Evaluation of the first GOCE static gravity field models using terrestrial gravity, vertical deflections and EGM2008 quasigeoid heights

C. Hirt

Western Australian Centre for Geodesy & The Institute for Geoscience Research, Curtin University of Technology, GPO Box U1987, Perth, WA 6845, Australia Email: <u>c.hirt@curtin.edu.au</u>

T. Gruber

Institute of Astronomical and Physical Geodesy, Technische Universität München, Arcisstrasse 21, 80333 München, Germany Email: <u>Thomas.Gruber@tum.de</u>

W.E. Featherstone

Western Australian Centre for Geodesy & The Institute for Geoscience Research, Curtin University of Technology, GPO Box U1987, Perth, WA 6845, Australia Email: <u>w.featherstone@curtin.edu.au</u>

Abstract: Recently, four global geopotential models (GGMs) were computed and released based on the first two months of data collected by the GOCE (Gravity field and steady-state Ocean Circulation Explorer) dedicated satellite gravity field mission. Given that GOCE is a technologically complex mission and different processing strategies were applied to real space-collected GOCE data for the first time, evaluation of the new models is an important aspect. As a first assessment strategy, we use terrestrial gravity data over Switzerland and Australia and astrogeodetic vertical deflections over Europe and Australia as ground-truth data sets for GOCE model evaluation. We apply a spectral enhancement method (SEM) to the truncated GOCE GGMs to make their spectral content more comparable with the terrestrial data. The SEM utilises the high-degree bands of EGM2008 and residual terrain model (RTM) data as a data source to widely bridge the spectral gap between the satellite and terrestrial data. Analysis of RMS (root mean square) errors is carried out as a function of (i) the GOCE GGM expansion degree and (ii) the four different GOCE GGMs. The RMS curves are also compared against those from EGM2008 and GRACE-based GGMs. As a second assessment strategy, we compare global grids of GOCE GGM and EGM2008 quasigeoid heights. In connection with EGM2008 error estimates, this allows location of regions where GOCE is likely to deliver improved knowledge on the Earth's gravity field. Our ground truth data sets, together with the EGM2008 quasigeoid comparisons, signal clear improvements in the spectral band ~160-165 to ~180-185 in terms of spherical harmonic degrees for the GOCE-based GGMs, fairly independently of the individual GOCE model used. The results from both assessments together provide strong evidence that the first two months of GOCE observations improve the knowledge of the Earth's static gravity field at spatial scales between ~125 and ~110 km, particularly over parts of Asia, Africa, South America and Antarctica, in comparison to the pre-GOCE-era.

Keywords: GOCE, validation, spectral enhancement, gravity, vertical deflections, EGM2008, GRACE

1 Introduction

ESA's (the European Space Agency's) GOCE (Gravity field and steady-state Ocean Circulation Explorer) is the first-ever satellite mission that deploys dedicated gradiometry in space to produce a homogeneous, highly-accurate, near-global map of the Earth's static gravity field (e.g., Rummel et al. 2002, Visser et al. 2002; Drinkwater et al. 2003). The main objective of the GOCE mission is to deliver a gravity field model at the ~1-2 cm accuracy level for geoid undulations and at the 1-2 mGal level for gravity anomalies (Rummel 2005) down to a target resolution of ~100 km, which equates to degree ~200 in terms of spherical harmonics. GOCE utilises space-borne gravity gradiometry in combination with GPS-based (Global Positioning System) high-low SST (satellite-to-satellite tracking) e.g., Rummel et al. (2002), Gruber (2008), Rummel et al. (2009).

Launched on 17 March 2009, GOCE is currently sensing the Earth's gravity field, and is expected to collect data until ~2012. GOCE is complementary to previous satellite gravity field missions, such as GRACE (Gravity Recovery And Climate Experiment; e.g., Reigber et al. 2005, Tapley et al. 2005), which significantly improved the knowledge of the low- to medium-frequency constituents of the Earth's static gravity field. Because of the complementary character of the missions, GOCE augmented by GRACE in the lower frequencies is expected to provide an even better picture of Earth's global gravity field than GOCE alone (Pail et al. 2010a).

Based on the first two months of GOCE observations, four GOCE-based global geopotential models (GGMs) were recently made available to the public via ESA (http://www.esa.int) and the IAG's (International Association of Geodesy's) ICGEM (International Centre for Global Earth Models, http://icgem.gfz-potsdam.de/ICGEM/) operated by GFZ German Research Centre for Geosciences Potsdam. The four new models (Table 1) differ in their maximum spherical harmonic degree and computation method (i.e., the way GOCE observations were modelled and processed to yield the GGM coefficients). As a further difference, auxiliary data sets (e.g., information from GRACE GGMs) were used as augmentation to some of the models (described later). This poses the question of the relative performance of the new GGMs: how closely do they approximate Earth's global gravity field and in which spectral bands do they deliver improved information?

Comparisons with independent gravity field functionals (as 'ground-truth') can be used to evaluate the performance of the new GOCE-based GGMs. Independent sources of gravity field information are, for instance, geoid undulations from GPS/levelling, gravity anomalies and astrogeodetic vertical deflections. The main problem with such comparisons is the different spectral content of the GGMs and terrestrial observations (e.g., Gruber 2009; Ihde et al. 2010). Because the spatial resolution of GGMs is limited by their maximum degree, there is always an omission error (i.e., neglect of high-degree spectral content) in the GGM-derived functionals. In contrast, terrestrial observations contain the full spectral signal.

Therefore, GGM evaluation usually requires the data sets involved (GGM functionals and terrestrial observations) to be comparable in terms of spectral content. There are (at least) two different strategies to solve this problem.

- 1. Low-pass filtering of the terrestrial data to remove their high-frequency constituents (e.g., Gruber 2004; Ihde et al. 2010).
- 2. Spectral enhancement of the GGM-based functionals in the high-and very-high-frequency range to become more comparable with the full-spectrum terrestrial observations (Pavlis et al. 2007; Gruber 2009; Hirt 2010; Hirt et al. 2010a; Gruber et al. 2011). For the spectral enhancement, the high-frequency bands of EGM2008 (Pavlis et al. 2008) and residual terrain model (RTM) data (e.g., Forsberg 1984; Hirt 2010) can be utilised.

The application of the spectral enhancement method (SEM) for GOCE GGM validation with scattered ground-truth data is one subject of this paper. Low-pass filtering using methods such as spectral filters, Gaussian filters or least-squares collocation (LSC) may perform well when the terrestrial data is available in some suitable arrangement, for instance, profiles (e.g., Ihde et al. 2010), grids, or at densely scattered locations. However, low-pass filtering is a delicate task in the case of very irregularly or scarcely distributed stations, if possible at all. In contrast, the SEM (Sect. 2) can be used for GGM validation *independent* of the spatial distribution of the terrestrial data. This is an important advantage when using irregularly distributed ground truth observations as is often the case with GPS/levelling, terrestrial gravimetry or astrogeodetic vertical deflections.

The present paper uses two complementary assessment techniques to evaluate the new GOCE-based GGMs. The first uses the SEM along with scattered terrestrial gravity over Switzerland and Australia and vertical deflection data over Europe and Australia (Sects. 2 to 4). The second compares global grids of EGM2008 and GOCE quasigeoid heights. Together with EGM2008 error estimates, world-wide insight into GOCE's performance is gained (Sect. 5).

The first assessment technique utilises observed ground-truth gravity and vertical deflections. Gravity observations (first radial derivatives of the disturbing potential) and vertical deflections (first horizontal derivatives of the disturbing potential) possess significant spectral power in the medium and high frequencies (Schwarz 1985, Jekeli 1999, Torge 2001, p. 274), so are useful complements to GGM evaluation using GPS/levelling points (Gruber 2009; Ihde et al. 2010; Gruber et al. 2011). The two vertical deflection data sets are particularly valuable, as they are fully independent of any GGM used in this paper. The deflections can be used to investigate in which spectral windows the GOCE-based models provide new information on the Earth's gravity field in our test areas.

