# Joint Power Control and Beamforming for Interference MIMO Relay Channel

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Abstract—In this paper, we consider an interference multipleinput multiple-output (MIMO) relay system where multiple source nodes communicate with their desired destination nodes with the aid of distributed relay nodes. An iterative algorithm is developed to minimize the total source and relay transmit power such that a minimum signal-to-interference-plus-noise ratio (SINR) threshold is maintained at each receiver. The proposed algorithm exploits the network beamforming technique at the relay nodes and the receive beamforming technique at the destination nodes to mitigate the interferences from the unintended sources in conjunction with transmit power control. In particular, we apply the semidefinite relaxation technique to transform the relay transmission power minimization problem into a semidefinite programming (SDP) problem which can be efficiently solved by interior point-based methods. Numerical simulations are performed to demonstrate the effectiveness of the proposed iterative algorithm.

## I. INTRODUCTION

In a large wireless network with many nodes, multiple source-destination links must share a common frequency band concurrently to ensure a high spectral efficiency of the whole network [1]. In such network, cochannel interference (CCI) is one of the main impairments that degrades the system performance. Developing schemes that mitigate the CCI is therefore important.

By exploiting the spatial diversity, multi-antenna technique provides an efficient approach to CCI mitigation [2]. When each source node has a single antenna and the destination nodes are equipped with multiple antennas, a joint power control and beamforming scheme is developed in [3] to meet the signal-to-interference-plus-noise ratio (SINR) threshold with the minimal transmission power. A joint transmit-receive beamforming and power control algorithm is proposed in [4], when the source nodes also have multiple antennas. Due to the transmit diversity, the total transmit power required in [4] is less than that in [3].

However, installing multiple antennas at mobile nodes may not always be feasible due to the size and power constraints. One possible approach to handle these practical restrictions is to apply the network beamforming technique [5] where multiple relay nodes work as virtual antennas to assist the communication between source and destination nodes.

The network beamforming scheme stems from the idea of cooperative diversity [6]-[8], where users share their commu-

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nication resources such as bandwidth and transmit power to assist each other in data transmission. In [9], a decentralized relay beamforming technique has been developed considering a network of one transmitter, one receiver, and several relay nodes each having a single antenna. In [10], a wireless ad hoc network consisting of d source-destination pairs and Rrelaying nodes each having a single antenna is considered, where the network beamforming scheme is used to meet the SINR threshold at all links with the minimal total transmission power consumed by all relay nodes.

However, [10] assumes that each source node uses its maximum available power. Such assumption degrades the system performance by not only in terms of transmit power, but also increases the interference to other users. So the beamforming and the power control problem should be considered jointly as in [3].

In this paper, we consider a two-hop interference multipleinput multiple-output (MIMO) relay system with L sourcedestination pairs communicating with the aid of K relay nodes so as to make communication possible over a long distance. Each of the source and the relay nodes are equipped with a single antenna whereas the destination nodes are equipped with multiple antennas. The amplify-and-forward scheme is used at each relay node due to its practical implementation simplicity. These relay nodes assist in CCI mitigation by performing distributed network beamforming.

We aim at developing a joint power control and beamforming algorithm such that the total transmission power consumed by all source nodes and relay nodes are minimized while maintaining the SINR at each receiver above a minimum threshold value. Compared with [10], we not only use the network beamforming technique at the relay nodes, but also apply the receive beamforming technique at the destination nodes to mitigate the CCI. Moreover, transmit power control is used in our algorithm to minimize the total transmit power, which is not considered in [10].

An iterative algorithm is proposed to jointly optimize the relay beamformer, the receive beamformers, and the source power. In each iteration, we first optimize the receive beamformers with fixed relay beamformer and source power. Then we update the source power such that the target SINR is just met with given relay and receive beamformers. Finally,



Fig. 1. Block diagram of an interference MIMO relay system.

we update the relay beamformer using fixed source power and receive beamformers. Since the relay beamforming optimization problem is nonconvex, we use semidefinite relaxation technique to transform the problem into a semidefinite programming (SDP) problem which can be efficiently solved by interior point-based methods. Numerical simulations are carried out to evaluate the performance of the proposed algorithm.

