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# Joint Source and Relay Optimization for Distributed MIMO Relay System

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***Abstract*—In this paper, we develop the optimal transmit beamforming vector and the relay amplifying factors for a multiple-input multiple-output (MIMO) relay communication system with distributed relay nodes. Using the optimal beamforming vector, an iterative joint source and relay beamforming algorithm is developed to minimize the mean-squared error (MSE) of the signal waveform estimation. Numerical simulations are carried out to demonstrate the performance of the proposed joint source and relay beamforming algorithm.**

***Index Terms*—Multiple-input multiple-output (MIMO), relay networks, parallel relay, beamforming.**

## I. Introduction

In order to establish a reliable wireless communication link, one needs to compensate for the effects of signal fading and shadowing. An efficient way to address this issue is to transmit signals through one or multiple relays [1]-[4]. Introducing multiple antennas at transmitting and receiving ends, which we call multiple-input multiple-output (MIMO) relay communication systems, can provide further improvement in terms of both spectral efficiency and link reliability [3]-[4]. Many works have studied the optimal relay amplifying matrix for MIMO relay channels. In [5] and [6], the optimal relay amplifying matrix is designed to maximize the mutual information (MI) between the source node and the destination node, assuming that the source covariance matrix is an identity matrix. In [7] and [8], the optimal relay amplifying matrix was designed to minimize the mean-squared error (MSE) of the signal waveform estimation at the destination.

However, few research have studied the jointly optimal source precoding matrix and the relay amplifying matrix for the source-relay-destination channel. In [9], the source covariance matrix and the relay amplifying matrix were jointly designed to maximize the source-destination MI. In [10], a unified framework was developed to jointly optimize the source precoding matrix and the relay amplifying matrix for a broad class of objective functions. All the works [3]-[10] focus on MIMO relay systems with a single relay node at each hop. In [11] and [12], the optimal source and relay matrices are designed for a multihop MIMO relay network with serial relays.

MIMO relay systems with multiple parallel relay nodes have been investigated in [13]-[15]. In [14], the authors investigated the jointly optimal structure of the source precoding matrix and the relay amplifying matrices considering a linear minimal MSE (MMSE) receiver at the destination. In [15], a non-linear receiver is used to design the matrices. On the other hand, a distributed relay network is investigated in [16] where multiple users and relays, each having a single antenna, are considered.

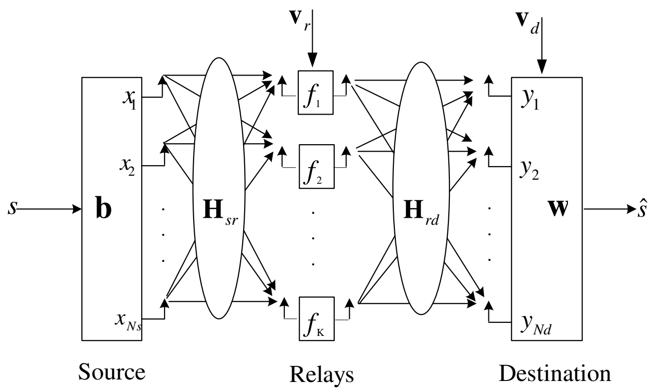


Fig. 1. Block diagram of a distributed MIMO relay communication system.

In this paper, we propose a jointly optimal source and relay beamforming algorithm which minimizes the MSE of the signal waveform estimation for single-antenna relay nodes in a MIMO relay communication system. In contrast to [13]-[15], where the receive power constraint at the destination node is considered, we consider in this paper the sum transmit power constraint throughout all relay nodes.

The rest of this paper is organized as follows. The system model is described in Section II. In Section III, we study the jointly optimal source and relay algorithm. Section IV shows the simulation results. Conclusions are drawn in Section V.