In addition to the four GOCE-based GGMs, we also include two GRACE-only GGMs and EGM2008 (Sect. 3), so as to identify improvements of the current knowledge of the Earth's gravity field coming from the new GOCE-based models. Analysis of the RMS (root mean square) residuals between the terrestrial data sets and spectrally enhanced GGM functionals (Sect. 4) indicates the relative performance of all GGMs involved, and identifies the actual spectral resolution of the models, which is the degree of signal loss (see Gruber et al. 2011); also see Sect. 4.

The second assessment technique is based on global comparisons between bandlimited EGM2008 and GOCE GGM quasigeoid heights (Sect. 5) and supplements the groundtruth comparisons of Sect. 4. When using identical spectral bands, this evaluation method does not require any modelling of omitted signals, thus avoiding the problem of different spectral content. Along with background knowledge of the global structure of EGM2008 commission errors (i.e., uncertainties in the GGM), the EGM2008 vs. GOCE comparisons indicate regions and continents where the GOCE models are likely to deliver improved gravity field data (Sect. 5).

Prior to describing the SEM and GOCE GGMs, it is useful to define the terms *long*, *medium*, *short and very-short* wavelength as used in this study to address the different spectral constituents of Earth's gravity field (see also Torge 2001, p. 74). The term *long* denotes here the spectral band of spherical harmonic degree 2...100 (wavelengths longer than ~400 km). *Medium* wavelengths range from ~400 km to ~200 km (spherical harmonic degrees of ~100 to about ~200, or a resolution down to ~100 km, which is the target resolution of GOCE). *Short* wavelengths cover the spherical harmonic degrees of ~200 to 2190 (wavelengths between ~200 km and ~18 km, or a spatial resolution between 100 km and 9 km). Finally, *very-short* denotes those features beyond the resolution of EGM2008, i.e., wavelengths less than ~18 km. It is acknowledged that other definitions exist in the geodetic literature, however, the terminology here is tailored to the GOCE validation with SEM.

2 The Spectral Enhancement Method

2.1 Principle

The basic idea of the SEM is to source relevant auxiliary data to bridge the spectral gap of the GOCE-based GGM functionals as far as possible, so that the terrestrial observations are better approximated and the omission error does not obscure the comparisons. In the SEM, the spectral gap between the GOCE-GGMs and the terrestrial observations is bridged partially by a combination of: (i) the high-degree spectral bands of EGM2008, and (ii) omission error estimates sourced from RTM data, providing information on the very-short wavelength gravity field constituents. Of course, there remains an omission error due to the limited RTM resolution.

The SEM strategy has been applied in Gruber et al. (2011) for the validation of GOCE-GGMs at GPS/levelling points, while Hirt et al. (2010a) used RTM omission error estimates (Hirt 2010) together with observed astrogeodetic vertical deflections to assess EGM2008. Pavlis et al. (2007) used SEM in the development of EGM2008 over areas where proprietary gravity anomaly data were only available, and the use of these data was only permitted to a spatial resolution of 15'x15'. Pavlis et al. (2007) used the RTM estimates to augment the spectral content of these anomalies from degree 721 to degree 2159 (in ellipsoidal harmonics).

A									
Source:	GGM	under evaluation	EGM2008		RTM				
Spectral band:	2	n ₁ +1	n ₂	n ₂ +1	~216,000				
В									
Source:	EGM2008	GGM under evaluation	der evaluation EGM2008		RTM				
Spectral band:	d: 2 n ₀ n ₀ +1		n ₁ n ₁ +1		n ₂ +1 ~216,00				
C									
Source:		EGM2008				RTM			
Spectral band:	2			n ₂	n ₂ +1	~216,000			

Fig. 1 Spectral enhancement principle for GGM evaluation. A (special case): GGM under evaluation substitutes EGM2008 spectral band $2...n_1$ (spectral enhancement from n_1+1 ...216,000). B (general case): GGM under evaluation substitutes EGM2008 spectral window $n_0+1...n_1$ (additional spectral enhancement in band $2...n_0$). C: EGM2008/RTM only solution (cf. Hirt et al. 2010a).

The SEM principle is exemplified in Fig. 1. The GOCE GGM under evaluation is expanded to spherical harmonic degree n_1 (e.g., 200), with EGM2008 used to recover the spectral band n_1+1 to n_2 (e.g., 2,190). Beyond degree n_2 , we use RTM omission error estimates to 'complete' the spectral content of the model as far as possible (e.g., ~216,000) with current data sets (Fig 1A).

The strategy to investigate a specific spectral band $(n_0+1 \text{ to } n_1)$ of the GOCE GGM is visualised in Fig 1B. Here, EGM2008 is not only used from n_1+1 to n_2 , but also in spectral band 2 to n_0 , with the GOCE GGM evaluated from n_0+1 to n_1 (e.g., 150 to 200). In other words, the GOCE-GGM substitutes EGM2008 within spectral window n_0+1 to n_1 . Varying the combination degrees n_0 and n_1 , as well as the GOCE-GGM, the RMS of the differences 'terrestrial observation minus GGM' can be used to indicate the performance of the different GGMs in any spectral window chosen.

Comparing against EGM2008/RTM 'benchmark' solutions (Fig. 1C) is another strategy to identify in which spectral bands the GOCE-based models deliver improved information on the Earth's static gravity field. However, this requires ground-truth data that are independent of all GGMs involved, which only holds for our vertical deflections (e.g., the gravity anomalies here were also used by the EGM2008 Development Team). The present study applies the SEM variants shown in Figs. 1A and 1C (also see Sect. 4).

2.2 RTM omission error estimates

To compute RTM omission error estimates, we construct the RTM (Forsberg 1984) as difference between the 3 arc second SRTM (Shuttle Radar Topography Mission) elevation model (ver4.1 by Jarvis et al. 2008) and the spherical harmonic expansion of the DTM2006.0 elevation data base (Pavlis et al. 2007), expanded to degree 2,160. Subtraction of DTM2006.0 spherical harmonic elevations from the SRTM elevations removes a large part of the 'spectral information' already implied by EGM2008 (expanded to degree 2,190), see Hirt

(2010) and Hirt et al. (2010a,b) for a fuller description. The parallel use of EGM2008 to degree $n_2=2,190$ and DTM2006.0 to degree 2,160 is recommended by EGM Development Team (Pavlis et al. 2008), and allows modelling of the EGM2008 omission error to a large extent in medium-elevated and rugged terrain (Hirt et al. 2010a, Hirt et al. 2010b).

The SRTM–DTM2006.0 residual topography possesses spectral energy beyond the resolution of EGM2008 (in spectral band $n_2+1 = 2,191$ to ~216,000, which is equivalent to the SRTM resolution of 3 arc seconds). The RTM elevation grid is then used for forward-modelling of the gravity field functionals (here vertical deflections and gravity disturbances), as implied by the residual topography [see Forsberg (1984); Nagy et al. (2000) and Hirt et al. (2010a) for the equations]. Our forward modelling is based on a uniform mass-density (2670 kg/m³) for the residual topography, which is why any density anomalies at scales beyond the EGM2008 resolution are not modelled by the RTM approach (akin to a commission error of the RTM omission error estimates).