The rest of this paper is organized as follows. In Section II, the system model of an interference MIMO relay network is introduced. The joint optimal power control and beamforming algorithm is developed in Section III. Section IV shows the simulation results which justify the significance of the proposed algorithm under various scenarios. Conclusions are drawn in Section V.

#### II. SYSTEM MODEL

We consider a two-hop interference MIMO relay system with L source-destination pairs as illustrated in Fig. 1. The communication links are supported by a network of K distributed relays so as to make communication possible over a long distance. All source nodes and relay nodes are equipped with a single antenna, whereas the *l*-th destination node is equipped with  $N_l$  antennas.

We assume that all relay nodes work in half-duplex mode, thus the communication between source-destination pairs is completed in two time slots. Moreover, the direct links between the source nodes and the destination nodes are not considered since we assume that these direct links undergo relatively larger path attenuations compared with the links via relays.

In the first time slot, the *l*-th source node transmits signal  $s_l$ . The received signal at the *k*-th relay node is given by

$$y_{r,k} = \sum_{l=1}^{L} h_{k,l} s_l + n_{r,k}, \qquad k = 1, \cdots, K$$
 (1)

where  $h_{k,l}$  is the channel coefficient between the *l*-th transmitting node and the *k*-th relay node and  $n_{r,k}$  is the additive Gaussian noise at the *k*-th relay node. Using vector notations, (1) can be expressed as

$$\mathbf{y}_r = \sum_{l=1}^L \mathbf{h}_l s_l + \mathbf{n}_r$$

where  $\mathbf{y}_r \triangleq [y_{r,1}, y_{r,2}, \cdots, y_{r,K}]^T$  and  $\mathbf{n}_r \triangleq [n_{r,1}, n_{r,2}, \cdots, n_{r,K}]^T$  are the received signal and the additive Gaussian noise vectors at all K relay nodes, respectively,  $\mathbf{h}_l \triangleq [h_{1,l}, h_{2,l}, \cdots, h_{K,l}]^T$  is the channel vector between the *l*-th source and all relay nodes, and  $(\cdot)^T$  stands for matrix or vector transpose.

The k-th relay multiplies its received signal by a complex coefficient  $f_k$  and transmits the amplitude- and phase-adjusted version of its received signal. Thus the  $K \times 1$  vector  $\mathbf{x}_r$  of the signals transmitted by all the relay nodes is given by

$$\mathbf{x}_r = \mathbf{F} \mathbf{y}_r \tag{2}$$

where  $\mathbf{F} \triangleq \operatorname{diag}(\mathbf{f})$  is the  $K \times K$  diagonal relay amplifying matrix with  $\mathbf{f} \triangleq [f_1, f_2, \cdots, f_K]^T$ . Here  $\operatorname{diag}(\mathbf{f})$  forms a diagonal matrix with the vector  $\mathbf{f}$  as the main diagonal and zeros elsewhere, whereas  $\operatorname{diag}(\mathbf{F})$  forms a vector with the main diagonal elements of the matrix  $\mathbf{F}$ . The received signal at the *l*-th destination node is obtained as the weighted sum of the received signals at each antenna element of that node, and is given by

$$y_{d,l} = \mathbf{w}_{l}^{H} \left( \mathbf{G}_{l} \mathbf{F} \mathbf{y}_{r} + \mathbf{n}_{d,l} \right)$$
$$= \mathbf{w}_{l}^{H} \left( \mathbf{G}_{l} \mathbf{F} \sum_{k=1}^{L} \mathbf{h}_{k} s_{k} + \mathbf{G}_{l} \mathbf{F} \mathbf{n}_{r} + \mathbf{n}_{d,l} \right)$$
(3)

where  $\mathbf{G}_l$  is the  $N_l \times K$  channel matrix between the relays and the *l*-th destination node,  $\mathbf{w}_l$  and  $\mathbf{n}_{d,l}$  are the beamforming weight vector and the additive Gaussian noise vector at the *l*-th destination node, respectively, and  $(\cdot)^H$  denotes matrix or vector Hermitian transpose. We assume that all noises are complex circularly symmetric with zero mean and variance  $\sigma_n^2$ .

