisons among the model and observations better indicate the model commission errors rather than possibly being swamped by the omission errors.

In this paper, we use a total of 1056 high-precision vertical deflections observed with zenith cameras over Switzerland, Germany, Portugal, Greece and some other European countries to assess EGM2008. Our vertical deflection data is the largest set that is currently available from zenith camera observations. Importantly, our vertical deflection data are totally independent of EGM2008, i.e., the data were used neither for the computation nor calibration of the EGM2008 model coefficients [*Pavlis* 2009, pers. comm.].

As opposed to previous studies assessing EGM2008 with vertical deflections, this paper models the signal omission error by means of RTM data, which greatly reduces the residuals among EGM2008 and the vertical deflections. The RTM data is constructed from SRTM (Shuttle Radar Topography Mission) elevations [*Farr et al.*, 2007] and a spherical harmonic reference surface (harmonic representation of the DTM2006.0 topography data base, cf. *Pavlis et al.*, [2007]) serving as an EGM2008-compatible long-wavelength reference.

2 Astrogeodetic vertical deflections

Astrogeodetic vertical deflections are defined as the angle between the physical plumbline and the ellipsoidal normal at points on or just above the Earth surface [e.g., *Torge*, 2001; *Featherstone and Lichti*, 2009]. Astrogeodetic instruments for star observation such as zenith cameras [e.g., *Hirt et al.*, 2010] are used for the observation of astronomical longitude Λ and latitude Φ (defining the direction of the plumbline) at points with known geodetic longitude λ and latitude φ (representing the ellipsoidal normal). Commonly, vertical deflections are expressed in terms of a North-South component (ξ) and an East-West component (η). The basic equations read [cf. *Jekeli*, 1999]:

$$\begin{aligned} \xi &= \Phi - \varphi + \frac{1}{2}\eta^2 \tan\varphi, \\ \eta &= (\Lambda - \lambda)\cos\varphi. \end{aligned} \qquad (1)$$

Astrogeodetic vertical deflections (ξ,η) from equation (1) are also known as surface vertical deflections or Helmert vertical deflections [cf. *Jekeli,* 1999; *Torge,* 2001; *Featherstone and Lichti,* 2009]. Vertical deflections from astronomical observations may be used for highly accurate determination of the geoid or quasigeoid using astrogeodetic levelling [cf. *Hirt and Flury*, 2008; *Hirt et al.* 2008]. In geophysics, vertical deflections are a useful source for interpretation and analysis of subsurface density anomalies [e.g., *Mönicke,* 1981; *Bürki,* 1989; *Somieski,* 2008].

At the University of Hanover (Germany) and ETH Zurich (Switzerland), analogue and digital zenith camera systems have been developed and used for the observation of astrogeodetic vertical deflections $(\xi,\eta)^{astro}$ in several European countries [see *Bürki*, 1989; *Hirt and Bürki,* 2002; *Hirt,* 2004; *Bürki et al.,* 2004, 2007; *Hirt et al.,* 2007, *Hirt et al.*, 2008, *Somieski et al.* 2007 for details]. The accuracy of vertical deflections from digital zenith camera observations was found to be 0.1" (arc seconds) [e.g., *Hirt and Seeber*, 2008], while vertical deflections from analogue zenith camera observations are less accurate with standard deviations of about 0.3-0.5" [*Bürki,* 1989]. For a discussion of error sources inherent in our astrogeodetic vertical deflections (e.g., star observations, star positions, and anomalous atmospheric refraction), we refer the reader to *Hirt and Seeber,* [2008] and *Bürki,* [1989].

The set of 1056 astrogeodetic vertical deflections $(\xi,\eta)^{astro}$ used in this study mainly originates from analogue and digital zenith camera observations. The TZK3 analogue zenith camera [*Bürki,* 1989] was used for the observation of 433 stations over Switzerland between 1983 and 2000 (Table 1). Between 2003 and 2008, the Hanover TZK2-D digital zenith camera [*Hirt*, 2004; *Hirt et al.,* 2010] and the Zurich DIADEM digital zenith camera [*Bürki* *et al.*, 2004, 2007; *Somieski*, 2008] were used for observation of 623 vertical deflections over other parts of Europe (Table 1).

The most important set is the Swiss national vertical deflection data set that consists of 101 and 433 evenly distributed stations. The data covers a very rugged part of the European Alps, as is seen in Figure 1A. Subsets of the Swiss national data set extend over adjacent regions of Italy, Germany, Liechtenstein, France and Austria and were partly provided by the state survey authorities of the neighbouring countries.

In the flatter parts of Northern Germany and the Netherlands, 175 stations are available from digital zenith camera observations (Figure 1B). Most of these stations are arranged along local traverses of 7-20 km length in areas where subterranean mass-density anomalies (principally salt domes) are present [*Hirt*, 2004; *Hirt and Seeber,* 2007]. Further vertical deflection data sets are available in the Harz Mountains, the most significant rugged area in Northern Germany (centred at ~51.9N, ~10.5E, cf. Figure 1B). Here, 120 stations form a 65 km long traverse that completely crosses the Harz Mountains with about a 700 m variation in elevation [*Hirt et al.*, 2008].

In the Bavarian Alps (Ester Mountains and Isar Valley), a total of 182 digital zenith camera vertical deflections extend over a local area of 25 km x 25 km (Figure 1C). Additional deflection data sets were collected in Southern Europe: 17 stations cover the whole of Portugal (Figure 1D) and 28 stations are located in the Aegean Sea region, Northern Greece (Figure 1E). The latter is particularly rare because the vertical deflection observations extend over numerous Greek islands [*Müller et al.*, 2007; *Somieski*, 2008].

In many test areas, our vertical deflection data is subject to local mass-density anomalies, such as, subterranean salt domes in Northern Germany [*Hirt and Seeber*, 2007], Pleistocene fillings of Alpine valleys [*Flury*, 2002], as well as lakes and glaciers [*Marti*, 1997]. These structures – occurring at scales of a few km (which is below the nominal EGM2008 resolution of ~10 km) – may influence the astrogeodetic vertical deflection field by a magnitude of 1” [see *Hirt*, 2004; *Hirt and Seeber*, 2007; *Hirt and Flury*, 2008]. This is akin to a commission error in the computation of the omission error in the RTM contribution to EGM2008. A part of our deflection data set covers the Ivrea area in Northern Italy [*Bürki*, 1989; *Marti*, 1997]. Here, the Ivrea Body as large-scale intra-crustal density anomaly influences the vertical deflection field with amplitudes as large as 30” [*Bürki*, 1989].

The complete set of 1056 vertical deflection stations does not homogeneously extend over Europe. However, the vertical deflection stations cover a range of different geographic regions as well as different terrain types (level, medium elevated and mountainous terrain, as well as islands). Because the majority of the observations are highly-accurate (about 0.1”, Table 1), this set of vertical deflections may be considered as ground truth for the comparison with EGM2008 and the RTM-modeled omission errors.

The geodetic coordinates of the (ξ, η) vertical deflection stations are provided in terms of geodetic longitude λ, geodetic latitude φ and ellipsoidal height h, referred to the European Terrestrial Reference System ETRS89 (http://etrs89.ensg.ign.fr/en/). All astrogeodetic vertical deflections used in this study are surface vertical deflections; as such, they correspond to Helmert’s definition [e.g., *Torge*, 2001; *Jekeli*, 1999]. The descriptive statistics of the complete astrogeodetic data set is given in Table 2.

3 Vertical deflections from EGM2008 and SRTM/DTM2006.0 RTM data

EGM2008 is used together with SRTM/DTM2006.0 RTM data for the computation of vertical deflections, which approximate the observed astrogeodetic deflections fairly well in terms of spectral content. However, the EGM2008 vertical deflections must correspond to Helmert's definition to be comparable with the astrogeodetic observations. In the following, we outline the steps necessary to compute surface vertical deflections from EGM2008 and show how they are enhanced by means of SRTM elevation data and DTM2006.0 spherical harmonic heights to reduce the omission error.