2.3 Computation scheme

The SEM can be applied for GGM evaluation with terrestrial gravity disturbances and vertical deflections using the following computation scheme. Naturally, it can also be applied to quasi/geoid heights from GPS/levelling (Gruber 2009; Hirt et al. 2010b; Gruber et al. 2011) or other 'ground-truth' data sources (e.g., torsion balance measurements or airborne gravimetry and airborne gravity gradiometry). The equations for the standard operations (e.g., spherical harmonic synthesis, forward modelling of terrain effects) are sufficiently and frequently detailed in the literature (e.g., Torge 2001; Forsberg 1984; Nagy et al. 2000; Hirt 2010), so are not duplicated here. For gravity observations, the SEM scheme reads:

- 1. Computation of gravity disturbances by subtracting the normal gravity at the ellipsoidal height of the station (Torge 2001 p. 117) from the observed gravity.
- 2. Synthesis of GGM gravity disturbances at the 3D coordinates of our gravity stations (geodetic latitude, longitude and ellipsoidal height), e.g., Torge (2001, p. 271). This is done by using the scattered point option of the harmonic_synth.f software (Holmes and Pavlis 2008). The synthesis computations are carried out separately for each GGM involved, and for a range of spectral windows from 2 to n₁ ∈ [2,3,..., n_max_model]. EGM2008 is included here so as to allow computation of 'EGM2008-only' solutions.
- 3. Computation of EGM2008 gravity disturbances by repeating step 2 with spectral windows n_1+1 to 2,190.
- 4. Addition of the gravity disturbances from step 2 (spectral band 2 to n_1) and the complementary EGM2008 disturbances (band n_1+1 to 2,190) from step 3. This gives a merger of gravity disturbances with spectral power in bands 2 to 2,190.
- 5. *Optional*: Construction of the GOCE GGM contributions in arbitrary spectral bands n_0+1 to n_1 (e.g., 120 to 240). For instance, this is achieved by adding a 'correction' EGM2008 (band 2 to n_0) minus GOCE GGM (band 2 to n_0) to the solutions of step 4.
- 6. Computation of the RTM gravity disturbances (spectral band 2,191 and well beyond). For each of the terrestrial stations, the required elevation grids (3 arc second SRTM and DTM2006.0) are extracted from our elevation data base (with the terrestrial station as centre and ~50 km grid radius). The RTM elevations are used for mass-

density forward modelling using the Curtin RTM software (Hirt et al. 2010a) based on the TC program (Forsberg 1984). For stations located 'within the residual topography', the [RTM] harmonic correction is applied, see also Forsberg and Tscherning (1981, p 7846) and Forsberg (1984, p. 40).

- 7. The sum of the RTM gravity disturbances (band 2191 and beyond) and synthesised gravity disturbances (band 2 to 2,190) from steps 4 and 5 gives our spectrally enhanced model gravity disturbances. They possess almost full spectral power and can be compared with the terrestrial observations.
- 8. RMS values are computed from the gravity disturbance differences 'observation minus model' (these are the results from step 1 minus results from step 7) and analysed as a function of (i) the GOCE GGM under evaluation and (ii) spectral band 2 to n_1 (or n_0+1 to n_1) used.

For vertical deflections, the scheme is very similar. However, step 1 is not necessary and the harmonic correction (Forsberg 1984, p.40) of step 6, does not need to be applied. Additionally, two small corrections are applied to the GGM vertical deflections, converting them from Molodensky's definition in spherical approximation to Helmert vertical deflections in ellipsoidal approximation [see Jekeli (1999) and Hirt et al. (2010a) for the equations and further explanations].

3 Data sets

3.1 GGMs

This study evaluates the first four GOCE-based GGMs which were made available in 2010 via ICGEM. In addition, we include two GRACE-only GGMs and EGM2008 in our analyses (Table 1) so as to identify any improvements of the new GOCE-models over GGMs from the pre-GOCE-era. Common to the four GOCE-based models is that the first two months of GOCE observations (collected between November 2009 and January 2010) were used as input data. However, they differ in the processing strategies applied, the spectral resolution (maximum degree of expansion) and, importantly, by the type and amount of a-priori information ('background models') used in the model construction.

An overview of the computation methods used for GOCE-based gravity field recovery is given in Rummel et al. (2004). Detailed descriptions of the underlying computation methods will become available in the near future (e.g., Pail et al. 2011). Table 1 lists all GGMs used in this study, including the ESA and ICGEM product names of the GOCE models where applicable (note that ESA and ICGEM use different names). For the sake of legibility, we abbreviate the ESA product names as shown in Table 1.

Table 1 Overview of the GGM	As used in this study. n_max	_model is the maximum spherical
harmonic degree of the model,	G=terrestrial gravity, $A = sa$	tellite altimetry.

	•		2
GGM	Model	Data	Method and
	resolution	sources	References
	n_max_model		
GOCO01S	224	GOCE,	Based on the time-wise
		GRACE	approach and combined
			with GRACE

			Pail et al. (2010a)
GOCE-DIR	240	GOCE,	Based on the direct
(ESA product name:		EIGEN-	approach
EGM_GOC_2_20091101T000000_20100		51C	Bruinsma et al. (2010),
110T235959_0002;		(GRACE,	Pail et al. (2011)
ICGEM product name:		CHAMP,G,	
GO_CONS_GCF_2_DIR)		A)	
GOCE-TIM	224	GOCE	Based on the time-wise
(ESA product name:			approach
EGM_GOC_2_20091101T000000_20100			Pail et al. (2010b),
111T000000_0002;			Pail et al. (2011)
ICGEM product name:			
GO_CONS_GCF_2_TIM)			
GOCE-SPW	210	GOCE,	Based on the space-wise
(ESA product name:		EGM2008	approach
EGM_GOC_2_20091030T005757_20100			Miggliaccio et al.
111T073815_0002;			(2010),
ICGEM product name:			Pail et al. (2011)
GO_CONS_GCF_2_SPW)			
GGM03s	180	GRACE	Based on GRACE inter-
			satellite ranging and
			SST
			Tapley et al. (2007)
ITG-GRACE2010s	180	GRACE	Based on GRACE inter-
			satellite ranging and
			SST
			Mayer-Gürr et al.
			(2010)
EGM2008	2,190	GRACE,	Based on ITG-
		G,A	GRACE03S GGM,
			altimetric and terrestrial
			data
			Pavlis et al. (2008)

GOCO01S (Pail et al. 2010a; GOCO Consortium 2010) is a combination of two months GOCE gradiometry data with seven years of GRACE observations (model ITG-GRACE2010s, Mayer-Gürr et al. 2010). GOCO01S makes use of the complementary character of the two missions: GRACE delivers superior information on the low- and medium- degree gravity field constituents, coarsely up to degree ~100, while GOCE is particularly sensitive to the medium wavelengths, to degree ~200, with a comparable spectral performance found in spectral band from 130 to 150 (Pail et al. 2010a). The GOCO01S model is constrained by Kaula's (1966) rule of thumb (power-decay of gravity field signals with degree) in their upper bands.

The GOCE-DIR model (Bruinsma et al. 2010; Pail et al. 2011) uses the direct computation approach and incorporates the EIGEN-51C combined GGM (Bruinsma et al. 2010) as a background model for the regularisation of the GOCE model. EIGEN-51C is based on six years of CHAMP (Challenging Mini-satellite Payload; e.g., Reigber et al. 2002)

and GRACE data and incorporates the DNSC08GRA global gravity anomaly data set (Andersen et al. 2010) that uses EGM2008 gravity over land. The regularisation has been done by applying the so-called spherical cap regularisation (Metzler and Pail 2005) with EIGEN-51C as pre-information over the polar caps. Due to this regularization approach, the GOCE-DIR model is 'reinforced' by EIGEN-51C. In the long wavelengths, EIGEN-51C utilises information from GRACE and CHAMP, while the medium- to short-wavelength information is sourced from DNSC08GRA (and hence EGM2008 terrestrial gravity over land areas). This is important to know for a correct interpretation of our evaluation results in Sect. 4. With the (restricted) information available to us, it is not possible to quantify to what extent GOCE observations and EIGEN-51C data contribute to the GOCE-DIR GGM in the various spectral bands.

GOCE-TIM (Pail et al. 2010b; Pail et al. 2011) is the only GGM that relies exclusively on GOCE observations (gradiometry and SST). Kaula (1966) regularisation is used to constrain the model's short wavelengths. Evaluation of GOCE-TIM (Sect. 4) provides pure feedback on the GOCE-only performance (as reached after two months) without influence of any other data sets.

The space-wise model GOCE-SPW (Migliaccio et al. 2010; Pail et al. 2011) not only uses the first two months of GOCE observations, but also EGM2008 (Pavlis et al. 2008) as a-priori information to strengthen the model in the long wavelengths, which is effectively the same as using ITG-GRACE03S (Mayer-Gürr 2007). In the medium wavelengths, however, GOCE-SPW can be considered a GOCE-only model.

The two GRACE-based satellite-only GGMs included in our analysis are GGM03S (Tapley et al. 2007) and ITG-GRACE-2010S (Mayer-Gürr et al. 2010). These two are based on ~4 years and ~7 years of GRACE data, respectively, and are used as a 'benchmark' of pre-GOCE satellite gravimetry.