## II. System Model

Fig. 1 illustrates a two-hop MIMO relay communication system consisting of one source node, $K$ parallel relay nodes, and one destination node. We assume that the source and the destination nodes have $N_s$ and $N_d$ antennas, respectively, whereas each relay node has a single antenna. Due to its merit of simplicity, we consider the amplify-and-forward scheme at each relay. The communication process between the source

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and destination nodes is completed in two time slots. In the first time slot, the modulated symbol $s$ is linearly precoded as

$$\mathbf{x} = \mathbf{b}s \tag{1}$$

where $\mathbf{b}$ is an $N_s \times 1$ transmit beamforming vector. The precoded vector $\mathbf{x}$ is then transmitted to the relay nodes. The received signal at the $i$th relay node can be written as

$$y_{r,i} = \mathbf{h}_{sr,i}\mathbf{x} + v_{r,i}, \qquad i = 1, \cdots, K \tag{2}$$

where $\mathbf{h}_{sr,i}$ is the $1 \times N_s$ channel vector between the source and the $i$th relay node, $y_{r,i}$ and $v_{r,i}$ are the received signal and the additive Gaussian noise at the $i$th relay node, respectively.

In the second time slot, the source node is silent, while each relay node transmits the amplified signal to the destination node as

$$x_{r,i} = f_i y_{r,i}, \qquad i = 1, \cdots, K \tag{3}$$

where $f_i$ is the amplifying coefficient at the $i$th relay node. The received signal vector at the destination node can be written as

$$\mathbf{y}_d = \sum_{i=1}^{K} \mathbf{h}_{rd,i} x_{r,i} + \mathbf{v}_d \tag{4}$$

where $\mathbf{h}_{rd,i}$ is the $N_d \times 1$ channel vector between the $i$th relay and the destination node, $\mathbf{y}_d$ and $\mathbf{v}_d$ are the received signal and the additive Gaussian noise vectors at the destination node, respectively.

Substituting (1)-(3) into (4), we have

$$\begin{aligned} \mathbf{y}_d &= \sum_{i=1}^{K} (\mathbf{h}_{rd,i} f_i \mathbf{h}_{sr,i} \mathbf{b}s + \mathbf{h}_{rd,i} f_i v_{r,i}) + \mathbf{v}_d \\ &= \mathbf{H}_{rd}\mathbf{F}\mathbf{H}_{sr}\mathbf{b}s + \mathbf{H}_{rd}\mathbf{F}\mathbf{v}_r + \mathbf{v}_d \end{aligned} \tag{5}$$

where we define

$$\begin{aligned} \mathbf{H}_{sr} &\triangleq \left[\mathbf{h}_{sr,1}^T, \mathbf{h}_{sr,2}^T, \cdots, \mathbf{h}_{sr,K}^T\right]^T \\ \mathbf{H}_{rd} &\triangleq \left[\mathbf{h}_{rd,1}, \mathbf{h}_{rd,2}, \cdots, \mathbf{h}_{rd,K}\right] \\ \mathbf{F} &\triangleq \text{diag}\big([f_1, f_2, \cdots, f_K]^T\big) \\ \mathbf{v}_r &\triangleq \left[v_{r,1}, v_{r,2}, \cdots, v_{r,K}\right]^T. \end{aligned}$$

Here $(\cdot)^T$ denotes the matrix (vector) transpose, diag$(\mathbf{a})$ stands for a diagonal matrix with the vector $\mathbf{a}$ as the main diagonal and zero elsewhere, $\mathbf{H}_{sr}$ is a $K \times N_s$ channel matrix between the source node and all relay nodes, $\mathbf{H}_{rd}$ is an $N_d \times K$ channel matrix between all relay nodes and the destination node, and $\mathbf{v}_r$ is obtained by stacking the noise terms at all the relays. We assume that all noises are independent and identically distributed (i.i.d.) with zero mean and unit variance.

The diagram of the equivalent MIMO relay system described by (5) is shown in Fig. 2. The received signal vector at the destination node can be equivalently written as

$$\mathbf{y}_d = \bar{\mathbf{h}}s + \bar{\mathbf{v}}$$

where we define $\bar{\mathbf{h}} \triangleq \mathbf{H}_{rd}\mathbf{F}\mathbf{H}_{sr}\mathbf{b}$ as the effective channel vector of the source-relay-destination link, and $\bar{\mathbf{v}} \triangleq \mathbf{H}_{rd}\mathbf{F}\mathbf{v}_r + \mathbf{v}_d$ as the equivalent noise vector.

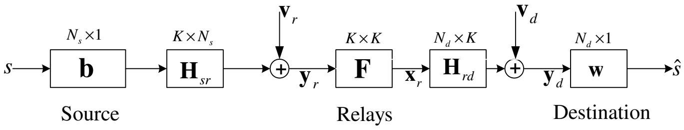


Fig. 2. Block diagram of the equivalent MIMO relay system.