3.1 Spherical harmonic synthesis

We start by converting the geodetic position (φ,λ,h) of the vertical deflection observation to geocentric polar coordinates (geocentric latitude $\bar{\varphi}$ and distance r between the observation point and the geocentre). The conversion is accomplished using global geocentric Cartesian coordinates (X,Y,Z) as auxiliary values, see, e.g., *Jekeli* [2006] or *Torge* [2001, p. 94] for the relevant equations. Then, the spherical harmonic series expansion of the disturbing potential T is evaluated [after, e.g., *Smith*, 1998; *Torge*, 2001, p. 215]

$$T(r,\theta,\lambda) = \frac{GM}{r}\sum_{n=2}^{n_{\max}^{EGM}}\left(\frac{a}{r}\right)^{n}\sum_{m=0}^{n}(\overline{\delta C}_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda)\bar{P}_{nm}(\cos\theta) \quad (2)$$

using the EGM2008 fully-normalized spherical harmonic coefficients $\bar{C}_{nm}$, $\bar{S}_{nm}$ along with the EGM2008 specific scaling parameters GM (geocentric gravitational constant) and a (semi major axis). In equation (2), n denotes the degree and m the order of the harmonic coefficients and $n_{\max}^{EGM}$ is the maximum degree of evaluation. The variable θ denotes geocentric co-latitude (θ = π/2-$\bar{\varphi}$) and $\bar{P}_{nm}(\cos\theta)$ are the fully-normalized associated Legendre functions [e.g., *Torge*. 2001, p. 71].

The term $\overline{\delta C}_{nm} = \bar{C}_{nm} - \bar{C}_{nm}^{GRS}$ expresses that the low-degree even zonal harmonics $\bar{C}_{nm}^{GRS}$ of the GRS80 (Geodetic Reference System 1980) normal gravity field must be subtracted from the $\bar{C}_{nm}$ zonal harmonic coefficients of EGM2008 (see, e.g., *Smith* [1998] for a detailed description). In equation (2), the zero-degree term (a vertical offset of a few dm, see, e.g., *Smith* [1998]; *Torge* [2001]) is neglected since it does not affect the vertical deflection values since they are the first horizontal derivatives of the disturbing potential.

EGM Development Team [2008] recommends to use EGM2008 to degree $n_{\max}^{EGM}$ = 2190. The coefficients of EGM2008 beyond degree 2159 arise from the conversion from ellipsoidal to spherical harmonics. These degrees are incomplete, but their inclusion is critical to reduce model errors in the high degrees, especially over areas near the poles (cf. *Holmes and Pavlis* [2007]).

Spherical harmonic vertical deflections (ξ^*, η^*) are obtained as derivatives of the disturbing potential T in direction of geocentric latitude $\bar{\varphi}$ (giving the North-South component ξ^*) and in direction of longitude λ (giving the East-West component η^*), cf. *Torge* [2001, p. 258], *Jekeli* [1999]:

$$\xi^* = -\frac{1}{\gamma r}\frac{\partial T}{\partial \bar{\varphi}}, \qquad (3)$$

$$\eta^{*} = -\frac{1}{\gamma r \cos\bar{\varphi}} \frac{\partial T}{\partial \lambda}. \quad (4)$$

Equations (2) - (4) are evaluated at the geodetic coordinates (φ,λ,h) of our 1056 stations using EGM2008 to maximum degree $n_{\max}^{EGM}$ = 2190 along with the high-degree synthesis software harmonic_synth.f [*Holmes and Pavlis* 2008]. It should be noted that, in practice, there is no difference between zero-tide and tide-free vertical deflections from EGM2008. This is directly seen by comparing (ξ^{*},η^{*}) vertical deflections computed from the tide-free and the zero-tide version of EGM2008. For details on the tidal systems, we refer to, e.g., *Ekman* [1989]; *Jekeli* [1999]; *Mäkinen and Ihde* [2009].

The values (ξ^{*},η^{*}) obtained from equations (3) and (4) are Molodensky vertical deflections in spherical approximation [cf. *Roland*, 2005; p. 7, *Jekeli*, 1999]. Molodensky's definition of vertical deflections uses the (curved) normal plumbline instead of the ellipsoidal normal as reference direction [cf. *Torge,* 2001; *Heiskanen and Moritz,* 1967].

3.2 Corrections

Two corrections are applied to the spherically approximated Molodensky vertical deflections (ξ^{*},η^{*}) in order to obtain EGM2008 Helmert vertical deflections $(\xi,\eta)^{EGM\,2008}$ in ellipsoidal approximation; these are:

$$\begin{aligned} \xi^{EGM\,2008} &= \xi^{*} + \delta\xi^{NC} + \delta\xi^{ell}, \\ \eta^{EGM\,2008} &= \eta^{*}. \end{aligned} \quad (5)$$

The terms $\delta\xi^{NC}$ (correction of the curvature of the normal plumb line) and $\delta\xi^{ell}$ (ellipsoidal correction) are explained next. Molodensky vertical deflections differ from Helmert vertical deflections by the curvature of the normal plumbline with respect to the ellipsoidal surface normal [cf. *Heiskanen and Moritz* 1967, p. 196]. The correction for the curvature of the normal plumb line $\delta\xi^{NC}$ concerns only the North-South component ξ^*. It is computed as a function of the ellipsoidal height *h* and ellipsoidal latitude φ [*Jekeli*, 1999]:

$$\delta\xi^{NC} = 0.17'' \cdot h\ [km] \cdot \sin 2\varphi \quad . \tag{6}$$

The correction $\delta\xi^{NC}$ reaches maximum values of about 0.3-0.5" in the mountainous areas of our study (Switzerland, Bavaria, $h \approx$ 2-3 km, $\varphi \approx 45°$), while it is insignificant in the low-elevated terrain. An additional correction is required because the ξ^* component is computed as partial derivative of the disturbing potential *T* with respect to geocentric latitude $\bar{\varphi}$ instead of geodetic latitude φ [cf. *Jekeli*, 1999]. In other words, equations (3) and (4) are spherical approximations in that the partial derivatives refer to the sphere instead of to the ellipsoid. In the longitude direction, there is no difference between the spherical and ellipsoidal approximation and, hence, no correction is required for the East-West component η^*. The ellipsoidal correction for the North-South component ξ^* reads [*Jekeli*, 1999]:

$$\delta\xi^{ell} = (\varphi - \bar{\varphi})\frac{\delta g}{\gamma} \tag{7}$$

where $(\varphi - \bar{\varphi})$ is the difference between geodetic and geocentric latitude, δg is the gravity disturbance (at the coordinates φ,λ,h) and γ is normal gravity on the ellipsoid (at latitude φ

and h = 0). The gravity disturbance δg is obtained as the radial derivative of the disturbing potential T (in spherical approximation), cf. *Torge* [2001, p. 271]:

$$\delta g = -\frac{\partial T}{\partial r}. \tag{8}$$

With maximum possible values of 690” for the latitude difference $(\varphi - \bar{\varphi})$ [see *Torge,* 2001, p. 95] and maximum values of gravity disturbances δg of about 200 mgal in the high European mountains, the ellipsoidal correction $\delta\xi^{ell}$ does not exceed values of about 0.15”.

For further, smaller corrections to vertical deflections from spherical harmonic synthesis, we refer to the study by *Jekeli* [1999]. Here, effects such as the tidal correction (i.e., conversion from the actual tide system to the mean tide system or from the mean tide system to the zero tide system) are not accounted for because the amplitudes are generally below 0.01-0.02”, as such without perceivable impact on our validation results.