From (3), the power of the received signal at the l-th destination node is given by

$$E\{y_{d,l}y_{d,l}^{*}\} = \sum_{k=1}^{L} p_{k}\mathbf{w}_{l}^{H}\mathbf{G}_{l}\mathbf{F}\mathbf{h}_{k}\mathbf{h}_{k}^{H}\mathbf{F}^{H}\mathbf{G}_{l}^{H}\mathbf{w}_{l} + \sigma_{n}^{2}\mathbf{w}_{l}^{H}\mathbf{G}_{l}\mathbf{F}\mathbf{F}^{H}\mathbf{G}_{l}^{H}\mathbf{w}_{l} + \sigma_{n}^{2}\mathbf{w}_{l}^{H}\mathbf{w}_{l}$$
(4)

$$=\sum_{k=1}^{L}p_{k}\mathbf{w}_{l}^{H}\boldsymbol{\psi}_{kl}\boldsymbol{\psi}_{kl}^{H}\mathbf{w}_{l}+\mathbf{w}_{l}^{H}\mathbf{C}_{l}\mathbf{w}_{l}$$
(5)

where  $(\cdot)^*$  denotes complex conjugate. Here we assume that  $E\{|s_l|^2\} = p_l, \psi_{kl} \triangleq \mathbf{G}_l \mathbf{F} \mathbf{h}_k$  is the equivalent vector channel response between the *k*th source node and the *l*-th destination node,  $\mathbf{C}_l \triangleq \sigma_n^2(\mathbf{G}_l \mathbf{F} \mathbf{F}^H \mathbf{G}_l^H + \mathbf{I}_{N_l})$  is the covariance matrix of the equivalent noise at the *l*-th receiver. Thus the SINR at the *l*-th destination node is given by

$$\Gamma_{l} = \frac{p_{l} \mathbf{w}_{l}^{H} \boldsymbol{\psi}_{ll} \boldsymbol{\psi}_{ll}^{H} \mathbf{w}_{l}}{\sum_{k \neq l}^{L} p_{k} \mathbf{w}_{l}^{H} \boldsymbol{\psi}_{kl} \boldsymbol{\psi}_{kl}^{H} \mathbf{w}_{l} + \mathbf{w}_{l}^{H} \mathbf{C}_{l} \mathbf{w}_{l}}.$$
(6)

Using (2), the transmit power consumed by all relay nodes can be expressed as

$$P_r = \operatorname{tr}(\mathrm{E}\{\mathbf{x}_r \mathbf{x}_r^H\}) = \operatorname{tr}(\mathbf{F}\mathbf{R}_y \mathbf{F}^H) = \mathbf{f}^T \mathbf{D}_y \mathbf{f}^*$$

where  $\operatorname{tr}(\cdot)$  denotes matrix trace,  $\mathbf{R}_{y} \triangleq \operatorname{E}\{\mathbf{y}_{r}\mathbf{y}_{r}^{H}\} = \sum_{l=1}^{L} p_{l}\mathbf{h}_{l}\mathbf{h}_{l}^{H} + \sigma_{n}^{2}\mathbf{I}_{K}$ , and  $\mathbf{D}_{y}$  is a diagonal matrix taking the main diagonal elements from  $\mathbf{R}_{y}$ . Here  $\mathbf{I}_{n}$  is an  $n \times n$  identity matrix. The total transmit power consumed by the whole network can be expressed as

$$P_T = P_r + \sum_{l=1}^{L} p_l = \mathbf{f}^T \mathbf{D}_y \mathbf{f}^* + \sum_{l=1}^{L} p_l.$$
(7)

