**Earth2012 – Spherical harmonic models of Earth's topography and potential**

**New Research**

Ellipsoidal topographic potential model released (June 2013)

**General**

Researchers of the Western Australian Geodesy Group have developed a suite of spherical harmonic models of Earth’s topography, rock-equivalent topography, Earth’s shape and implied topographic potential. Our Earth2012 models are complete to degree and order 2160. The spherical harmonic models available here are based on the SRTM V4.1, SRTM30_PLUS and ETOPO data, and can be used in applications such as forward-modelling, Bouguer anomaly computation and evaluation of geopotential models. The Earth2012 models presented and updated here were used in recent studies on GOCE gravity fields and Earth’s topographic potential. Please acknowledge any use by citing:


For a technical description of the evaluation of gravity effects from our degree-2160 topographic potential model see:


**Topography models**

Three spherical harmonic models of Earth’s topography are available that represent the physical surface of our planet, with or without ocean water masses, and the solid part of Earth (without water/ice).

- **Earth’s surface (with water):**
  Harmonic model of the interface between Earth and its atmosphere, providing heights above mean sea level of the terrain and ice over land and zero heights over the oceans.
Earth's surface (without water):

Harmonic model of the Earth’s topography without ocean water masses. This model provides heights of the terrain and of ice over land and bathymetric depths over the oceans, Caspian Sea and major inland lakes (Superior, Michigan, Huron, Erie, Ontario and Baikal).

Earth's surface (without water and ice):

Harmonic model of the Earth’s topography without icesheets and without ocean water masses. This model provides heights of the terrain over land, bathymetric depths over the oceans, Caspian Sea and major inland lakes, and bedrock heights over Antarctica and Greenland.

Rock-equivalent topography model RET2012

RET2012 is a spherical harmonic model of Earth's rock-equivalent topography (RET) to degree and order 2160, whereby the water masses of the oceans and major lakes, as well as the ice sheets are compressed to layers equivalent to topographic rock of 2670 kg m$^{-3}$. RET2012 is the updated and improved version of the degree-360 RET2011 which was used to
evaluate new-generation gravity fields from the GOCE satellite mission (Hirt et al. 2012, JGR). RET2012 takes into account

1. the ocean water bodies (based on SRTM30plus bathymetry) with a mass-density of 1030 kg m\(^{-3}\), and
2. ice shields of Antarctica and Greenland (based on ETOPO bedrock and ice thickness) with a mass-density of 927 kg m\(^{-3}\), and
3. the water bodies of Earth’s major lakes (Superior, Michigan, Huron, Erie, Ontario and Baikal) and Caspian Sea (based on SRTM30plus inland bathymetry) with a mass-density of 1030 kg m\(^{-3}\).

- **Earth’s rock-equivalent topography**
  Harmonic model of Earth’s rock-equivalent topography to degree and order 2160.

[Click for Larger Image (1.7 MB)]
Earth2012.RET2012.SHCto2160.zip (46MB)

**Shape models**

The shape models provide radii (distance from the surface point to the geocenter) thus representing the shape of planet Earth. Our shape models were derived by adding EGM96 geoid undulations (transforming SRTM sea level heights to SRTM ellipsoidal heights) and ellipsoidal radii of the GRS80 (Moritz 2000) ellipsoid to the topography models. EGM96 was selected as geoid model because it was used as vertical reference in the development of SRTM elevation models. The following four shape models are available:

- **Earth’s shape (with water):**
  Harmonic shape model of the interface between Earth and its atmosphere, providing radii of the terrain and ice over land and of the sea level over the oceans.

[Click for Larger Image (850 KB)]
Earth2012.shape_air.SHCto2160.zip (46MB)
• **Earth’s shape (without water):**
Harmonic shape model of the Earth without ocean water masses. This model provides radii of the terrain and ice over land and of the sea bed over the oceans.

[Click for Larger Image (1.3MB)]
Earth2012.shape_bathy.SHCto2160.zip (46MB)

• **Earth’s shape (without water and ice):**
Harmonic shape model of the Earth without icesheets and without ocean water masses. This model provides radii of the terrain over land, of the sea bed over the oceans and inland lakes and of bedrock heights over Antarctica and Greenland.

[Click for Larger Image (1.4MB)]
Earth2012.shape_bathy_bed.SHCto2160.zip (46MB)

• **Earth’s rock-equivalent topography as shape model:**
Harmonic shape model of Earth’s rock-equivalent topography to degree and order 2160.