Finally, EGM2008 (Pavlis et al. 2008) serves as a source for omission error estimates for the SEM technique (Sect. 4). Importantly, EGM2008 is also used as a 'standard' over well-surveyed areas against which the performance of the new GOCE-solutions are tested (cf. Sects. 4 and 5). EGM2008 is a combined model based on ITG-GRACE03S (a predecessor of ITG-GRACE-2010S, cf. Mayer-Gürr 2007) and 5 arc minute surface gravity data from a variety of sources (Pavlis et al. 2008). EGM2008 delivers highly-accurate information of the spectral band 2 to 2190 and is widely considered as the current state-of-the-art of high-resolution global gravity field modelling (e.g., Newton's Bulletin 2009).

3.2 Terrestrial comparison data

Although the SEM can be applied fairly independently of the arrangement of the terrestrial (ground-truth) stations, some basic conditions should be met to facilitate a 'useful' GGM evaluation. First, it is preferable to use terrestrial data sets consisting of several 100 (or many more) stations, thus making the results of the RMS analysis reasonably independent of individual errors of the terrestrial observations and the SEM-modelled omission error corrections, as well as being statistically significant. Second, given the spatial resolution of the GOCE GGMs is ~100 km, it is desirable for the data sets to cover a larger area (country or even continental scale), to make the analysis spectrally representative. Finally, the comparison data should be independent of the GOCE-based GGMs. It is beneficial if the

comparison data is also independent of the data sets sourced by the SEM technique to model the omission errors (here the EGM2008 GGM and RTM). All data sets described next are accompanied by accurate ellipsoidal heights (directly observed with GPS or constructed from spirit-levelling plus regional quasigeoid heights), as required for the computation of gravity disturbances from observed gravity and harmonic synthesis (cf. Sect. 2.3).

3.2.1 Terrestrial gravity

The present study utilises the national gravity data set of Switzerland (Marti 2004) and a selected subset of gravity stations from the Australian national gravity data base (Murray 1997). The Swiss data set consists of 31,598 stations (release of 2003) and was kindly provided by swisstopo (national surveying agency of Switzerland). The gravity station coverage (Fig. 2) extends over the whole of Switzerland including all types of topography (from the low-elevated Northern parts of Switzerland to the rugged Swiss Alps in the South). Subsets also cover adjacent regions of neighbouring countries. The accuracy of the Swiss gravity is at the 0.1 mgal level or better (U. Marti, pers. comm. 2010). With an area of roughly 200 km x 350 km covered, this data set provides a sufficiently large area to provide feedback on the GOCE models.



Fig. 2 Location of 31,598 gravity stations over Switzerland



Fig. 3 Location of 28,551 recent gravity stations over Australia

As second set, we selected a subset of 28,551 gravity stations over Australia (Fig. 3), acquired during the last decade, from the Geoscience Australia (GA) land gravity data base (http://www.gadds.ga.gov.au), which consists of roughly ~1.5 million observations (end of 2010). Most of the 28,551 stations were collected in areas of resource prospectivity in dense grids (visible as 'patches' in Fig. 3) with a spatial resolution of 2-4 km (Featherstone et al. 2011). As opposed to many poorly documented 'historical' gravity sets held in the GA land gravity data base, the selected data is coordinated with GPS and at the 0.1 mGal accuracy level, so it can be considered to be sufficiently reliable. There exist another ~250,000 GPS-coordinated gravity stations in the GA data base, but these were not included because they do not cover other areas than those shown in Fig. 3. Though mostly arranged in patches (Fig 3), the selected data extends over an area of coarsely ~2000 km x ~3000 km. Covering a large part of Australia as a significant land mass in the Southern Hemisphere, we consider this data set to play an important role in the evaluation of the new GOCE data sets. The inspection and inclusion of further extracts of the GA land gravity data base is intended as a future task.

3.2.2 Vertical deflections

Sets of astrogeodetic vertical deflections are available over Europe and Australia for this study. The European vertical deflection set (Fig. 4) originates exclusively from zenith camera observations (e.g., Bürki et al. 2007, Hirt et al. 2010c). It consists of 1,056 astrogeodetic stations observed with analogue zenith cameras (433 stations) and digital zenith cameras (623 stations) over the last three decades. The most important part of the European vertical deflections are ~540 stations covering the whole of Switzerland (provided by swisstopo and the Swiss Geodetic Commission). Other subsets of digital zenith camera measurements cover parts of Portugal and Greece (provided by B. Bürki, ETH Zürich), the Bavarian Alps (Hirt and Flury 2008), parts of Northern Germany and the Netherlands. We refer to Hirt et al. (2010a) for full details on this data set. The accuracy of the digital zenith camera vertical

deflections is 0.1'' (Hirt and Seeber 2008) while the data sets originating from analogue observations are accurate to 0.3''-0.5'' (Hirt et al. 2010a, Hirt et al. 2010c).



Fig. 4 Location of 1,056 vertical deflections over Europe



Fig. 5 Location of 1,064 vertical deflections over Australia

The 1,056 European vertical deflections is the largest astrogeodetic data set available from zenith camera observations so far. Though the station coverage is not optimum for the purpose of GGM validation (e.g., parts of the European continent remain uncovered, subsets of vertical deflections are arranged locally along densely-spaced profiles), the set was already used for an assessment of EGM2008 (Hirt et al. 2010a). This study demonstrated the suitability of the European deflection data set for GGM evaluation. Another set of ~300 vertical deflections (not used here) was recently observed for GOCE validation with a digital zenith camera in Germany. These stations, located along two ~500 km profiles, were low-

pass filtered with a Gaussian filter for comparisons with recent GGMs (Voigt et al. 2009; Ihde et al. 2010).

Secondly, a set of 1,080 astrogeodetic vertical deflections is used over Australia, provided by GA and Landgate (the Western Australian State geodetic agency). This set covers the whole of the Australian continent (Fig. 5). The Australian vertical deflection data set originates from 'historic' observations, carried out around four decades ago (Featherstone and Morgan 2007; Featherstone and Lichti 2009). The accuracy of the Australian vertical deflections has been "cautiously estimated to be 1 arc second" (Featherstone and Lichti 2009). Comparison of the Australian vertical deflections with EGM2008/RTM-based vertical deflections revealed a total of 16 clear outliers (discrepancies larger than 6 arc seconds), which were removed prior to its use for GOCE model evaluation. Though not as accurate as the European set, our experiments will demonstrate that the Australian deflections are capable of providing some valuable feedback on the new GOCE models (Sect 4.2).

4. Evaluation results

From our terrestrial comparison data, only the vertical deflections are independent of all GGMs used here and therefore suited to identify – in comparison to a pure EGM2008/RTM solution (Fig. 1C) – if and in which spectral bands the first GOCE models improve the gravity field in our test areas. As opposed to this, large parts of the Swiss and Australian gravity data were used in the construction of EGM2008. As an immediate consequence, the EGM2008/RTM solution is 'biased' towards the gravity data and not suited to serve as a 'fair benchmark' for evaluation of the new GOCE-based data sets in our SEM scheme. Also, the GOCE-DIR model is not independent from the terrestrial gravity data sets because of the use of EIGEN-51C in the model development (cf. Sect. 3). In case of existing inter-dependencies, however, the relative comparison between the RMS curves is used to determine the actual spectral content of the new GOCE models.

The subsequent analyses are based on the SEM variants shown in Figs. 1A and 1C. The statistics of the differences between the 'observed' gravity disturbances (Sect. 2.3, step 1) and the model values from EGM2008 (band 2 to 2190) and EGM2008 (band 2 to 2190) augmented by RTM (Fig. 1C) are reported in Table 2. The RTM augmentation reduces the RMS errors to ~4.5 mGal for the Swiss data (~89 % reduction) and to ~2.7 mGal for the Australian data (~21% reduction). For the sake of completeness, we also list the descriptive statistics of the differences 'observed deflection minus model deflection' (Table 3). By virtue of RTM augmentation, the RMS errors are reduced to the 1 arc second level and below (~65% reduction over Europe, ~10% over Australia). The *variations* of the RMS differences due to using GOCE-based GGMs instead of EGM2008 in the spectral window 2...n₁ are analysed in Sects. 4.1 and 4.2.