## III. JOINT POWER CONTROL AND BEAMFORMING

In this section, we design the optimal transmit power vector  $\mathbf{p} \triangleq [p_1, p_2, \cdots, p_L]^T$ , the relay beamforming vector  $\mathbf{f}$  and receive beamforming vectors  $\mathbf{w}_l, l = 1, \cdots, L$ , such that a target SINR threshold  $\gamma_l, l = 1, \cdots, L$ , is maintained at the destination nodes with the minimal  $P_T$ . The optimization problem can be written as

$$\min_{\mathbf{p},\mathbf{f},\{\mathbf{w}_l\}} P_T \tag{8}$$

s.t. 
$$\Gamma_l \ge \gamma_l, \qquad l = 1, \cdots, L$$
 (9)

where  $\{\mathbf{w}_l\} \triangleq \{\mathbf{w}_l, l = 1, \cdots, L\}.$ 

# A. Receive Beamforming

The optimal  $\mathbf{w}_l$ ,  $l = 1, \dots, L$ , for fixed  $\mathbf{p}$  and  $\mathbf{f}$  can be obtained such that it minimizes the sum of noise and interference at the receiver under the condition of constant gain for the user of interest, which can be written as

$$\min_{\mathbf{w}_l} \sum_{k \neq l}^{L} p_k \mathbf{w}_l^H \boldsymbol{\psi}_{kl} \boldsymbol{\psi}_{kl}^H \mathbf{w}_l + \mathbf{w}_l^H \mathbf{C}_l \mathbf{w}_l$$
(10)

s.t. 
$$\mathbf{w}_l^H \boldsymbol{\psi}_{ll} = 1.$$
 (11)

Using the Lagrangian multiplier method, the solution to the problem (10)-(11) is given by

$$\mathbf{w}_{l} = \frac{\boldsymbol{\Phi}_{l}^{-1} \boldsymbol{\psi}_{ll}}{\boldsymbol{\psi}_{ll}^{H} \boldsymbol{\Phi}_{l}^{-1} \boldsymbol{\psi}_{ll}}$$
(12)

where  $\mathbf{\Phi}_l \triangleq \sum_{k \neq l}^{L} p_k \psi_{kl} \psi_{kl}^{H} + \mathbf{C}_l$ , and  $(\cdot)^{-1}$  denotes matrix inversion.

## B. Optimal Transmit Power Allocation

To obtain optimal **p** with given beamforming vectors **f** and  $\mathbf{w}_l, l = 1, \dots, L$ , we reformulate the problem (8)-(9) as

$$\min_{\mathbf{p}} \quad \sum_{l=1}^{L} p_l + c \tag{13}$$

s.t. 
$$\frac{p_l[\mathbf{H}]_{l,l}}{\sum_{k\neq l}^L p_k[\mathbf{H}]_{k,l} + \bar{n}_l} \ge \gamma_l, \quad l = 1, \cdots, L \quad (14)$$

where  $c \triangleq \mathbf{f}^T \mathbf{D}_y \mathbf{f}^*$ , **H** is an  $L \times L$  covariance matrix such that  $[\mathbf{H}]_{k,l} = \mathbf{w}_l^H \psi_{kl} \psi_{kl}^H \mathbf{w}_l$  and  $\bar{n}_l = \mathbf{w}_l^H \mathbf{C}_l \mathbf{w}_l$ . In an optimal power allocation, the transmit power of each user is set to the minimum required level such that the target SINR is just met. Thus the optimal power solution to the problem (13)-(14) is given by

$$\mathbf{p} = (\mathbf{I}_L - \mathbf{H})^{-1}\mathbf{u} \tag{15}$$

where  $[\tilde{\mathbf{H}}]_{l,k} = \begin{cases} 0, & k = l \\ \gamma_l[\mathbf{H}]_{k,l}/[\mathbf{H}]_{l,l}, & k \neq l \end{cases}$ , and **u** is an  $L \times 1$  vector whose *l*-th element is given by  $\gamma_l \bar{n}_l/[\mathbf{H}]_{l,l}, l = 1, \cdots, L$ .