[Click For Larger Image (1.2MB)]
Earth2012.shape_RET2012.SHCto2160.zip (46MB)

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**Topopotential Models**

**Ellipsoidal topopotential model dV_ELL_RET2012**
We have developed a new topographic potential model in ellipsoidal approximation using a new formalism based on Claessens (2005; 2006)

- **Earth's ellipsoidal topographic potential** to d/o 2190
  Spherical harmonic gravitational potential coefficients of RET2012 to degree and order 2190 in ellipsoidal approximation dV_ELL_RET2012_SHCto2190.zip (48MB)

A manuscript describing the creation of this model has been submitted for publication; details will be made available in due time. This model represents the ellipsoidal topographic potential of Earth based on the RET2012 topography as input data set. The model-defining constants are \( R = 6378137.0 \) m and \( GM = 3.986005E14 \) m³ s⁻². Akin EGM2008, it features additional spherical harmonic potential coefficients in spectral band of degrees 2161-2190. The dV_ELL_RET2012 model can be used to synthesise gravity effects due to topography at the surface of the GRS80 ellipsoid or any other point in space. All coefficients to degree and order 2190 must be taken into account in the gravity synthesis if the full resolution of the dV_ELL_RET2012 model is sought.

Here is a short guide on the use of the spherical and ellipsoidal topographic potential models: Guide_topopotential_models.pdf

**Spherical topopotential model dV_SPH_RET2012**

We have transformed the RET2012 topography to the implied gravitational potential in spherical approximation by expanding the topographic gravitational potential into the first ten powers of the RET2012 topography; see e.g. Hirt and Kuhn (2012) for the mathematical formalism.

- **Earth’s spherical topographic potential** to d/o 2160
  Spherical harmonic gravitational potential coefficients of RET2012 to degree and order 2160 in spherical approximation dV_SPH_RET2012_SHCto2160.zip (48MB)

Note that the model-defining constants for dV_SPH_RET2012 are \( R = 6378137.0 \) m and \( GM = 3.986005E14 \) m³ s⁻², and the underlying mass-density assumptions are 2670 kg m⁻³ for the topographic masses. The spherical harmonic coefficients can be evaluated with high-degree synthesis software (e.g., harmonic_synth by Holmes and Pavlis (2008) or Shtools by Wieczorek (2011), in order to compute topographic gravity effects. See also our guide on the recommended use of dV_SPH_RET2012: Guide_topopotential_models.pdf

Gravity effects from our dV_SPH_RET2012 model are in better agreement with observed satellite gravity (from the GOCE mission, model GOCE-TIM3) than with the previous version RET2011 used in Hirt et al. (2012), see Fig. 1.
Fig. 1. Global comparison of topography-implied gravity from RET2012 and the previous version RET2011 with measured gravity from the Gravity field and steady-state Ocean Circulation Explorer mission GOCE (release GOCE-TIM3 from Pail et al. 2011). Left: Reduction rates (amount of topography signals captured by the GOCE satellite), Right: correlation between gravity effects as a function of the spherical harmonic degree, over a near world-wide evaluation area.

Fig. 2 shows how the first ten integer powers of the RET2012 topography contribute to Earth's topopotential in spherical approximation (dV_SPH_RET2012) as a function of the harmonic degree. Some of the higher-order powers make a non-negligible contribution at high degrees for some applications which is why they are taken into account here.

Fig. 2 Contribution of the first ten powers of the RET2012 topography to the topographic potential, shown in terms of dimensionless potential degree variances as a function of the harmonic degree. Black line shows total contribution, Figure from Hirt and Kuhn (2012)
Details and comparisons

Format

The spherical harmonic coefficients (SHCs) of all models are provided in ASCII format, where the format is in all cases

\[ i \quad j \quad C_{ij} \quad S_{ij} \]

with \( i \) degree, \( j \) order, \( C_{ij} \) and \( S_{ij} \) are the 4-pi fully-normalized SHCs. The SHCs of the topographic potential model \( dV_{RET2012} \) are unitless, and the SHCs of all other models are in unit metres.

Data Sources

All models presented here rely on;

- SRTM V4.1 hole-filled 250m resolution release by CGIAR-CSI Consortium for Spatial Information within the coverage of the Shuttle Radar Topography Mission,
- SRTM30_PLUS bathymetry (by University of California) over the oceans and major lakes, and
- NOAA’s ETOPO1 ice and bed-rock data over Antarctica and Greenland.

We selected the SRTM V4.1 release because this data is frequently deployed in our recent research projects. SRTM30plus provides topographic heights in high latitudes, outside the actual SRTM data coverage. ETOPO1 is used over Greenland and Antarctica as data source for terrain heights, ice thickness and bedrock heights (Fig. 3).

Applications

The spherical harmonic topography models provided here use partially different data than other available products such as the EGM2008-accompanied height model DTM2006 (see comparison in Fig. 5). Our models may prove useful for applications such as;
• **Gravity forward-modelling:** Our spherical harmonic topography models are rigorously consistent with the SRTM V4.1 release over land and SRTM30_PLUS over sea. They can be used as long-wavelength reference surface for residual terrain modelling (RTM) of short-scale gravity effects over land, marine and coastal regions.