## III. Optimal Source and Relay Design

In this section we develop the optimal transmit beamforming vector $\mathbf{b}$ and the relay amplifying matrix $\mathbf{F}$ to minimize the MSE of the signal waveform estimation. Using a linear receiver, the estimated signal waveform vector at the destination node is given by

$$\hat{s} = \mathbf{w}^H \mathbf{y}_d \tag{6}$$

where $\mathbf{w}$ is an $N_d \times 1$ weight vector, and $(\cdot)^H$ denotes the matrix (vector) Hermitian transpose.

The MMSE approach tries to find a weight vector $\mathbf{w}$ that minimizes the statistical expectation of the signal waveform estimation given by

$$\text{MSE} = \text{E}\big[|\hat{s} - s|^2\big] \tag{7}$$

where E$[\cdot]$ denotes statistical expectation. We assume that the source signal satisfies E$[|s|^2] = 1$. Substituting (6) into (7), we find that the $\mathbf{w}$ which minimizes (7) can be written as

$$\mathbf{w} = (\bar{\mathbf{h}}\bar{\mathbf{h}}^H + \bar{\mathbf{C}})^{-1}\bar{\mathbf{h}} \tag{8}$$

where $(\cdot)^{-1}$ denotes the matrix inversion, and $\bar{\mathbf{C}}$ is the equivalent noise covariance matrix given by

$$\bar{\mathbf{C}} = \mathbf{H}_{rd}\mathbf{F}\mathbf{F}^H\mathbf{H}_{rd}^H + \mathbf{I}_{N_d}. \tag{9}$$

Here $\mathbf{I}_n$ is an $n \times n$ identity matrix.

Substituting (8) back into (7), we obtain the minimal MSE as a function of $\mathbf{b}$ and $\mathbf{F}$, given by

$$\text{MSE} = 1 - \bar{\mathbf{h}}^H(\bar{\mathbf{h}}\bar{\mathbf{h}}^H + \bar{\mathbf{C}})^{-1}\bar{\mathbf{h}}. \tag{10}$$

Applying the matrix inversion lemma $(\mathbf{A} + \mathbf{BCD})^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{B}(\mathbf{D}\mathbf{A}^{-1}\mathbf{B} + \mathbf{C}^{-1})^{-1}\mathbf{D}\mathbf{A}^{-1}$, (10) can be written as

$$\text{MSE} = \big(1 + \bar{\mathbf{h}}^H\bar{\mathbf{C}}^{-1}\bar{\mathbf{h}}\big)^{-1}. \tag{11}$$

From (3), the total transmission power consumed by all relay nodes can be expressed as

$$\text{tr}\big(\text{E}[\mathbf{x}_r\mathbf{x}_r^H]\big) = \text{tr}\big(\mathbf{F}\left[\mathbf{H}_{sr}\mathbf{b}\mathbf{b}^H\mathbf{H}_{sr}^H + \mathbf{I}_K\right]\mathbf{F}^H\big) \tag{12}$$

where tr$(\cdot)$ stands for matrix trace.

Using (12), the joint source and relay optimization problem can be formulated as

$$\min_{\mathbf{F},\mathbf{b}} \quad \text{MSE} \tag{13}$$

$$\text{s.t.} \quad \mathbf{b}^H\mathbf{b} \le P_s \tag{14}$$

$$\text{tr}\big(\mathbf{F}\left[\mathbf{H}_{sr}\mathbf{b}\mathbf{b}^H\mathbf{H}_{sr}^H + \mathbf{I}_K\right]\mathbf{F}^H\big) \le P_r \tag{15}$$

where (14) is the transmit power constraint at the source node, and (15) is the sum transmit power constraint throughout all relay nodes. Here $P_r > 0$ and $P_s > 0$ are the corresponding power budget. The problem (13)-(15) is highly nonconvex and a closed-form expression of the optimal $\mathbf{F}$ and $\mathbf{b}$ is intractable. In this paper, we develop an iterative algorithm to optimize $\mathbf{F}$ and $\mathbf{b}$.