## C. Relay Beamforming

Equation (4) can be rewritten as

$$E\{y_{d,l}y_{d,l}^*\} = \sum_{k=1}^{L} p_k \mathbf{f}^T \operatorname{diag}(\mathbf{w}_l^H \mathbf{G}_l) \mathbf{h}_k \mathbf{h}_k^H \operatorname{diag}(\mathbf{G}_l^H \mathbf{w}_l) \mathbf{f}^* \\ + \sigma_n^2 \mathbf{f}^T \operatorname{diag}(\mathbf{w}_l^H \mathbf{G}_l) \operatorname{diag}(\mathbf{G}_l^H \mathbf{w}_l) \mathbf{f}^* + \sigma_n^2 \mathbf{w}_l^H \mathbf{w}_l.$$

Thus the desired signal power of the l-th link can be expressed as

$$P_{s,l} = p_l \mathbf{f}^T \operatorname{diag}(\mathbf{w}_l^H \mathbf{G}_l) \mathbf{h}_l \mathbf{h}_l^H \operatorname{diag}(\mathbf{G}_l^H \mathbf{w}_l) \mathbf{f}^* = \mathbf{f}^T \mathbf{R}_l \mathbf{f}^*$$

where  $\mathbf{R}_l \triangleq p_l \operatorname{diag}(\mathbf{w}_l^H \mathbf{G}_l) \mathbf{h}_l \mathbf{h}_l^H \operatorname{diag}(\mathbf{G}_l^H \mathbf{w}_l)$ . The interference power of the *l*-th link is given by

$$P_{in,l} = \sum_{k \neq l}^{L} p_k \mathbf{f}^T \operatorname{diag}(\mathbf{w}_l^H \mathbf{G}_l) \mathbf{h}_k \mathbf{h}_k^H \operatorname{diag}(\mathbf{G}_l^H \mathbf{w}_l) \mathbf{f}^*$$
$$= \mathbf{f}^T \mathbf{Q}_l \mathbf{f}^*$$

where  $\mathbf{Q}_l \triangleq \sum_{k \neq l}^{L} p_k \operatorname{diag}(\mathbf{w}_l^H \mathbf{G}_l) \mathbf{h}_k \mathbf{h}_k^H \operatorname{diag}(\mathbf{G}_l^H \mathbf{w}_l)$ . Finally, the total noise power at the *l*-th destination node is

$$P_{n,l} = \sigma_n^2 \mathbf{f}^T \operatorname{diag}(\mathbf{w}_l^H \mathbf{G}_l) \operatorname{diag}(\mathbf{G}_l^H \mathbf{w}_l) \mathbf{f}^* + \sigma_n^2 \mathbf{w}_l^H \mathbf{w}_l$$
$$= \mathbf{f}^T \mathbf{D}_l \mathbf{f}^* + \sigma_n^2 \mathbf{w}_l^H \mathbf{w}_l$$

where  $\mathbf{D}_l \triangleq \sigma_n^2 \operatorname{diag}(\mathbf{w}_l^H \mathbf{G}_l) \operatorname{diag}(\mathbf{G}_l^H \mathbf{w}_l)$ . And the SINR  $\Gamma_l$  in (6) can be expressed as

$$\Gamma_l = \frac{P_{s,l}}{P_{in,l} + P_{n,l}} = \frac{\mathbf{f}^T \mathbf{R}_l \mathbf{f}^*}{\mathbf{f}^T (\mathbf{Q}_l + \mathbf{D}_l) \mathbf{f}^* + \sigma_n^2 \mathbf{w}_l^H \mathbf{w}_l}$$