3.3 Construction of RTM data

We use residual terrain model (RTM) data for computing omission errors in order to enhance the spectral content of EGM2008 vertical deflections, recalling that these are more sensitive to the higher frequencies [cf. *Jekeli 1999*]. EGM2008 vertical deflections $(\xi,\eta)^{EGM\,2008}$, as obtained from equations (3) and (4), do not possess the full spectral power – as opposed to astrogeodetic vertical deflections. This is because the spherical harmonic series expansion (equation 2) is truncated at maximum degree $n_{\max}^{EGM}$ = 2190, thus neglecting high-frequency spectral signals of Earth’s gravity field with wavelengths of 5’ (~10 km in latitude direction) or shorter. This effect – known as signal omission error [e.g., *Torge*, 2001, *Gruber* 2009] – can reach amplitudes of some arc seconds for vertical deflections [e.g. *Torge*, 1981; *Hirt*, 2010].

A considerable part of the high-frequency spectrum of vertical deflections is generated by the topography [e.g., *Forsberg and Tscherning*, 1981]. RTM data, i.e. detailed elevation data referred to a smooth (long-wavelength) reference surface, is capable of reconstituting the high frequencies of the gravity field [*Forsberg*, 1984, 1994]. Constructing the reference surface consistent with the maximum degree n_{max} of the EGM2008 vertical deflections allows us to use the RTM method to compute signal omission errors [cf. *Hirt*, 2010]. Estimates of signal omission errors may be used to augment the EGM2008 in the very high degrees beyond the truncation of the series expansion in equation (2). This then allows for a more objective assessment of EGM2008 since the omission error has been reduced to some extent.

The freely available 3 arc second SRTM (Shuttle Radar Topography Mission) elevation data set by CGIAR-CSI (Consortium for Spatial Information of the Consultative Group for International Agricultural Research) [*Jarvis et al.*, 2008] was selected as a detailed elevation data set for the omission error computation. Version 4.1 of this elevation data set is a post-processed SRTM release with the data gaps (i.e., no data areas present in the original SRTM releases) filled applying a range of sophisticated interpolation methods [*Reuter et al.*, 2007]. Some of the gaps in rugged terrain (representing problem areas in earlier SRTM releases, e.g., *Denker*, [2004]; *Marti*, [2004]) have been filled by means of auxiliary data sets instead of simple interpolation [*Reuter et al.*, 2007]. This leads to considerably improved SRTM elevation data in mountainous areas such as our test areas in the European Alps. It is acknowledged that SRTM is a digital surface model containing heights of vegetated areas;

hence it is not a digital terrain model. Nevertheless, SRTM data set is a valuable data source that allows the computation of precise gravity field effects [e.g., *Tsoulis et al*., 2009; *Hirt*, 2010]. For accuracy analyses of the SRTM elevation data sets, the reader is referred to, e.g., *Marti* [2004]; *Rodigruez et al*. [2005]; *Jarvis et al.* [2008].

The global topographic database DTM2006.0 created by the EGM2008 Development Team [cf. *Pavlis et al*., 2007] is used as a long-wavelength reference surface for the construction of the RTM. The spherical harmonic expansion of the DTM2006.0 elevation data base, complete to degree and order 2190, supplements EGM2008. It was computed by means of spherical harmonic analysis of the global SRTM model, bathymetric data and further elevation data sets [*Pavlis et al.,* 2007]. The spherical harmonic expansion of the heights (+) above mean sea level (MSL) and depths (-) below MSL of the DTM2006.0 global topographic database is available complete to degree and order 2190, and comprises a set of about 2.4 million pairs of fully normalized height coefficients $\overline{HC}_{nm}, \overline{HS}_{nm}$ that give $H^{DTM\,2006.0}$ elevations using

$$H^{DTM\,2006.0}(\theta,\lambda) = \sum_{n=0}^{n_{\max}^{DTM}} \sum_{m=0}^{n} (\overline{HC}_{nm} \cos m\lambda + \overline{HS}_{nm} \sin m\lambda) \overline{P}_{nm}(\cos\theta) \tag{9}$$

where $n_{\max}^{DTM}$ is the maximum degree of evaluation, (θ,λ) are geocentric co-latitude and geodetic longitude, and $\overline{P}_{nm}(\cos\theta)$ are the fully-normalized associated Legendre functions [cf. *EGM Development Team* 2008]. Equation (9) can be evaluated, e.g., with the harmonic_synth.f software [*Holmes and Pavlis,* 2008]. The spherical harmonic expansion of the DTM2006.0 elevation database was successfully used for the computation of RTM-implied gravitational information during EGM2008 model construction [*Pavlis et al*., 2007], but this was not for degrees beyond 2190 as is done in this study.

RTM elevations *z* are formed as differences SRTM elevations H^{SRTM} minus DTM2006.0 elevations $H^{DTM\,2006.0}$. We use the appropriate maximum degrees for the computation of EGM2008 deflections [equations (2)-(8)] and DTM2006.0 spherical harmonic heights [equation (9)], cf. Section 4 for details. As a consequence, the spectral content implied by EGM2008 is widely removed from the SRTM data.

DTM2006.0 spherical harmonic heights consistently supplement EGM2008 on land areas and may be used for precisely filtering SRTM data. At or near the coastlines (e.g., our test sites in Portugal or specifically in Greece), however, the use of DTM2006.0 for constructing RTM data from SRTM is limited. This is because DTM2006.0 contains bathymetry on ocean surfaces, as opposed to SRTM where the ocean heights are zero. This inconsistency may be diminished (but not eliminated) by setting the DTM2006.0 heights on ocean surfaces to zero, which was done in this study.

As a first alternative solution, the SRTM elevation data set (with the ocean heights set to zero) may be converted to spherical harmonic coefficients using spherical harmonic analysis and used as long-wavelength RTM reference. As a second alternative, DTM2006.0 may be used together with precision bathymetry in ocean areas, yielding a consistent RTM data set. However, such an advanced application of the RTM technique at land-sea transitions is beyond the scope of the present study and remains as a future task.

3.4 Omission error computation (RTM vertical deflections)

The RTM elevation grid is used for the omission error computation. We make use of the prism method, which is described by several authors [e.g. *Forsberg and Tscherning,* 1981; *Forsberg,* 1984; *Tsoulis* 1999, *Nagy et al.,* 2000, 2002]. The RTM elevation z of each grid node represents a rectangular prism (mass element) for which the gravitational potential can be calculated analytically [see *Nagy et al.,* 2000, 2002]. The horizontal derivatives of the gravitational potential in the North-South (or East-West) direction give the RTM effect for deflection component ξ (or η), respectively. The numerical integration (summation) is performed over all prisms within some distance (explained later) around the computation point in order to compute the RTM vertical deflections $(\xi,\eta)^{RTM}$ in radians [after *Forsberg,* 1984; *Nagy et al*., 2000, 2002]:

$$
\begin{aligned}
\xi^{RTM} &= -\frac{1}{\gamma}\sum_{1}^{k} G\rho \left|\left|\left| y\ln(z+r) + z\ln(y+r) - x\tan^{-1}\frac{yz}{xr} \right|_{x_1}^{x_2}\right|_{y_1}^{y_2}\right|_{z_1}^{z_2}, \\
\eta^{RTM} &= -\frac{1}{\gamma}\sum_{1}^{k} G\rho \left|\left|\left| z\ln(x+r) + x\ln(z+r) - y\tan^{-1}\frac{xz}{yr} \right|_{x_1}^{x_2}\right|_{y_1}^{y_2}\right|_{z_1}^{z_2}.
\end{aligned}
\qquad (10)
$$

Here, G denotes the Universal gravitational constant, ρ the density of the topography, γ normal gravity, and r is the distance between the point (x, y, z) and the computation point, which is the origin of the coordinate system used for the calculation [cf. *Nagy et al*., 2000]. The limits $(x_1, y_1, z_1, x_2, y_2, z_2)$ define the geometry of the each prism. Equation (10) is evaluated by substituting (x, y, z) with the limits $(x_1, y_1, z_1, x_2, y_2, z_2)$ in all combinations, giving 24 terms [cf. *Nagy et al.,* 2000]. We use equation (10) with $z_1 = 0$ and $z_2 = z_{RTM} = H^{SRTM} - H^{DTM\,2006.0}$, so that the prism height $z_2 - z_1$ represents the residual elevations z_{RTM}.