### A. Optimal Relay Factors

For given beamforming vector $\mathbf{b}$ satisfying (14), we optimize the relay matrix $\mathbf{F}$ by solving the following optimization problem

$$\min_{\mathbf{F}} \quad \text{MSE} \tag{16}$$
$$\text{s.t.} \quad \text{tr}\big(\mathbf{F}\big[\mathbf{H}_{sr}\mathbf{b}\mathbf{b}^H\mathbf{H}_{sr}^H+\mathbf{I}_K\big]\mathbf{F}^H\big) \le P_r. \tag{17}$$

Let us introduce

$$\bar{\mathbf{h}}_s \triangleq \mathbf{H}_{sr}\mathbf{b}. \tag{18}$$

Substituting (18) back into (16)-(17), we can rewrite the optimization problem as

$$\min_{\mathbf{F}} \big[1+\bar{\mathbf{h}}_s^H\mathbf{F}^H\mathbf{H}_{rd}^H(\mathbf{H}_{rd}\mathbf{F}\mathbf{F}^H\mathbf{H}_{rd}^H+\mathbf{I}_{N_d})^{-1}\mathbf{H}_{rd}\mathbf{F}\bar{\mathbf{h}}_s\big]^{-1} \tag{19}$$
$$\text{s.t.} \sum_{i=1}^{K}|f_i|^2\big(|\bar{h}_{s,i}|^2+1\big) \le P_r \tag{20}$$

where $\bar{h}_{s,i}$ stands for the $i$th element of $\bar{\mathbf{h}}_s$. Problem (19)-(20) is equivalent to

$$\max_{\mathbf{F}} \ \bar{\mathbf{h}}_s^H\mathbf{F}^H\mathbf{H}_{rd}^H(\mathbf{H}_{rd}\mathbf{F}\mathbf{F}^H\mathbf{H}_{rd}^H+\mathbf{I}_{N_d})^{-1}\mathbf{H}_{rd}\mathbf{F}\bar{\mathbf{h}}_s \tag{21}$$
$$\text{s.t.} \ \sum_{i=1}^{K}|f_i|^2\big(|\bar{h}_{s,i}|^2+1\big) \le P_r. \tag{22}$$

Since the objective function (21) is still a complicated function of $\mathbf{F}$, in the following, we optimize an upper-bound of (21). The problem can be rewritten as

$$\max_{\mathbf{F}} \ \bar{\mathbf{h}}_s^H\mathbf{F}^H\mathbf{H}_{rd}^H\mathbf{H}_{rd}\mathbf{F}\bar{\mathbf{h}}_s \tag{23}$$
$$\text{s.t.} \ \sum_{i=1}^{K}|f_i|^2\big(|\bar{h}_{s,i}|^2+1\big) \le P_r. \tag{24}$$

Let $\mathbf{f} \triangleq [f_1, f_2, \cdots, f_K]^T$ denote the diagonal elements of $\mathbf{F}$ and define $\mathbf{D}_s \triangleq \text{diag}(\bar{\mathbf{h}}_s)$, so that

$$\mathbf{F}\bar{\mathbf{h}}_s = \mathbf{D}_s\mathbf{f}. \tag{25}$$

Now by substituting (25) in (23)-(24), we can express the maximization problem as follows

$$\max_{\mathbf{f}} \quad \mathbf{f}^H\mathbf{D}_s^H\mathbf{H}_{rd}^H\mathbf{H}_{rd}\mathbf{D}_s\mathbf{f} \tag{26}$$
$$\text{s.t.} \quad \mathbf{f}^H\mathbf{A}\mathbf{f} \le P_r \tag{27}$$

where $\mathbf{A} \triangleq \mathbf{D}_s\mathbf{D}_s^H + \mathbf{I}_K$. Defining $\bar{\mathbf{f}} \triangleq \mathbf{A}^{\frac{1}{2}}\mathbf{f}$, problem (26)-(27) can be equivalently written as

$$\max_{\bar{\mathbf{f}}} \quad \bar{\mathbf{f}}^H\mathbf{A}^{-\frac{H}{2}}\mathbf{D}_s^H\mathbf{H}_{rd}^H\mathbf{H}_{rd}\mathbf{D}_s\mathbf{A}^{-\frac{1}{2}}\bar{\mathbf{f}}$$
$$\text{s.t.} \quad \bar{\mathbf{f}}^H\bar{\mathbf{f}} \le P_r.$$