With given  $\mathbf{p}$  and  $\mathbf{w}_l, l = 1, \dots, L$ , the problem (8)-(9) can be formulated as an SDP problem by introducing  $\mathbf{X} = \mathbf{f}^* \mathbf{f}^T$ and is given by

$$\min_{\mathbf{X}} \quad \mathrm{tr}(\mathbf{D}_{y}\mathbf{X}) \tag{16}$$

s.t. 
$$\operatorname{tr}(\mathbf{T}_{l}\mathbf{X}) \geq \gamma_{l}\sigma_{n}^{2}\mathbf{w}_{l}^{H}\mathbf{w}_{l}, \quad l = 1, \cdots, L$$
 (17)

$$\mathbf{X} \succeq \mathbf{0} \tag{18}$$

$$\operatorname{rank}(\mathbf{X}) = 1 \tag{19}$$

where  $\mathbf{T}_l \triangleq \mathbf{R}_l - \gamma_l(\mathbf{Q}_l + \mathbf{D}_l)$ ,  $\mathbf{X} \succeq 0$  means that  $\mathbf{X}$  is a positive semidefinite (PSD) matrix, and rank(·) denotes the rank of a matrix. Note that, in the problem (16)-(19), the cost function is linear in  $\mathbf{X}$ , the trace constraints are linear inequalities in  $\mathbf{X}$ , and the PSD matrix constraint is convex. However, the rank constraint on  $\mathbf{X}$  is not convex. Interestingly, the problem (16)-(19) can be solved by the semidefinite relaxation technique [11] as explained in the following. First we drop the rank constraint (19) to obtain the following relaxed SDP problem which is convex in  $\mathbf{X}$ .

$$\min_{\mathbf{X}} \quad \mathrm{tr}(\mathbf{D}_{y}\mathbf{X}) \tag{20}$$

s.t. 
$$\operatorname{tr}(\mathbf{T}_{l}\mathbf{X}) \geq \gamma_{l}\sigma_{n}^{2}\mathbf{w}_{l}^{H}\mathbf{w}_{l}, \quad l = 1, \cdots, L$$
 (21)

$$\mathbf{X} \succeq \mathbf{0}. \tag{22}$$

SDP problems like (20)-(22) can be conveniently solved by using interior point methods at a complexity order that is at most  $O((L + K^2)^{3.5})$  [12]. We use CVX MATLAB toolbox for disciplined convex programming [13] to obtain the optimal X. Due to the relaxation, the matrix X obtained by solving the SDP problem will not necessarily be rank one in general. If it is, then its principal eigenvector will be the optimal solution to the original problem. Otherwise, we have to use alternative techniques such as randomization [11], [14], [15] to obtain a (suboptimal) rank-one solution from X. Different randomization techniques have been studied in the literature [11], [12], [14], [15]. The one we choose can be summarized as in Table I. Note that when  $rank(\mathbf{X}) > 1$ , at least one of the constraints in (9) will be violated after the randomization operation. However, a feasible relay beamforming vector can be obtained by simply scaling f so that all the constraints are satisfied.

#### TABLE I RANDOMIZATION TECHNIQUE FOR SEMIDEFINITE RELAXATION APPROACH

- 1) Let  $\mathbf{X} = \mathbf{U} \mathbf{\Sigma} \mathbf{U}^H$  be the eigenvalue decomposition of  $\mathbf{X}$ .
- Choose a  $K \times 1$  random vector **v** whose elements are independent random variables, uniformly distributed on the unit circle in the complex plane, i.e.,  $[\mathbf{v}]_k = e^{j\theta_k}, k = 1, \dots, K$ , where  $\theta_k$  is independent and uniformly distributed on  $[0, 2\pi)$ .
- 3) Choose  $\mathbf{f} = \mathbf{U} \mathbf{\Sigma}^{1/2} \mathbf{v}$  which ensures that  $\mathbf{f}^H \mathbf{f} = \operatorname{tr}(\mathbf{X})$ .