Because RTM elevations z oscillate between positive and negative values [e.g., *Forsberg and Tscherning*, 1981; *Forsberg*, 1984], the summation of RTM effects [equation (10)] needs to be performed only over k prisms within some radius R around the computation point. The radius depends on the roughness and oscillations of the RTM elevations. We determined the required integration radius empirically by comparisons of RTM vertical deflections from a range of integration radii with those computed from an 80 km integration radius, serving as 'reference'. For most stations and a radius R = 50 km, the differences were found to be below 0.05" [*Hirt*, 2010]. This indicates the required area of evaluation to obtain fairly stable values of RTM vertical deflections $(\xi,\eta)^{RTM}$. The numerical integration [equation (10)] was performed with software based on the Gravsoft program TC [*Forsberg*, 1984], using a standard rock density ρ of 2.67 x 10^3 kg m^{-3}.

RTM vertical deflections $(\xi,\eta)^{RTM}$ as obtained from equation (10) contain a significant part of the high frequency gravity field spectrum beyond the spherical harmonic degree $n_{\max}^{DTM}$. As such, they represent estimates of the EGM2008 omission error, but there is a commission error in this because mass-density variations in the residual topography are not modelled by the constant-density assumption. Through a simple combination (addition), EGM2008 vertical deflections $(\xi,\eta)^{EGM2008}$ are spectrally enhanced by RTM deflections $(\xi,\eta)^{RTM}$ to give EGM2008/RTM deflections $(\xi,\eta)^{EGM2008/RTM}$:

$$\begin{aligned}\xi^{EGM2008/RTM} &= \xi^{EGM2008} + \xi^{RTM}, \\ \eta^{EGM2008/RTM} &= \eta^{EGM2008} + \eta^{RTM}.\end{aligned} \tag{11}$$

The descriptive statistics of the data sets $(\xi,\eta)^{EGM2008}$ [from equations (2)-(8)] ,$(\xi,\eta)^{RTM}$ [equations (9), (10)] and $(\xi,\eta)^{EGM2008/RTM}$ [equation (11)], respectively, at our 1056 astrogeodetic stations are listed in Table 3. The RTM vertical deflections $(\xi,\eta)^{RTM}$ reach significant values (maximum amplitudes of about 15" and RMS values of 2.6"-2.7"). This indicates the magnitude of the EGM2008 omission error for vertical deflections, over the locations of the 1056 sites tested here.

4 Comparisons

4.1 Astrogeodetic deflections vs. EGM2008/RTM

For our first numerical test, we follow the recommendation of the *EGM Development Team* [2008] to use EGM2008 to degree $n_{\max}^{EGM}$ = 2190 and DTM2006.0 to degree $n_{\max}^{DTM}$ = 2160. The latter is used as input for the computation of RTM vertical deflections $(\xi,\eta)^{RTM}$, cf. Section 3.4. We compared our astrogeodetic vertical deflections $(\xi,\eta)^{astro}$ with the EGM2008 vertical deflections $(\xi,\eta)^{EGM2008}$ ($n_{\max}^{EGM}$=2190) and with the EGM2008/RTM deflections $(\xi,\eta)^{EGM2008/RTM}$ ($n_{\max}^{EGM}$=2190 and $n_{\max}^{DTM}$ = 2160).

The complete descriptive statistics of the differences $(\xi,\eta)^{astro}$ – $(\xi,\eta)^{EGM2008}$ and $(\xi,\eta)^{astro}$ – $(\xi,\eta)^{EGM2008/RTM}$, respectively, are compiled in Table 4 for the complete data set and, further to this, for all subsets which were defined in Table 1. The RMS values from the differences Astro-EGM2008/RTM reflect – in essence – two error sources: (1) EGM 2008 commission errors (uncertainty from the spherical harmonic model coefficients only) and (2) the impact of any short-scale (below 5') density anomaly [cf. *Forsberg* 1984] with respect to the standard rock density ρ used for the computation of RTM vertical deflections.

Further to these error sources, the SRTM elevations and the astrogeodetic observations represent minor sources of uncertainty which are neglected in the sequel. The uncertainty of the astrogeodetic observations is on the order of 0.1" for many of our stations, see Sect. 2. The impact of errors in the SRTM elevations on the RTM vertical deflections used in our study is estimated to be below 0.2" (RMS) based on analysis of vertical deflections differences computed from differences between SRTM and national elevation data in the European Alps.

The comparisons show that the maximum differences between astrogeodetic and EGM2008 deflections of around 15" are reduced to a level of 5" using RTM data as augmentation for EGM2008. Similarly, the RMS errors (around 3" for both deflection components over Europe) diminish to the level of 1" by using EGM2008/RTM deflections. The improvement rates given in the last column of Table 4 show that about 65% of the RMS errors between the astrogeodetic observations and the EGM2008 deflections are explained by the RTM vertical deflections. The effectiveness of the RTM data for reducing the discrepancies between astrogeodetic deflections and EGM2008 is also illustrated by the distribution of $(\xi,\eta)^{astro}$ – $(\xi,\eta)^{EGM\,2008}$ and $(\xi,\eta)^{astro}$ – $(\xi,\eta)^{EGM\,2008/RTM}$ residuals, respectively in Figure 2, and in Figure 3 showing the residuals as a function of the terrain roughness.

A detailed analysis of the descriptive statistics of the Astro-EGM2008/RTM differences for our subsets (Tables 1 and 4) shows the following:

- Supplementing EGM2008 with RTM data generally improves the agreement (RMS values) in all test areas, both for the North-South component ξ and the East-West component η, with improvement rates varying between 2% and 81%. There is a tendency for larger improvement rates in rugged terrain than in low-elevated terrain. Even in the relatively flat Northern Germany test area, RTM data slightly improves the agreement between EGM2008 and the astrogeodetic vertical deflections.

- In mountainous Switzerland, the RMS values based on the analogue zenith camera observations (about 1.35" for both components) are larger than those based on the much more accurate digital zenith camera observations (RMS of about 1.1"). As such, the comparisons using the 433 analogue zenith camera observations reflect not only the above mentioned error sources, but also the larger observation noise of the old analogue observations (assumed to be on the level 0.3-0.5", cf. *Bürki*, [1989]).

- For the other test areas (level Northern Germany and Netherlands, the rugged areas Harz mountains, Bavarian Alps and Portugal), the RMS errors are as low as 0.4"-0.8", which is a very good agreement between the astrogeodetic ground truth and the EGM2008/RTM vertical deflections. A correlation between terrain roughness and the discrepancies Astro-EGM2008/RTM is not evident from our data. This observation is supported by a plot of the differences astrogeodetic deflections $(\xi,\eta)^{astro}$ – EGM2008/RTM deflections $(\xi,\eta)^{EGM2008/RTM}$ as a function of the terrain roughness (cf. Figure 3).

- Relatively small improvement rates were obtained over Greece (Islands of the North Aegean Sea). Here, we found the lowest overall RMS agreement of around 1.4" for both deflection components. This behaviour may be a manifestation that DTM2006.0 is less suited for filtering SRTM heights at near or coastal zones, even after setting the DTM2006.0 heights to zero in ocean areas (see above). It is acknowledged that, particularly in the Greece test area, the inconsistency between DTM2006.0 and SRTM is evident. This is because of the steep bathymetry (North Aegean Trough), found near the astrogeodetic observation sites on a number of small islands [e.g., *Somieski,* 2008].

4.2 Comparisons with EGM2008 commission error estimates

Another interesting aspect of our EGM2008 assessment involves the comparison among the official, i.e. formally stated, EGM2008 commission error estimates and the RMS errors from our Astro-EGM2008/RTM comparisons. *EGM Development Team* [2009] has published standard deviations for point values of vertical deflections (and of other gravity field functionals, but which are not relevant here) which were computed from the EGM2008 input data [cf. *Pavlis et al.,* 2008] based on a special error propagation technique described by *Pavlis and Saleh* [2004]. Importantly, these commission error estimates account for the geographic location of the computation points and, hence, do not merely represent a global estimate of the commission error that can be computed based on variance propagation of the standard deviations of the spherical harmonic coefficients [e.g. *Koch,* 2005].