Introducing $\mathbf{Z} \triangleq \mathbf{A}^{-\frac{H}{2}}\mathbf{D}_s^H\mathbf{H}_{rd}^H\mathbf{H}_{rd}\mathbf{D}_s\mathbf{A}^{-\frac{1}{2}}$, we obtain

$$\max_{\bar{\mathbf{f}}} \quad \bar{\mathbf{f}}^H\mathbf{Z}\bar{\mathbf{f}} \tag{28}$$
$$\text{s.t.} \quad \bar{\mathbf{f}}^H\bar{\mathbf{f}} \le P_r. \tag{29}$$

The Lagrangian of the optimization problem (28)-(29) can be written as

$$\mathcal{L} \ = \ -\bar{\mathbf{f}}^H\mathbf{Z}\bar{\mathbf{f}} + \mu_1(\bar{\mathbf{f}}^H\bar{\mathbf{f}} - P_r) \tag{30}$$

where $\mu_1 \ge 0$ is the Lagrangian multiplier associated with the constraint (29). Taking the derivative of $\mathcal{L}$ with respect to $\bar{\mathbf{f}}^H$ and letting the result be 0, it can be shown that the optimal $\bar{\mathbf{f}}$ satisfies the following equation

$$\mathbf{Z}\bar{\mathbf{f}} = \mu_1\bar{\mathbf{f}}.$$

Thus $\bar{\mathbf{f}} = \sqrt{P_r}\text{eig}(\mathbf{Z})$, where $\text{eig}(\mathbf{Z})$ stands for the principal eigenvector of $\mathbf{Z}$.

### B. Joint Source and Relay Optimization

For a fixed $\mathbf{F}$, problem (13) - (15) can be expressed as below to optimize the beamforming vector $\mathbf{b}$

$$\min_{\mathbf{b}} \quad \big(1+\mathbf{b}^H\mathbf{\Psi}_1\mathbf{b}\big)^{-1} \tag{31}$$
$$\text{s.t.} \quad \mathbf{b}^H\mathbf{b} \le P_s \tag{32}$$
$$\mathbf{b}^H\mathbf{\Psi}_2\mathbf{b} \le \bar{P}_r \tag{33}$$

where we define

$$\begin{aligned}\mathbf{\Psi}_1 &\triangleq \mathbf{H}_{sr}^H\mathbf{F}^H\mathbf{H}_{rd}^H\mathbf{H}_{rd}\mathbf{F}\mathbf{H}_{sr}\\ \mathbf{\Psi}_2 &\triangleq \mathbf{H}_{sr}^H\mathbf{F}^H\mathbf{F}\mathbf{H}_{sr}\\ \bar{P}_r &\triangleq P_r - \text{tr}(\mathbf{F}\mathbf{F}^H).\end{aligned}$$

The problem (31)-(33) is equivalent to

$$\max_{\mathbf{b}} \quad \mathbf{b}^H\mathbf{\Psi}_1\mathbf{b} \tag{34}$$
$$\text{s.t.} \quad \mathbf{b}^H\mathbf{b} \le P_s \tag{35}$$
$$\mathbf{b}^H\mathbf{\Psi}_2\mathbf{b} \le \bar{P}_r. \tag{36}$$

The Lagrangian function associated with the problem (34)-(36) can be written as

$$\mathcal{L} = -\mathbf{b}^H\mathbf{\Psi}_1\mathbf{b} + \mu_2(\mathbf{b}^H\mathbf{b} - P_s) + \mu_3(\mathbf{b}^H\mathbf{\Psi}_2\mathbf{b} - \bar{P}_r). \tag{37}$$

Here $\mu_2 \ge 0$ and $\mu_3 \ge 0$ are the Lagrangian multipliers associated with the constraints (35) and (36), respectively. The problem (34)-(36)can be solved by using KKT conditions [17] that can be expressed as

$$\frac{\partial\mathcal{L}}{\partial\mathbf{b}^H} = 0 \tag{38}$$
$$\mu_2(\mathbf{b}^H\mathbf{b} - P_s) = 0 \tag{39}$$
$$\mu_3(\mathbf{b}^H\mathbf{\Psi}_2\mathbf{b} - \bar{P}_r) = 0 \tag{40}$$
$$\mathbf{b}^H\mathbf{b} \le P_s \tag{41}$$
$$\mathbf{b}^H\mathbf{\Psi}_2\mathbf{b} \le \bar{P}_r \tag{42}$$
$$\mu_2 \ge 0, \quad \mu_3 \ge 0. \tag{43}$$