The relatively low RMS values of the differences 'observation minus EGM2008/RTM' (Tables 2 and 3) indicates a reasonably good quality of the terrestrial data and the SEM-based omission error modelling. It should be noted that the RMS values reflect any errors of the data sets involved, so the EGM2008 commission errors in the range n_1 to 2190, commission errors in the RTM corrections (e.g., due to unmodelled mass-density anomalies) and observational errors of the terrestrial data sets. Because of the presence of

these error sources, the use of the new GOCE-GGMs (in place of EGM2008) is *not* expected to lead to a *significant decrease* of the RMS values listed in Tables 2 and 3.

Table 2 Descriptive statistics of the gravity disturbance differences observation minus model, units in mGal.

Area	#points	Differences	Min	Max	Mean	RMS
Switzerland	31,598	Obs – EGM2008	-224.70	95.09	-18.18	±39.50
	31,598	Obs-EGM2008/RTM	-98.92	32.31	-1.57	±4.48
Australia	28,551	Obs – EGM2008	-34.70	34.43	-1.04	±3.40
	28,551	Obs - EGM2008/RTM	22.12	24.93	-0.92	±2.68

Table 3 Descriptive statistics of the vertical deflection differences observation – model, units in arc seconds. xi = North-South vertical deflection component, eta = East-West vertical deflection component. 16 outliers were removed from the Australian vertical deflection set.

Area	#points	Differences	Comp.	Min	Max	Mean	RMS
Europe	1,056	Obs-EGM2008	xi	-15.00	15.54	-0.03	±3.02
	1,056	Obs-EGM2008/RTM	xi	-4.74	5.37	-0.04	±1.04
	1,056	Obs – EGM2008	eta	-11.67	15.62	0.32	±2.97
	1,056	Obs-EGM2008/RTM	eta	-4.33	4.90	0.15	±1.05
Australia	1,064	Obs-EGM2008	xi	-4.89	3.33	-0.15	±0.78
	1,064	Obs-EGM2008/RTM	xi	-4.93	2.62	-0.15	±0.67
	1,064	Obs-EGM2008	eta	-4.98	3.78	-0.09	±1.03
	1,064	Obs-EGM2008/RTM	eta	-5.24	4.14	-0.10	±0.96

To identify the spherical harmonic degree where the new GOCE-GGMs lose spectral power (denoted here as signal loss), we introduce an arbitrary threshold

$$th_{signal_loss} = 1.1 \cdot RMS(EGM2008only), \qquad (1)$$

i.e., a threshold value that is 10% larger than the RMS value of the differences 'observation - EGM2008-only solution'. Visually, signal loss manifests as steep slopes of the RMS curves shown in Figs. 6 to 9. The RMS values are computed as a function of n_1 (ranging from 2 to n_max_model) and compared against th_{signal_loss} for any n_1 . If the threshold is exceeded more than once, we consider the smallest value n_1 to be the spherical harmonic degree of signal loss under our criterion. Next, the RMS curves are presented and discussed for our four test areas. The spherical harmonic degrees of signal loss, as obtained from the numerical criterion in Eq. (1) are reported in Table 4.

4.1 Results using terrestrial gravity

Figure 6 visualises the SEM evaluation results using the Swiss gravity data. The RMS values of the gravity disturbance differences 'observation minus model' are shown as a function of the spherical harmonic band 2 to n_1 and the seven GGMs evaluated (Table 1). All the GGMs

show comparable performance up to degree ~140. GGM03S (green-solid line) and ITG-GRACE2010S (green-dotted line) experience signal loss at degree 142 and 170, respectively. The signal loss is seen by the rapidly increasing RMS values in Fig 6.



Fig. 6 RMS of the gravity disturbance differences over Switzerland as a function of the GGM used in spectral window $2...n_1$, Units in mGal.

In contrast, steep slopes are visible in Fig. 6 for the GOCE models GOCO01S (redsolid line), GOCE-TIM (blue-solid line) and GOCE-SPW (blue-dotted line) near degree ~185 (Table 4). This shows that the GOCE-based models improve knowledge in the spectral band ~170 to ~185 with respect to GRACE. For GOCE-DIR (red-dotted line), a similarly sudden signal loss cannot be observed over the whole spectral band. However, this is not necessarily attributable to GOCE, because EIGEN-51C terrestrial gravity anomalies were used to reinforce the GOCE-DIR model in the high-degrees. The somewhat worse performance of GOCE-DIR in comparison to EGM2008 in the short wavelengths (to degree 240) could be related to the increasing noise of GOCE observations, but this cannot be proven with the data available to us. It should be noted that oscillations of the RMS values are visible all over the low and medium and short degrees with varying amplitudes (cf. Sect. 4.3).



Fig. 7 RMS of the gravity disturbance differences over Australia as a function of the GGM used in spectral window $2...n_1$, Units in mGal.

The SEM evaluation results are shown in Fig. 7 for the Australian gravity data set. In comparison to the Swiss data, the models experience signal loss roughly 15 spherical harmonic degrees 'earlier', e.g. ITG-GRACE2010S near degree 154 instead of 170 and the GOCE-based GGMs GOCO01S, GOCE-TIM and GOCE-SPW at degree ~170 instead of ~185 (see Table 4). Despite this somewhat enigmatic shift, which may be due to the uncertain GPS reference frames used to coordinate the gravity surveys (Featherstone et al. 2011), it can be seen that the three GOCE-based models provide improved information on the band ~155 to ~170 in comparison to the GRACE-only GGMs. Again, GOCE-DIR experiences only a slowly progressing signal loss in its upper band ~170 to 240, which is explained by the terrestrial data included in its construction.

Both over Switzerland and Australia, EGM2008 exhibits for most of the spectral degrees the 'best' RMS performance. However, this should not be over-interpreted, recalling the inter-dependencies between EGM2008 and the terrestrial gravity data. The observed similar behaviour of those GGMs using GOCE-only data in their upper bands (GOC001S, GOCE-TIM and GOCE-SPW) suggest a very comparable performance of the underlying GOCE computation methods 'time-wise-approach' and 'space-wise-approach'. For all of the GGMs involved, a fairly similar performance is observed for the long wavelengths (degrees 2 to ~100).

4.2 Results using vertical deflections

Figure 8 shows the RMS values of the vertical deflection differences 'model minus observation' for our European data set as a function of the GGM used in spherical harmonic band 2 to n_1 . Again, the SEM technique (Sect. 2 and Fig. 1A) was applied to reduce the discrepancies between the data sets as much as possible. From Fig 8A (North-South vertical deflection component xi), the GRACE-only models GGM03S and ITG-GRACE2010S experience signal loss between degrees ~160 to ~170. GOCO01S, GOCE-TIM and GOCE-

SPW exhibit signal loss near degree ~200, which suggests superior GOCE performance in band ~170 to almost ~190 to ~200. The GOCE-DIR model does not experience any signal loss, as is seen in Fig. 8A. The RMS values of the GOCE models in spectral window ~150 to ~190 are mostly smaller than those of EGM2008 (red line). Recalling that our vertical deflections are independent of all GGMs used, this indicates an improved gravity field knowledge due to the new GOCE models even in a densely and well-surveyed area such as Europe.

Analysis of the vertical deflection residuals in the East-West direction (eta component) does not give a similarly clear picture (Fig. 8). The steep gradients for the GRACE-based models are further apart (degrees ~145 for GGM03S and ~170-175 for the ITG model), with the latter not clearly separated from slopes of the GOCE models (around ~185). Thus, the eta-component shows only a slight improvement of gravity field knowledge in the according bands. In the spectral band 100-150, the RMS values of the GOCE-based models are somewhat smaller than those of the GRACE-only models and EGM2008.



Fig. 8 RMS of the vertical deflection differences over Europe as a function of the GGM used in spectral window $2...n_1$. Top: Vertical deflection component xi, Bottom: component eta. Units in arc seconds.



Fig. 9 RMS of the vertical deflection differences over Australia as a function of the GGM used in spectral window $2...n_1$. Top: Vertical deflection component xi, Bottom: component eta. Units in arc seconds.