The EGM2008 commission error estimates are available in terms of 5’ x 5’ grids and refer to the spectral band 2-2159 [*EGM Development Team,* 2009]. Figure 4 shows the EGM2008 commission error estimates for vertical deflection component ξ^* [equation (3)], together with the location of astrogeodetic stations over Europe. For most of our stations, the EGM2008

ξ^* commission error varies between 0.4" and 0.8". As the EGM2008 η^* commission error estimates are almost identical to the ξ^* error estimates (statistics of the differences over the European area in Figure 4: min/max/mean/RMS: -0.56 / 0.41 / 0.00 / 0.03"), the η^* commission error estimates are not shown.

A numerical comparison among the EGM2008 ξ^*, η^* commission error estimates (mean standard deviations for our various test areas) with the RMS errors from the differences Astro–EGM/RTM is shown in Table 5. We recall that the Astro–EGM/RTM RMS are "combined" (joint) estimates of the EGM2008 commission error *and* of any local mass-density anomaly not modelled from our RTM data (akin to a commission error of the omission error estimates). Using the formally stated EGM2008 commission error estimates, we obtain rough estimates of the average signal strength $\sigma(\text{local density})$ of short-scale density anomalies:

$$\sigma^2(\text{local density}) \approx RMS^2(\text{ASTRO} - \text{EGM/RTM}) - \sigma^2(\text{EGM commission}) \qquad (12)$$

We evaluated equation (12) using all digital zenith camera observations (0.1" accuracy), without the Greece data (excluding the impact of the previously addressed RTM inconsistencies for islands near deep ocean troughs) and without the analogue zenith camera observations (removing the impact of lower observational accuracy). Based on these 595 high-precision astrogeodetic observations (Table 5), $\sigma(\text{local density})$ is found to be approximately 0.4" for both vertical deflection components. These values indicate the average signal strength (amplitude) of unmodelled topographic mass-density anomalies in our

RTM vertical deflections at scales shorter than 5' (degree 2160), e.g. salt domes, lakes, valley fillings and all other local density anomalies.

It is acknowledged that these values are coarse estimates because the $\sigma(\text{EGM commission})$ values are certainly not free of uncertainty and because of further sources of error (e.g., SRTM and DTM2006.0 heights), which were not modelled in equation (12). Given that short-scale density anomalies may influence surface vertical deflections by about 1" magnitude [a cautious estimation based on *Hirt,* 2004; *Hirt and Seeber,* 2007; *Hirt and Flury,* 2008; *Hirt et al*., 2008], our comparison: (1) indicates fairly realistic estimates of the average signal strength of mass-density anomalies $\sigma(\text{local density})$ at short scales; and (2) does not provide evidence that the EGM2008 commission error estimates are too optimistic.

4.3 Analysis of different combination degrees

Further insight into the quality of EGM2008 over Europe is gained from a set of experimental computations. In addition to the results based on a spherical harmonic degree of $n_{\max}^{EGM}$ = 2160 and $n_{\max}^{DTM}$ = 2190, we used other maximum degrees $n_{\max}^{EGM}$ (360, 720, …, 2160 and 2190) for the spherical harmonic synthesis of EGM2008 [equation (2)] and, applied the *same* degree ($n_{\max}^{DTM}$ = $n_{\max}^{EGM}$) to the DTM2006.0 computation [equation (9)]. Further to this, we used EGM96 [*Lemoine et al.,* 1998] up to its limiting degree of 360. The RMS values of the comparisons $(\xi,\eta)^{astro} - (\xi,\eta)^{EGM}$ and $(\xi,\eta)^{astro} - (\xi,\eta)^{EGM/RTM}$, respectively are reported in Table 6.

The comparisons between EGM96 and EGM2008 with $n_{\max}^{EGM}$ = 360 show similar RMS values for the differences Astro-EGM96 and Astro-EGM2008, respectively, amounting to 5.0-5.9".

Owing to the use of RTM data, however, significantly smaller values are observed for the ξ-component (2.3” for EGM2008 instead of 3.3” for EGM96). The η-component is improved slightly from 2.4” (EGM96) to 2.25” (EGM2008). These results show that the long-wavelength part of the Earth’s gravity field is better modelled by EGM2008 than by EGM96 in our European test areas, which is most probably due to GRACE (Gravity Recovery and Climate Experiment) satellite observations used in EGM2008 for the low degrees up to 180 [cf. *Pavlis et al.,* 2008]. This conclusion is a corroboration of similar findings by *Gruber* [2009], who analysed GNSS/levelling data over Europe. Importantly, it is the RTM augmentation applied to EGM2008 and EGM96, respectively, that has allowed us to detect the improvement of EGM2008 over EGM96 based on astrogeodetic vertical deflections.

Further evaluations using $n_{\max}^{EGM}$ = 360, 720, 1080, 1440, 1800 and 2160 show a steadily improving agreement of EGM2008 (and EGM2008/RTM solution) with the astrogeodetic deflections. This demonstrates that the EGM2008 spherical harmonic coefficients are significant even in the medium and high degrees (360…2190) [cf. *Jekeli*, 1999]. It should be noted that for the spherical harmonic degrees 1440 to 2160, the Astro-EGM2008/RTM comparisons show a quite similar agreement of about 1.1”. This demonstrates, first, that this spectral window of the vertical deflections is dominated by the topography. Second, these results suggest that our RTM data implies fairly similar information as EGM2008 does in the high spherical harmonic degrees 1441-2160.

Using the EGM2008 gravitational model to degree $n_{\max}^{EGM}$ = 2190 and the DTM2006.0 topographic model to degree 2160 for the computation of RTM vertical deflections ($n_{\max}^{DTM}$ =2160) gives the best agreement with the astrogeodetic deflections (RMS differences of 1.05” for both components). The agreement is slightly better than the results obtained from $n_{\max}^{EGM} = n_{\max}^{DTM}$ =2160 and $n_{\max}^{EGM} = n_{\max}^{DTM}$ =2190, respectively. We consider this as empirical endorsement of the ‘official’ recommendation [*EGM Development Team* 2008] to use “EGM2008 gravitational model to degree 2190, with the parallel use of [the DTM2006.0] elevation expansion to degree 2160”.

5 Conclusions

Our comparisons of EGM2008 (to degree 2160) with 1056 vertical deflections over Europe showed RMS differences of around 3”. Enhancing EGM2008 with RTM data as an estimate of the signal omission error greatly reduced the RMS errors to the level of 1“ for both vertical deflection components. Considering that any short-scale (below the EGM2008 resolution of ~10 km) density anomalies (occurring with amplitudes of about 1“) are not modelled from the RTM data, the overall agreement among the astrogeodetic observations and EGM2008 augmented by RTM is assessed to be very good over Europe.

Our experimental computations of EGM2008, EGM96 and RTM data show that EGM2008 is an improvement over EGM96 in the spherical harmonic degrees 2-360, which is attributed to the use of GRACE data. Furthermore, the agreement between EGM2008 and the astrogeodetic deflections is found to be better the higher the maximum degree of EGM2008 used. The best agreement between the astrogeodetic data and EGM2008 only is reached for a spherical harmonic expansion degrees 2160 and 2190 with RMS values of about 3“. For the combined EGM2008/RTM data, however, the best agreement (RMS values around 1.1“) can be attained for lower maximum degrees of 1440, and an expansion of EGM2008 to degree 2160 does not lead to a further, significant improvement. This suggests that RTM data is capable of delivering similar information as EGM2008 within the spectral window 1441-2160 in Europe.

Owing to its considerable quality, EGM2008 may be used in combination with RTM data for the prediction of surface vertical deflections. Over Europe, an overall prediction accuracy of the order of 1“ may be expected, without the need to carry out astronomical measurements. Of course, it is acknowledged that the accuracy for vertical deflection predictions at a particular site may be degraded by the presence of local mass-density anomalies.