Now we set out to solve (38)-(43). If $\mu_2 > 0$ and $\mu_3 = 0$, then the optimization problem (34)-(36) can be rewritten as

$$\max_{\mathbf{b}} \quad \mathbf{b}^H \mathbf{\Psi}_1 \mathbf{b} \tag{44}$$

$$\text{s.t.} \quad \mathbf{b}^H \mathbf{b} = P_s. \tag{45}$$

Thus we solve $\mathbf{b}$ as

$$\mathbf{b} = \sqrt{P_s}\, \mathrm{eig}(\mathbf{\Psi}_1). \tag{46}$$

If $\mathbf{b}$ in (46) satisfies the constraint (36), then (46) is the optimal solution to the problem (34)-(36). Otherwise, if $\mu_3 > 0$ and $\mu_2 = 0$, then the optimization problem (34)-(36) can be rewritten as

$$\max_{\mathbf{b}} \quad \mathbf{b}^H \mathbf{\Psi}_1 \mathbf{b} \tag{47}$$

$$\text{s.t.} \quad \mathbf{b}^H \mathbf{\Psi}_2 \mathbf{b} = \bar{P}_r. \tag{48}$$

Then we solve $\mathbf{b}$ as

$$\mathbf{b} = \alpha\, \mathrm{eig}(\mathbf{\Psi}_2^{-1}\mathbf{\Psi}_1) \tag{49}$$

where $\alpha = \sqrt{\bar{P}_r / (\mathrm{eig}^H(\mathbf{\Psi}_2^{-1}\mathbf{\Psi}_1)\mathbf{\Psi}_2 \mathrm{eig}(\mathbf{\Psi}_2^{-1}\mathbf{\Psi}_1))}$. If $\mathbf{b}$ in (49) satisfies the constraint (35), then (49) is the optimal solution to the problem (34)-(36).

Finally, if $\mu_2 > 0$ and $\mu_3 > 0$, then we have from (38) that

$$\mathbf{\Psi}_1 \mathbf{b} = \mu_2 \mathbf{b} + \mu_3 \mathbf{\Psi}_2 \mathbf{b}$$

which can be equivalently written as

$$\left(\mathbf{I}_{N_s} + \frac{\mu_3}{\mu_2}\mathbf{\Psi}_2\right)^{-1} \mathbf{\Psi}_1 \mathbf{b} = \mu_2 \mathbf{b}. \tag{50}$$

Thus the optimal $\mathbf{b}$ can be obtained as

$$\mathbf{b} = \sqrt{P_s}\, \mathrm{eig}((\mathbf{I}_{N_s} + \lambda \mathbf{\Psi}_2)^{-1}\mathbf{\Psi}_1) \tag{51}$$

where $\lambda > 0$ can be found by substituting (51) back into (48) and solving the obtained nonlinear equation.

Now the original problem (13)-(15) can be solved in an iterative fashion. In each iteration, we first fix $\mathbf{b}$ and update $\mathbf{f}$ by solving the problem (26)-(27). Then we update $\mathbf{b}$ with fixed $\mathbf{F}$ through solving the problem (34)-(36). The procedure of the proposed iterative algorithm is listed in Table I, where $\varepsilon$ is a small positive number close to zero. Since each update may only reduce or maintain, but can not increase the MSE, a monotonic convergence of this iterative algorithm follows directly from this observation.

TABLE I
PROCEDURE OF SOLVING THE PROBLEM (13)-(15) BY THE PROPOSED ITERATIVE ALGORITHM

1) Initialize the algorithm with random $\mathbf{b}^{(0)}$; Set $n = 0$.
2) Solve the subproblem (26)-(27) using given $\mathbf{b}^{(n)}$ to obtain $\mathbf{f}^{(n)}$.
3) Solve the subproblem (34)-(36) using $\mathbf{F}^{(n)}$ to obtain $\mathbf{b}^{(n)}$.
4) if $\|\mathbf{b}^{(n+1)} - \mathbf{b}^{(n)}\| \le \varepsilon$, then end.
   Otherwise, let $n := n + 1$ and go to step 2.