Focussing on the [less accurate] Australian vertical deflections (Fig. 9), the GOCEbased models GOCO01S, GOCE-TIM and GOCE-SPW experience signal loss near spherical harmonic degree ~180-190 (both for xi and eta). Opposed to this, the GRACE-only model ITG-GRACE2010S shows steep slopes near degree ~165. The EIGEN-51C augmented GOCE-DIR model shows a similar behaviour and, additionally, only a slight deterioration in their RMS values in the spectral band 180 to 240. This again demonstrates improvements conferred by GOCE in the spectral window ~165 to ~185. It is remarkable that the ~40+ year old Australian vertical deflections 'see' the advancements in gravity field knowledge coming from only two months of GOCE observations. As such, old and seemingly inaccurate 'ground-truth' data (but if in sufficient quantity) can still be useful for GGM validation.

4.3 Synthesis of results

From the range of observations made in the GGM comparisons (Table 4), we can surmise the following: our gravity observations and vertical deflection data sets are capable of sensing improvements delivered by the new GOCE gravity field models, even if they are based only on two months of in-orbit data. Over our ground-truth areas, the first four GOCE models advance knowledge of the Earth's static gravity field. Depending on the gravity field functional and test area, the degrees of the improved spectral band vary, but it can be broadly stated that the improved spectral band begins near degree \sim 160-165 and ends near degree \sim 180-190 (Table 4).

Table 4 Actual model resolution (as indicated by the ground-truth comparisons). The listed values are the spherical harmonic degrees where the models experience signal loss as computed with the numerical criterion in Eq. (1) n.o. = not observed.

GGM \ Dataset	Swiss Gravity	Australian Gravity	European deflections xi, eta	Australian deflections xi, eta
GOCO01s	186	172	198, 186	182, 188
GOCE-DIR	224	221	n.o.	n.o.
GOCE-TIM	184	168	209, 185	180, 188
GOCE-SPW	185	170	197, 186	181, 188
GGM03s	142	134	161, 144	153, 140
ITG-GRACE2010s	170	154	172, 174	166, 162

The variations in the model resolutions (Table 4) suggest the possibility of a hemisphere-dependent performance of GOCE (cf. European and Australian results). However, additional results from the second assessment technique using global EGM2008 quasigeoid grids do *not* indicate a worsened GOCE performance over the Southern Hemisphere continents (Sect. 5). Therefore, the observed variations in the resolution of the GOCE models (Table 4) are attributed to the test data sets (e.g., station distribution, area covered) and not to GOCE. This finding is also considered a justification to use *different* ground truth areas and functionals holistically, so as to obtain a better picture of the actual model signal content.

From any of our comparisons, comparable performance is observed for GOCO01S, GOCE-TIM and GOCE-SPW over the whole spectrum. Significant differences among these methods are not indicated by our comparisons, and they are unable to discriminate between them. Finally, the GOCE-DIR model outperforms the other three-GOCE-based models in the spectrum beyond ~190, where it loses only little spectral power. This is expected as it is augmented with terrestrial data, whereas Kaula's (1966) rule of thumb dictates that satellite-only GGMs cannot sense the high-frequency gravity field. The GOCE-DIR performance must be balanced against the fact that the EIGEN-51C model is used as reinforcement and, hence, the main data source in this short-wavelength window is assumed to be terrestrial gravity. As a consequence, there is no 'fair' comparison possible among the performance of

the direct computation approach and the time-wise or space-wise approach to GOCE gravity field recovery.

The terrestrial gravity data sets (Figs. 6 and 7) showed a slightly better or equal performance of GOCO01S with respect to GOCE-TIM and GOCE-SPW for spectral degrees ~100 and beyond. With the exception of vertical deflection component eta over Australia, this is also seen from the vertical deflection comparisons (Fig. 8 and 9). These observations indicate that the combined GOCE/GRACE gravity field modelling, as described by Pail et al. (2010a) yields more accurate models of Earth's gravity field than GOCE alone.

Our comparisons involving vertical deflections show for any of the new GOCE-based GGMs (and for ITG-GRACE2010S) RMS values which are mostly similar and often smaller than those of the EGM2008-only/RTM solutions (Fig. 8 to 10). Given that vertical deflections are truly independent, this indicates a tendency of the new GOCE models (and the ITG model) having more than comparable quality than EGM2008 in the tested spectral bands.

Finally, all RMS curves in Figs. 6 to 9 are subject to oscillations with a wavelength of coarsely ~10 spherical harmonic degrees. Striking examples are Fig. 6 (Swiss gravity), spherical harmonic degree 145, Fig 8A (European deflections, component xi), degrees 135 and 175 and many more occurrences are visible over the whole range of n_1 -combination degrees. Given that the RTM contributions are always the same, these oscillations provide some evidence that the spectral enhancement of GGMs (use of the GGM to degree n_1) with EGM2008 (use of EGM2008 from n_1+1 onwards) is *not* a rigorous technique (see also Featherstone 2007 p. 76). A discussion of the mechanisms that cause these oscillations is beyond the scope of the present study.

5. Comparisons using global EGM2008 quasigeoid heights

A characteristic of the 'ground-truth' comparisons presented in Sect. 4 is that the comparison data sets are restricted to the geographical regions of some limited extent. Although the above experiments justify the assumption that the GOCE models improved knowledge in the spectral band ~165-185 globally (and not only over our test areas), an alternative EGM2008-based assessment technique is used, serving to support the SEM-based ground-truth comparisons in Sect. 4.

5.1 Concept

The basic idea of the EGM2008-based assessment is to compare global grids of quasigeoid heights (other functionals or the disturbing potential itself could be used) from EGM2008 and GOCE in identical spectral bands, e.g., 2 to 180. The omitted signals (beyond the band's upper degree) should be similar, so largely cancel in the quasigeoid differences between EGM2008 and GOCE. Therefore, no omission error modelling is required when comparing identical bands.

Importantly, the comparisons between EGM2008 and GOCE GGMs under evaluation are facilitated by the global distribution of the EGM2008 commission errors (EGM Development Team 2009) derived from an error propagation technique described in Pavlis and Saleh (2004). Figure 10 shows the EGM2008 quasigeoid commission error as a function of the geographic location (data from EGM Development Team 2009). Though Pavlis et al. (2008) computed the commission errors for spectral band 2 to 2160, we assume that Fig. 10

provides a good enough indication on the global distribution of EGM2008 commission errors for low/medium degree spectral bands (e.g., 2 to 180). This is justified by the fact that most of the spectral energy of quasi/geoids are in the lower and medium degrees (Torge 2001, p. 274f; Schwarz 1985).

Figure 10 shows that EGM2008 quasigeoid heights are predicted to be accurately known (~5 cm) over most parts of Europe, North America and Australia, where dense and available terrestrial gravity data sets were used in EGM2008 (Pavlis et al. 2008). Over these three continents, and the oceans, we consider EGM2008-quasigeoid heights to have 'ground-truth' quality for the purpose of GOCE model evaluation. In contrast to this, commission errors of ~15 cm and larger (Fig. 10) reveal large parts of Asia, Africa, South America and Antarctica as regions with poor, restricted or no terrestrial gravity data coverage, and where the EGM Development Team had to use 'fill-in' anomalies (Pavlis et al. 2007; Pavlis et al. 2008). As such, over large parts these continents, EGM2008 quasigeoid heights do not represent decent ground truth.



Fig. 10 EGM2008 quasigeoid height commission error, unit is metres

The global map of EGM2008 commission errors (Fig. 10) represents valuable background information for GOCE model evaluation from EGM2008-GOCE residuals. This is because the commission error knowledge allows discrimination between EGM2008 and GOCE errors by establishing following simple three-case-scheme for interpretation of EGM2008 minus GOCE differences:

- Small differences, occurring over regions with low EGM2008 commission errors (e.g., Europe), indicate that two months of GOCE is already competitive with EGM2008.
- Large differences, occurring over regions with significant EGM2008 commission errors (e.g., Himalaya) *indicate* GOCE may improve over EGM2008. However, since there are no ground truth data in these regions, it is only possible to make an inference.
- High-frequency difference patterns (cantaloupe effect) occurring homogenously over the whole of the Earth and becoming more pronounced with larger truncation degree do not relate to EGM2008 quality. Instead, they are attributable to GOCE and reflect the degree of signal loss (cf. Fig. 6 to 9).