As future work, our approach to augmenting a spherical harmonic model in the high degrees with RTM data may be extended to other gravity field quantities, e.g. gravity anomalies or disturbances and geoid/quasigeoid heights. This would enable a better validation of Earth Geopotential Models, like EGM2008, from terrestrial observations (as shown here with vertical deflections). Particularly in mountainous regions with scarce gravity data coverage or in rugged areas without precise geoid/quasigeoid models, our approach is expected to reduce EGM omission errors, thus improving predictions of gravity field functionals.

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Table 1. Overview of the European test areas with astrogeodetic vertical deflections from zenith camera observations

Area	**Terrain characteristics**	**Heights [m]**	**Stations**	**Observation period**	**Instruments**	**Main references**
Switzerland	medium elevated – mountainous	290-2800	101	2003-2008	DIADEM, TZK2-D	*Müller et al.*, [2004], *Bürki et al.* [2005]
Switzerland	medium elevated – mountainous	60-3580	433	1983-2000	TZK3 (analogue camera)	*Buerki*, [1989], *Marti*, [1997]
Northern Germany, Netherlands	level terrain	0-80	175	2004-2006	TZK2-D	*Hirt*, [2004] *Hirt and Seeber*, [2007]
Harz Mountains	medium elevated	80-830	120	2006	TZK2-D	*Hirt et al.*, [2008]
Bavarian Alps	mountainous	650-1480	182	2004-2005	TZK2-D	*Hirt and Flury*, [2008]
Portugal	medium elevated	20-1430	17	2004	DIADEM	*Somieski et al.*, [2007]
Greece	Islands	0-30	28	2005-2006	DIADEM	*Somieski*, [2008]

Table 2. Descriptive statistics of the 1056 astrogeodetic vertical deflections $(\xi,\eta)^{astro}$. Units are arc seconds.

		Component ξ				Component η			
Data set	**Variables**	**Min**	**Max**	**Mean**	**RMS**	**Min**	**Max**	**Mean**	**RMS**
Astrogeodetic	$(\xi,\eta)^{astro}$	-33.20	30.59	5.64	11.48	-22.20	37.33	1.56	7.34

Table 3. Descriptive statistics of the 1056 EGM2008 vertical deflections $(\xi,\eta)^{EGM\,2008}$ (evaluated to degree 2190), the RTM vertical deflections $(\xi,\eta)^{RTM}$ (with a degree 2160 DTM2006.0 reference surface) and the EGM2008/RTM vertical deflections $(\xi,\eta)^{EGM\,2008/RTM}$. Units are arc seconds.

		Component ξ				Component η			
Data set	**Variables**	**Min**	**Max**	**Mean**	**RMS**	**Min**	**Max**	**Mean**	**RMS**
EGM (2190)	$(\xi,\eta)^{EGM\,2008}$	-30.86	30.86	5.66	11.33	-18.70	32.02	1.23	6.97
RTM (2160)	$(\xi,\eta)^{RTM}$	-14.45	16.21	0.03	2.67	-11.12	13.87	0.17	2.57
EGM2008/RTM	$(\xi,\eta)^{EGM\,2008/RTM}$	-31.05	31.25	5.69	11.46	-23.31	33.86	1.41	7.33

Table 4. Descriptive statistics of the comparison of astrogeodetic vertical deflections with EGM2008 and with EGM2008/RTM vertical deflections. Com. = Vertical deflection component, Imp. = Improvement rate of the RMS in percent between the Astro–EGM2008

comparison and the Astro–EGM2008/RTM comparison. The test areas are the same as in Table 1. Units of vertical deflections are arc seconds.

			Astro–EGM2008				**Astro–EGM2008/RTM**				**Imp.**
Area/subset	**Com.**	**Stations**	**Min**	**Max**	**Mean**	**RMS**	**Min**	**Max**	**Mean**	**RMS**	**%**
Europe (all)	ξ	1056	-15.00	15.54	-0.02	3.02	-4.74	5.37	-0.05	1.05	65.4
	η	1056	-11.67	15.62	0.33	2.97	-4.33	4.90	0.15	1.05	64.6
Swiss (digital)	ξ	101	-15.00	8.23	-1.07	3.77	-2.67	2.93	-0.29	1.12	70.3
	η	101	-6.01	6.98	0.41	2.92	-2.01	2.91	0.25	1.12	61.5
Swiss (analogue)	ξ	433	-13.31	15.54	0.30	3.66	-4.74	5.37	0.10	1.36	62.7
	η	433	-11.67	15.62	0.11	3.76	-4.33	4.90	0.03	1.37	63.7
N. Germany	ξ	175	-0.35	1.59	0.35	0.53	-0.47	0.89	0.17	0.40	25.4
	η	175	-0.56	1.23	0.33	0.70	-0.68	1.27	0.21	0.69	1.5
Harz	ξ	120	-2.19	2.37	-0.12	0.95	-1.12	1.63	-0.10	0.54	43.1
	η	120	-2.39	1.19	-0.22	0.79	-1.32	0.78	-0.05	0.36	54.8
Bavaria	ξ	182	-8.75	6.77	-0.40	3.41	-1.34	1.06	-0.36	0.75	78.0
	η	182	-6.55	8.77	1.08	3.28	-0.57	1.62	0.44	0.62	81.1
Portugal	ξ	17	-1.98	2.96	0.39	1.35	-1.41	0.63	-0.06	0.56	58.7
	η	17	-0.71	4.90	0.70	1.39	-0.76	0.68	0.05	0.40	71.4
Greece	ξ	28	-3.92	2.74	-0.66	1.84	-3.77	2.53	-0.55	1.39	24.4
	η	28	-4.47	4.93	0.70	2.51	-1.84	2.64	0.49	1.46	41.7

Table 5. RMS values of the Differences Astro–EGM/RTM vs. mean EGM commission errors. Units are arc seconds.

Data set		Astro–EGM/ RTM		EGM commission errors	
Area	**Stations**	**RMS(ξ)**	**RMS(η)**	**Mean σ(ξ)**	**Mean σ(η)**
Europe (all)	1056	1.05	1.05	0.71	0.71
Europe (without 2 and 7)	595	0.71	0.71	0.60	0.60
1 Swiss (digital)	101	1.12	1.12	0.89	0.90
2 Swiss (analog)	433	1.36	1.37	0.86	0.87
3 Northern Germany	175	0.40	0.69	0.33	0.33
4 Harz Mountains	120	0.54	0.36	0.52	0.53
5 Bavaria	182	0.75	0.62	0.76	0.75
6 Portugal	17	0.56	0.40	0.46	0.47
7 Greece	28	1.39	1.46	0.66	0.66

Table 6. RMS values of the $(\xi,\eta)^{astro} - (\xi,\eta)^{EGM}$ and $(\xi,\eta)^{astro} - (\xi,\eta)^{EGM/RTM}$, comparisons, respectively, as a function of the EGM (EGM2008 or EGM96) and the spherical harmonic

degree used for EGM2008 expansion (n_{max}^{EGM}) and DTM2006.0 expansion (n_{max}^{DTM}). Units are arc seconds.