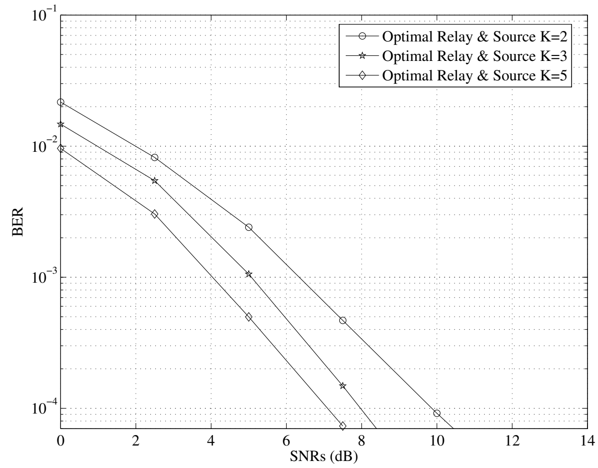


Fig. 3. Example 1. BER versus $\mathrm{SNR_s}$ with varying $K$. $N_s = N_d = 3$, $\mathrm{SNR_r}$ = 20dB.

## IV. SIMULATIONS

In this section, we study the performance of the proposed optimal joint source and relay beamforming algorithm for distributed MIMO relay systems. All simulations are conducted in a flat Rayleigh fading environment using BPSK constellations, and the noises are i.i.d. Gaussian random variables with zero mean and unit variance. The channel matrices have zero-mean entries with variances $\sigma_s^2/N_s$ and $\sigma_r^2/K$ for $\mathbf{H}_{sr}$ and $\mathbf{H}_{rd}$, respectively. We vary the signal-to-noise ratio (SNR) in the source-to-relay link $\mathrm{SNR_s}$ while fixing the SNR in the relay-to-destination link $\mathrm{SNR_r}$ to 20dB. We transmit 1000 randomly generated bits in each channel realization, and the bit-error-rate (BER) results are averaged through 200 random channel realizations.

In the first example, we study the effect of the number of relays to the system BER performance using the proposed algorithm. We choose $N_s = N_d = 3$. Fig. 3 shows the BER performance with $K = 2$, 3, and 5. It can be seen that at BER = $10^{-3}$, we achieve a 2.5-dB gain by increasing from $K = 2$ to $K = 5$.

In the second example, we simulate a distributed MIMO relay system with $N_s = N_d = 5$. Fig. 4 shows the BER performance with $K = 2$, 3, and 5. It can be seen that at BER = $10^{-3}$, we achieve a 2-dB gain by increasing from $K = 2$ to $K = 5$.

## V. CONCLUSIONS

In this paper, we have developed the optimal source and relay beamforming vectors for MIMO relay communication systems with distributed relay nodes. The proposed algorithm minimizes the MSE of the signal waveform estimation. Simulation results demonstrate the performance of the algorithm.

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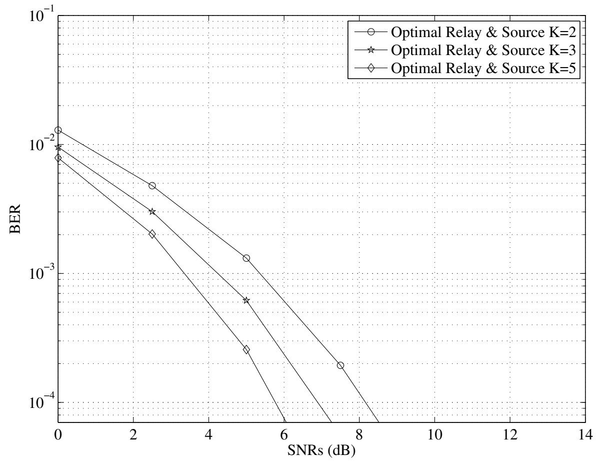


Fig. 4. Example 2. BER versus $\mathrm{SNR_s}$ with varying $K$. $N_s = N_d = 5$, $\mathrm{SNR_r}$ = 20dB.

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