5.2 Results

We computed global grids of quasigeoid heights (as point values) at a resolution of 20 arc min (540 x 1080 grid points) from all GGMs listed in Table 1 and for spectral bands 2 to 5, 2 to 10 up to 2 to n_max_model (e.g., band 2 to 210 in case of GOCE-SPW). Using identical spectral bands, quasigeoid differences between EGM2008 and each GOCE model and EGM2008 and the two GRACE-based models were computed. Six selected results are presented and interpreted next.

Figure 11 (top) shows the quasigeoid differences between EGM2008 and the pure GOCE model GOCE-TIM in bands 2 to 180. The descriptive statistics of the differences are reported in Table 5 for various regions, e.g., Earth, land and oceans, continents and selected areas of Asia, Africa and South America, and, additionally, without the polar areas not measured by GOCE (i.e., $|\varphi| \ge 83.3^{\circ}$) due to GOCE's inclination of 96.7°.

- A good mutual agreement between both models is observed over most areas where EGM2008 quasigeoid heights possess 'ground-truth quality', i.e., Europe (RMS of 7 cm), Australia (8 cm), North America (8 cm) and most parts of the oceans (RMS of 10 cm, Table 5).
- Large discrepancies are found over the EGM2008 'problem regions', these are Antarctica (RMS of 23 cm), South America (RMS of 34 cm, and 40 cm over the Andes/Amazonas Basin region), Asia (19 cm, going up to RMS of 42 cm and maximum difference of ~2 to 3 m over the Himalaya region) and Africa (23 cm, going up to 30 cm over Central Africa), cf. Table 5 and Fig. 11 (top).

Both observations almost perfectly correlate with the EGM2008 commission error map in Fig. 10. *Together*, this provides strong evidence of GOCE adding to knowledge of the Earth's gravity field over wide parts of Antarctica, South America, Africa and Asia, while simultaneously confirming the gravity field knowledge over the EGM2008 'high-quality' areas. A rigorous *proof* of improvements over the EGM2008 'problem' areas, however, would require ground-truth data, which are not available to us.

Further to this, the difference patterns in Fig. 11 show a homogeneous GOCEperformance over both hemispheres, which is an important observation in view of the expected improvements of gravity field knowledge for all Southern-Hemisphere continents (see also Sect 4.3). Figure 11 (bottom) shows the EGM2008 and GOCE-TIM quasigeoid residuals in spectral band 2 to 210. In comparison to Fig. 11 (top), additional high-frequency patterns appear over all continents and oceans, increasing the RMS differences, e.g., from 7 to 15 cm (Europe), 8 to 15 cm (Australia) and 10 to 17 cm (oceans); see Table 5. This shows the attenuation of GOCE-captured gravity field signals in bands 181 to 210. As an aside, Fig. 11 exhibits large differences for the polar regions (which cannot be observed due to GOCE's orbit inclination) and discrepancies between GOCE-TIM and EGM2008 over the Southern Ocean (~140°E, ~50°S), near the South Magnetic Pole, which have been found to be related to the GOCE yy-gravity-gradients (Pail 2011, pers. comm.).

Figure 12 (top) shows the EGM2008 and seven-year GRACE-only ITG-GRACE2010S quasigeoid differences in bands 2 to 160 (see Table 6 for the descriptive statistics). The difference patterns bear strong resemblance with those from EGM2008 minus GOCE-TIM (Fig. 11 top), revealing the ITG-GRACE2010S improvements delivered for the EGM2008 'problem regions'. Figure 12 (bottom) shows the EGM2008 minus ITG-GRACE2010S differences in bands 2 to 180. From Table 6, the RMS increases from 8 cm to 24 cm over the oceans and from 7 cm to ~20 cm over Europe and North America. The noise-patterns visible in Fig. 12 (bottom) reflect GRACE signal loss in bands 161 to 180; also see Table 6.

The cross-comparison between Fig. 11 (GOCE-only results) and Fig. 12 (GRACE-only results) demonstrates improvements of GOCE with respect to the latest GRACE gravity field in bands ~160...180 over *the well-surveyed regions* (seen by the good agreement with EGM2008). Additionally, the cross-comparison provides strong evidence that the two months of GOCE observations have delivered new information in bands ~160...180 (and slightly beyond) over *the poorly-surveyed regions* of our planet.

Finally, the comparison between EGM2008 and GOCO01S (Fig. 13 and Table 7) shows in bands 2...180 and 2...210 slightly smaller residuals than the comparison between EGM2008 and GOCE-TIM (Fig 11). For bands 2...180, the RMS values drop, e.g., for Australia and North America from 8 cm to 7 cm, and for Greenland from 12 cm to 11 cm (cf. Tables 5 with 7). This is another indication that GOCE/GRACE combined gravity field solutions (as is the case with GOCO01S) allows more accurate gravity field modelling than GOCE alone.

Table 5 Descriptive statistics of the differences EGM2008 minus GOCE-TIM (2...180) and EGM2008 minus GOCE-TIM (2...210) for various regions of Earth. The regions are the whole Earth, land, oceans (with $|\phi| < 90^{\circ}$ and $|\phi| < 83.3^{\circ}$, corresponding to GOCE's inclination) continents/land masses and selected areas Andes/Amazon Basin ($-20^{\circ} \le \phi \le 10^{\circ}$, $-80^{\circ} \le \lambda \le -50$), Central Africa ($-20^{\circ} \le \phi \le 10^{\circ}$, $10^{\circ} \le \lambda \le 40^{\circ}$) and Himalayas ($20^{\circ} \le \phi \le 40^{\circ}$, $65^{\circ} \le \lambda \le 100^{\circ}$). NZ= New Zealand. The second column lists the percentage area covered by the region (whole of the Earth = 100%). Unit of descriptive statistics is metres.

		Min	Max	Mean	RMS	Min	Max	Mean	RMS
Earth	100	-3.59	6.82	0.05	0.58	-3.90	6.93	0.05	0.60
Land	29.5	-3.59	6.82	0.10	0.83	-3.90	6.93	0.10	0.86
Ocean	70.5	-2.85	3.98	0.03	0.38	-2.67	4.23	0.03	0.40
Earth ($ \phi < 83.3^{\circ}$)	99.3	-3.59	2.62	0.00	0.14	-3.90	3.35	0.00	0.21
Land (ϕ < 83.3°)	29.1	-3.59	2.62	0.00	0.20	-3.90	3.35	0.00	0.28
Ocean (φ < 83.3°)	70.2	-1.17	0.95	0.00	0.10	-1.55	1.27	0.00	0.17
Australia	1.5	-0.32	0.27	0.01	0.08	-0.58	0.61	0.00	0.15
NZ, Oceanian Islands (φ<-12°)	0.1	-0.44	0.34	-0.03	0.14	-0.91	0.68	-0.01	0.25
South America	3.6	-2.03	2.62	0.00	0.34	-2.55	3.35	0.00	0.40
North America	4.4	-0.65	0.61	0.00	0.08	-0.80	0.77	0.00	0.14
Greenland	0.4	-0.39	0.37	0.00	0.12	-0.65	0.63	0.01	0.16
Europe	1.4	-0.35	0.37	0.00	0.07	-0.65	0.63	0.00	0.15
Africa	5.9	-1.58	2.06	0.00	0.23	-1.89	2.35	0.00	0.28
Asia	9.5	-3.59	2.18	0.00	0.19	-3.90	2.58	0.00	0.23
Antarctica (φ < 83.3°)	2.4	-1.01	1.06	-0.01	0.23	-1.76	1.60	-0.01	0.39
Andes/Amazon Basin	2.2	-2.03	2.62	-0.01	0.40	-2.55	3.35	-0.01	0.46
Central Africa	2.2	-1.58	2.06	0.00	0.30	-1.89	2.35	0.00	0.34
Himalayas	1.5	-3.59	2.18	0.00	0.42	-3.90	2.58	0.00	0.46

Table 6 Descriptive statistics of the differences EGM2008 minus ITG-GRACE2010s (2..160) and EGM2008 minus ITG-GRACE2010s (2..180) for various regions of Earth, unit is metres.