• **Bouguer anomalies:** RET2012 and the topography-implied potential model may be useful as a complement to EGM2008, e.g., for the computation of EGM2008-based spherical harmonic Bouguer anomalies or gravity disturbances.

• **Evaluation of geopotential models:** The topography-implied potential model is suitable for topography-based evaluations of Earth geopotential models, e.g., from the dedicated satellite gravity field missions GRACE and GOCE (Fig. 1).

### Summary of procedures applied

Here is a general summary of processing steps applied to create the spherical harmonic topography and topopotential models.

1. **SRTM Grid merging.** The SRTM V4.1 release was merged with the SRTM30_PLUS bathymetry at 7.5 arc second resolution, which required interpolation of the 30_PLUS resolution from 30 to 7.5 arc seconds. The SRTM V4.1 embedded sea mask was used to create the merger, with SRTM30 plus information used at all ocean cells and outside the V4.1 data coverage. The inland lakes were extracted from the differences between V4.1 and 30 PLUS through application of region growing algorithms.

2. **Downsampling.** Reduction of the SRTM V4.1/30PLUS merger (7.5 arc seconds), and ETOPO1 (60 arc seconds) to a common resolution of 2 arc minutes. This was accomplished by applying box-filters to derive cell-mean values (SRTM:16x16 cell means, ETOPO: 2x2 means).

3. **ETOPO grid merging.** ETOPO information (ice heights, bed rock heights) was combined with the downsampled SRTM mosaic at 2 arc min resolution, likewise information on the major inland lakes.

4. **Rock-equivalent topography.** Ocean and lake water masses and major ice sheets masses were compressed into topographic rock (density of 2670 kg m⁻³). This was done at 7.5 arc second resolution level for the SRTM30plus bathymetry (repetition of step 1). Ocean depths (from SRTM30plus) where reduced by a factor of 0.614, likewise the depths of the Caspian Sea, the great inland seas of North America (Superior, Michigan, Huron, Erie, Ontario) and Asia(Baikal). Ice-sheets of Antarctica and Greenland, as implied by the ETOPO1-bed and ice products were made rock-equivalent with the procedures and values described in Hirt et al. (2012).

5. **Grid transformation.** To this point, the 2x2 arc minute grids were prepared in terms of geodetic coordinates and centre-of-cell registration (5400 x 10800 cell values). For the spherical harmonic analysis, geocentric coordinates and grid-line registration was required. We have therefore transformed all grids from geodetic to geocentric latitude and from centre-of-cell to grid-line registration, requiring (bicubic) interpolation of the data.

6. **Spherical harmonic analysis.** We expanded all grids into spherical harmonics using Driscoll and Healy’s 1994 algorithm, as implemented in the SH-Tools package (Wieczorek 2011). In Driscoll and Healy’s algorithm, the grid resolution determines the maximum harmonic degree, which is 2699 for our 2 arc minute grids. We did not use the coefficients beyond degree and order 2160, while those complete to 2160 are provided above.

7. **Topopotential expansions.** Transformation of the spherical harmonic RET2012 topography to its potential required to take into account higher-order powers of the
topography. RET2012 heights to degree 2160 were synthesized in terms of 2x2 arc minute grids, powers (squared, cubed, up to power 10) formed, harmonically analysed, and transformed to the RET2012 gravitational potential coefficients (see Wieczorek 2007).

Difference plots - internal comparison

Fig 4 shows the internal comparison between our four Earth2012 models in the representations topography/air, topography/bathymetry, topography/bathymetry/bed rock and rock-equivalent topography (RET2012).

Fig 4 Differences between our four Earth2012 models expanded to degree and order 2160, unit in metres

Difference plots - external comparison
The external comparison with the DTM2006.0 (Pavlis et al. 2012) spherical harmonic model (also known as “EGM2008 topography”) shows a fairly good agreement of few metres over large parts of the continents (Fig 5 left). This shows that the procedures applied to generate our models and DTM2006.0 are compatible, and thus provides an independent check. Larger regional differences are due to the different data sets processed, with the most striking differences present over the oceans, showing the inclusion of depth soundings in our models. Local differences over land areas reflect different elevation data releases (e.g., SRTM) used by the two groups.

Acknowledgements and references

• Jarvis, A., H.I. Reuter, A. Nelson, E. Guevara, 2008, Hole-filled SRTM for the globe Version 4, available from the CGIAR-CSI SRTM 90m Database, 250 m release by Andrew Jarvis. Link
• Wieczorek M (2011) SHTools 2.6 - Tools for working with spherical harmonics. Link

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