Gravity field model			**Astro–EGM**		**Astro–EGM/ RTM**		**Improvement**	
Model	n_{max}^{EGM}	n_{max}^{DTM}	**RMS (ξ)**	**RMS (η)**	**RMS(ξ)**	**RMS(η)**	**%(ξ)**	**%(η)**
EGM2008	**2190**	**2160**	3.02	2.97	1.05	1.05	65.4	64.6
EGM2008	**2190**	**2190**	3.02	2.97	1.12	1.14	62.8	61.5
EGM2008	**2160**	**2160**	3.03	2.96	1.10	1.06	63.8	64.1
EGM2008	**1800**	**1800**	3.45	3.20	1.11	1.05	68.0	67.3
EGM2008	**1440**	**1440**	4.08	3.62	1.14	1.09	72.0	69.9
EGM2008	**1080**	**1080**	4.21	3.83	1.23	1.17	70.8	69.6
EGM2008	**720**	**720**	4.69	4.27	1.54	1.44	67.2	66.2
EGM2008	**360**	**360**	5.52	5.02	2.28	2.25	58.8	55.2
EGM96	**360**	**360**	5.88	5.02	3.30	2.42	43.8	51.8

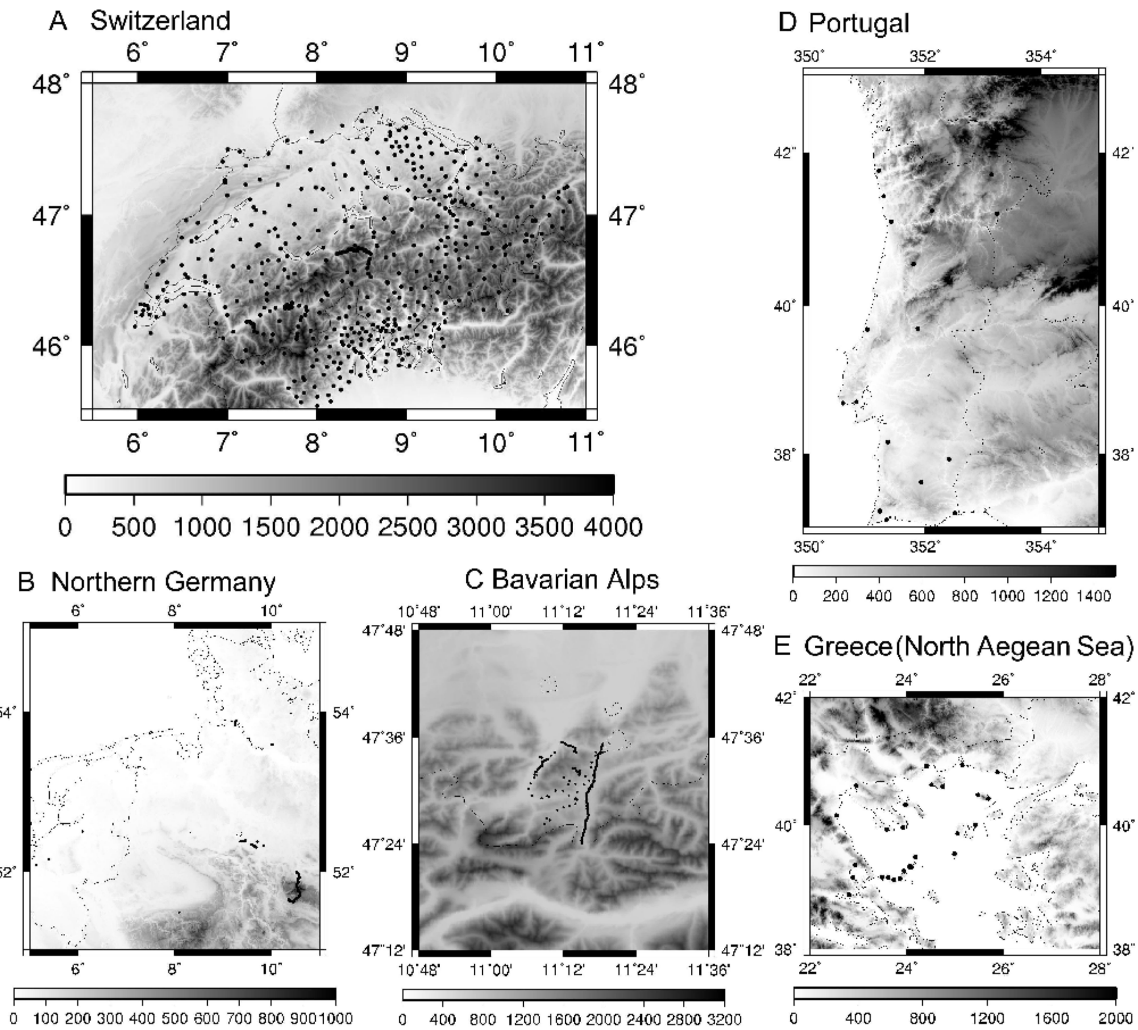

Figure 1. Test areas with vertical deflection data. A: Switzerland (and neighbour countries Italy, Liechtenstein, Austria, France and Germany). B: Northern Germany with Harz Mountains and the Netherlands. C: Bavarian Alps (Ester Mountains, Isar Valley). D: Portugal. E: Greece (North Aegean Sea). Coordinates in terms of ETRS89 latitude and longitude. Elevation data is from SRTM, unit is metre.

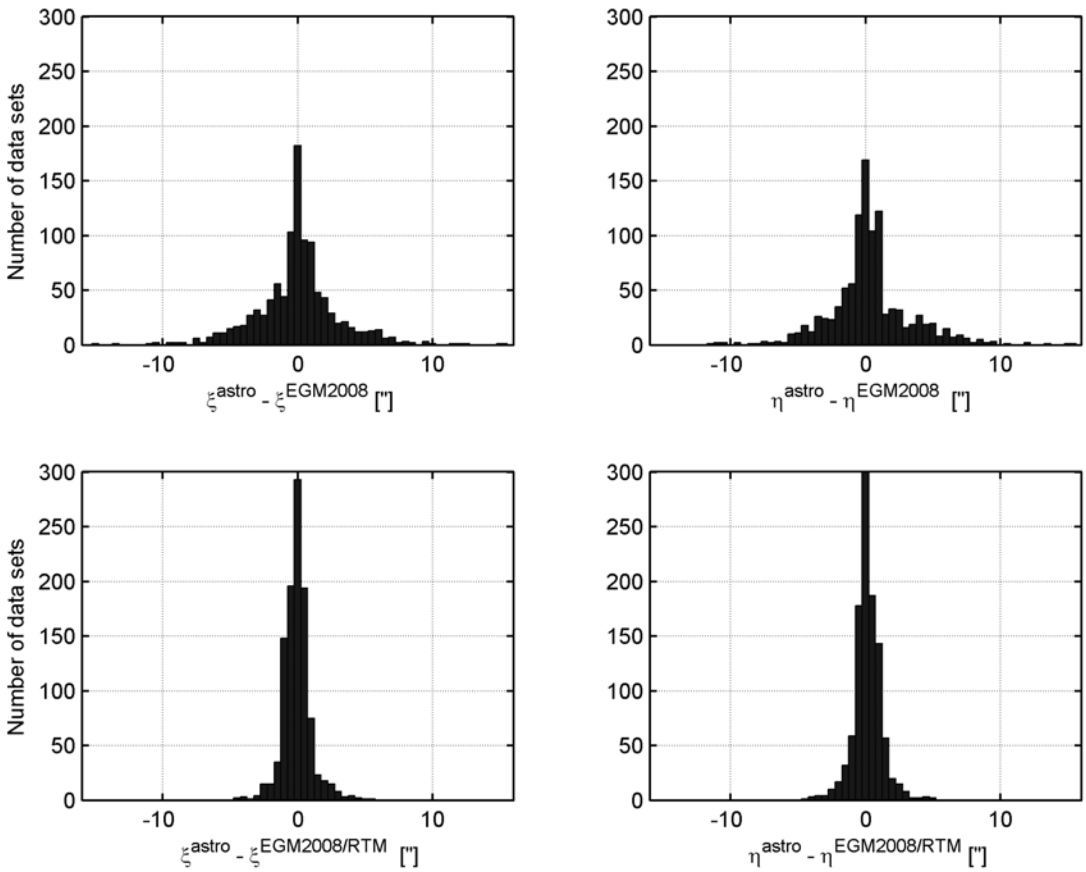

Figure 2. Histogram of the differences $(\xi,\eta)^{astro} - (\xi,\eta)^{EGM\,2008}$ (top) and $(\xi,\eta)^{astro} - (\xi,\eta)^{EGM\,2008/RTM}$ (bottom) at 1056 stations in Europe. Units are arc seconds.