Area	%	EGM2008-itg-grace2010s (2160)			EGM2008-itg-grace2010s (2180)			
		Min	Max	Mean	RMS	Min	Max Mean	RMS
Earth	100	-3.16	2.14	0.00	0.11	-3.33	2.57 0.00	0.26
Land	29.5	-3.16	2.14	0.00	0.15	-3.33	2.57 0.00	0.28
Ocean	70.5	-0.75	0.69	0.00	0.08	-1.28	1.32 0.00	0.24
Australia	1.5	-0.32	0.29	0.00	0.08	-1.01	0.94 0.00	0.29
NZ, Oceanian Islands (φ<-12°)	0.1	-0.22	0.21	-0.01	0.09	-0.59	0.50 -0.01	0.21
South America	3.6	-1.73	2.14	0.00	0.30	-2.20	2.57 0.00	0.41
North America	4.4	-0.49	0.37	0.00	0.07	-0.88	0.78 0.00	0.19
Greenland	0.4	-0.54	0.40	0.00	0.10	-0.90	0.72 0.00	0.20
Europe	1.4	-0.28	0.31	0.00	0.07	-0.72	0.65 0.00	0.21
Africa	5.9	-1.46	1.46	0.00	0.21	-1.83	2.25 0.00	0.31
Asia	9.5	-3.16	1.89	0.00	0.18	-3.33	2.40 0.00	0.28
Antarctica ($ \phi < 83.3^\circ$)	2.4	-0.54	0.57	0.00	0.10	-1.25	1.76 0.00	0.31
Andes/Amazon Basin	2.2	-1.73	2.14	0.00	0.35	-2.20	2.57 0.00	0.44
Central Africa	2.2	-1.46	1.46	0.00	0.27	-1.83	2.25 0.00	0.38
Himalayas	1.5	-3.16	1.89	0.00	0.39	-3.33	2.40 0.00	0.46

Region	%	EGM2008-GOCO01S (2180)			EGM2008-GOCO01S (2210)				
		Min	Max	Mean	RMS	Min	Max	Mean	RMS
Earth	100	-3.59	2.56	0.00	0.13	-3.88	3.28	0.00	0.23
Land	29.5	-3.59	2.56	0.00	0.19	-3.88	3.28	-0.01	0.32
Ocean	70.5	-1.12	0.71	0.00	0.08	-1.52	1.09	0.00	0.17
Earth ($ \phi < 83.3^{\circ}$)	99.3	-3.59	2.56	0.00	0.13	-3.88	3.28	0.00	0.21
Land ($ \phi < 83.3^{\circ}$)	29.1	-3.59	2.56	0.00	0.19	-3.88	3.28	0.00	0.28
Ocean (φ < 83.3°)	70.2	-1.12	0.71	0.00	0.08	-1.52	1.09	0.00	0.16
Australia	1.5	-0.28	0.26	0.00	0.07	-0.52	0.61	0.00	0.15
NZ, Oceanian Islands (φ<-12°)	0.1	-0.42	0.25	-0.02	0.12	-0.90	0.69	0.00	0.24
South America	3.6	-2.04	2.56	0.00	0.33	-2.55	3.28	0.00	0.39
North America	4.4	-0.68	0.52	0.00	0.07	-0.77	0.72	0.00	0.14
Greenland	0.4	-0.45	0.36	0.00	0.11	-0.69	0.58	0.00	0.16
Europe	1.4	-0.33	0.37	0.00	0.07	-0.63	0.61	0.00	0.14
Africa	5.9	-1.51	2.03	0.00	0.23	-1.87	2.33	0.00	0.27
Asia	9.5	-3.59	2.19	0.00	0.18	-3.88	2.62	0.00	0.23
Antarctica ($ \phi < 83.3^{\circ}$)	2.4	-0.93	1.00	0.00	0.22	-1.74	1.60	0.00	0.39
Andes/Amazon Basin	2.2	-2.04	2.56	0.00	0.39	-2.55	3.28	0.00	0.45
Central Africa	2.2	-1.51	2.03	0.00	0.30	-1.87	2.33	0.00	0.34
Himalayas	1.5	-3.59	2.19	0.00	0.42	-3.88	2.62	0.00	0.46

Table 7 Descriptive statistics of the differences EGM2008 minus GOCO01S (2..180) and EGM2008 minus GOCO01S (2..210) for various regions of Earth, unit is metres.



Fig.11 Top: EGM2008 minus GOCE-TIM quasigeoid heights in spectral band 2 to 180. Bottom: EGM2008 minus GOCE-TIM quasigeoid heights in spectral band 2 to 210. Units are metres



Fig.12 Top: EGM2008 minus ITG-GRACE2010S quasigeoid heights in spectral band 2 to 160. Bottom: EGM2008 minus ITG-GRACE2010S quasigeoid heights in spectral band 2 to 180. Units are metres



Fig.13 Top: EGM2008 minus GOCO01S quasigeoid heights in spectral band 2 to 180. Bottom: EGM2008 minus GOCO01S quasigeoid heights in spectral band 2 to 210. Units are metres

6. Conclusions

We have evaluated the first four GOCE-based GGMs in comparison to two GRACE-only GGMs and EGM2008. As a first assessment method, we used terrestrial gravity and vertical deflection data sets (Sects. 2 to 4). To overcome the problem of different spectral content, we applied a spectral enhancement method (SEM), where EGM2008 and RTM data are used to better match the spectral bands. Adding omission error estimates from EGM2008 and RTM data greatly reduces the discrepancies between the GGMs and terrestrial observations. As a second assessment method, we compared global grids of EGM2008 and GOCE-based/GRACE-based quasigeoid heights in different spectral bands (Sect. 5). This is a simple and powerful evaluation technique for global GOCE evaluation. The method circumvents the problem of different spectral content and benefits from the high-quality of EGM2008 over well-surveyed areas.

The terrestrial gravity and vertical deflection data sets turned out to be very sensitive to indicate the degree beyond which signal loss occurs; however, direct insight into the GOCE-performance is gained only over the data areas and not outside. Complementarily to this, the global EGM2008-based quasigeoid comparisons give world-wide insight into the GOCE-performance. Because of the different data coverage (regional vs. global), gravity field functionals (gravity, deflections and quasigeoid) and different handling of signal omission (modelling vs. avoidance) the results from both assessment strategies are used together to evaluate the first GOCE gravity field models.

As the main outcome of this study, any of the evaluated GOCE-based GGMs seem to advance the knowledge of Earth's global gravity field in comparison to ITG-GRACE2010S as a benchmark of the 'pre-GOCE-era'. Both the vertical deflections and the terrestrial gravity measurements 'sensed' improvements in the spectral band ~160-165 to ~180–185. The spectral band was confirmed by comparisons involving global grids of EGM2008 quasigeoid heights. Given that spherical harmonic degrees 180 to 185 are equivalent to a spatial resolution of ~107 to ~110 km, after two months, GOCE is already near the targeted resolution of ~100 km. The prospects for the complete mission are therefore good.

The comparisons against ground-truth vertical deflections and gravity anomalies showed GOCO01S, GOCE-TIM and GOCE-SPW to experience signal loss near degrees ~180-185, which is below their nominal model resolution (spherical harmonic degree 210 and 224, respectively). This suggests that the models are somewhat underpowered in their high frequencies, which is expected from the decay of the gravity field with altitude. Given that the evaluated first GOCE-GGMs are based on only two months of data, improvements may be possible for future models based on more observations.

The GOCE-based models showed – up to their degree of signal loss – a performance fairly equivalent to EGM2008 over areas with very good pre-GOCE gravity field knowledge. This observation was made not only from the comparisons using ground-truth vertical deflections and terrestrial gravity (Sect. 4), but consistently also from EGM2008 quasigeoid heights over Australia, Europe, North America and the oceans, where EGM2008's quality is high (Sect 5). Complementarily, we found large discrepancies between EGM2008 and GOCE quasigeoid heights over parts of South America, Africa, Asia and Antarctica, which are the EGM2008 'problem' areas (Sect. 5). Holistically, these observations provide strong evidence

that the two months of GOCE observations improve the gravity field knowledge of our planet in the spherical harmonic window of degrees ~160-165 to ~180–185.

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