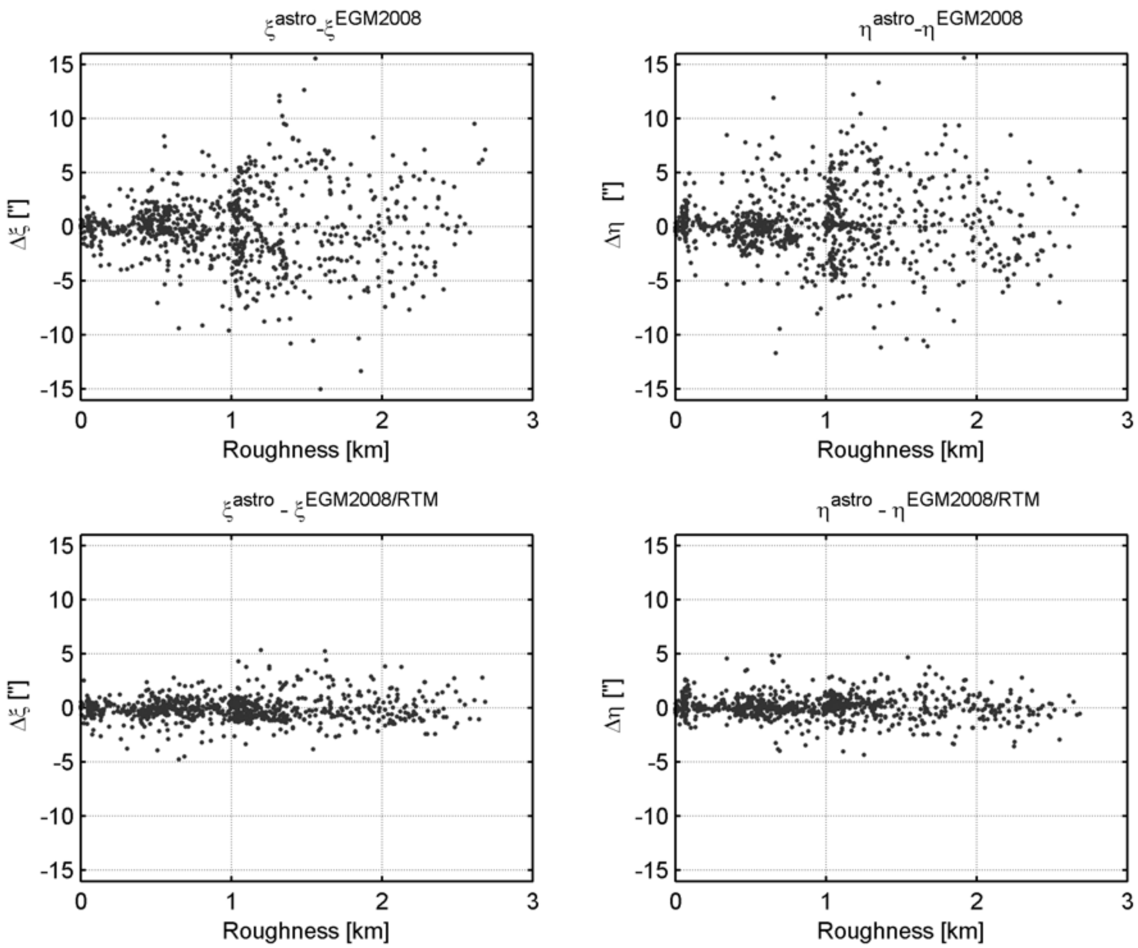

Figure 3. 1056 differences $(\xi,\eta)^{astro} - (\xi,\eta)^{EGM\,2008}$ (top) and $(\xi,\eta)^{astro} - (\xi,\eta)^{EGM\,2008/RTM}$ (bottom) as a function of the terrain roughness. The terrain roughness was computed as RMS of the adjacent SRTM elevations within a radius of 1 km around each station. Units of deflections are arc seconds.

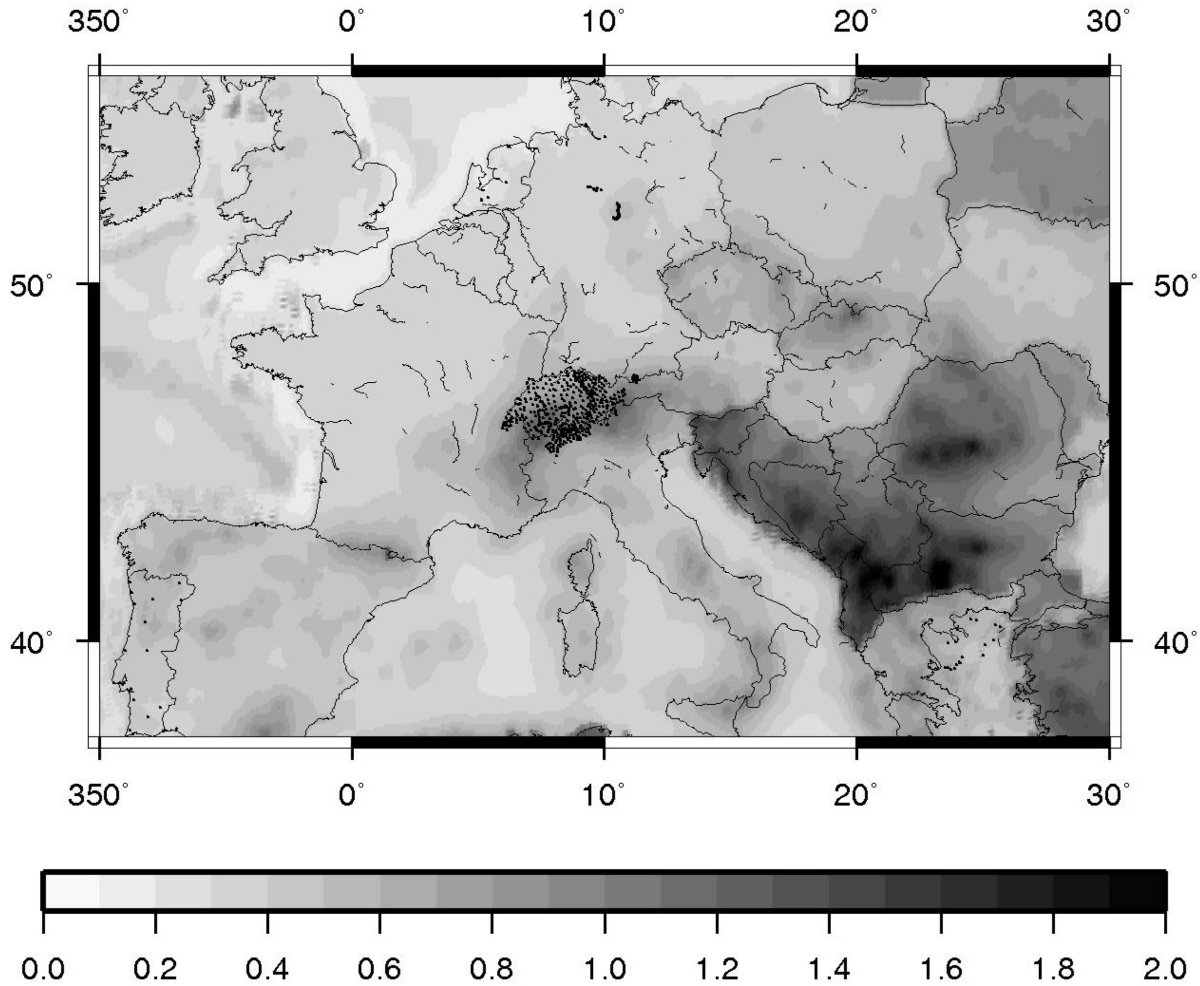

Figure 4. Formally stated EGM2008 commission errors for vertical deflections component ξ in arc seconds (source: Pavlis and Saleh [2004] and *EGM Development Team* [2009]) and location of the 1056 astrogeodetic vertical deflections.