Now the original total transmit power optimization problem (8)-(9) can be solved by an iterative algorithm as shown in Table II. Here  $\varepsilon$  is a small positive number close to zero up to which convergence is acceptable and the superscript (n)denotes the number of iterations. It can be easily shown, as in [3], that starting with random  $\mathbf{p}^{(0)}$  and  $\mathbf{f}^{(0)}$ , the algorithm in Table II converges to the optimal solution.



- 1) Initialize the algorithm with an arbitrary power vector  $\mathbf{p}^{(0)}$  and a randomly generated relay beamforming vector  $\mathbf{f}^{(0)}$ ; Set n = 0. Solve the subproblem (10)-(11) using given  $\mathbf{p}^{(n)}$ , and  $\mathbf{f}^{(n)}$  to obtain
- 2)  $\mathbf{w}_{l}^{(n)}, l = 1, \cdots, L$ , as in (12).
- 3) Solve the subproblem (13)-(14) with fixed  $\mathbf{f}^{(n)}$  and  $\mathbf{w}_{l}^{(n)}$  to obtain power vector  $\mathbf{p}^{(n+1)}$  as in (15).
- Solve the subproblem (20)-(22) using known  $\mathbf{w}_{l}^{(n)}$ ,  $l = 1, \dots, L$ , and 4)  $\mathbf{p}^{(n+1)}$  to obtain **X**.
  - a) Use the randomization technique in Table I to obtain f.
  - Find the most violated constraint in the original problem (8)-(9) b) using such f.
  - Scale f so that the most violated constraint is satisfied with c) equality to obtain  $\mathbf{f}^{(n+1)}$ .
- 5) If max  $|\mathbf{p}^{(n+1)} \mathbf{p}^{(n)}| \le \varepsilon$ , then end. Otherwise, let n := n + 1 and go to step 2.

#### **IV. NUMERICAL EXAMPLES**

In this section, we study the performance of the proposed joint power control and beamforming algorithm for an interference MIMO relay channel through numerical simulations.



Fig. 2. Total power versus target SINR.  $\sigma_h^2 = 15$ ,  $\sigma_q^2 = 10$ , K = 20.

The source nodes and the relay nodes are equipped with a single antenna while destination nodes are all equipped with  $N_l = N_d$ ,  $l = 1, \dots, L$ , antennas. In each simulation, the channel matrices have entries generated as i.i.d. complex Gaussian random variables with zero mean and variances  $\sigma_h^2$  and  $\sigma_g^2$  for  $\mathbf{h}_l$  and  $\mathbf{G}_l$ ,  $l = 1, \cdots, L$ , respectively. For simplicity, we assume  $\gamma_l = \gamma$ ,  $l = 1, \cdots, L$ , and  $\sigma_n^2 = 1$  in all simulations. All simulation results are averaged over 200 independent channel realizations.

For the proposed algorithm, the procedure in Table II is carried out in each simulation to obtain the optimal power vector **p**, relay beamforming vector **f**, and receive beamforming vectors  $\mathbf{w}_l, l = 1, \cdots, L$ . To initialize the algorithm in Table II, we randomly generate the relay beamforming vector f with an arbitrary transmit power vector p.

In the first example, we compare the performance of the proposed joint optimal algorithm with optimal power control (WPC) with the relay-only optimal algorithm with no power control (NPC) proposed in [10]. For the NPC scheme, we used the optimal **f** and  $\mathbf{w}_l, l = 1, \dots, L$ , from the first iteration and the initial  $\mathbf{p}^{(0)}$  of the joint optimal algorithm. That is, in the NPC scheme it is assumed that the *l*th source uses its maximum available transmit power  $p_l^{(0)}$ ,  $l = 1, \dots, L$ . We compare the performance of these two algorithms for two different network setups namely,  $N_d = 4, L = 4$  and  $N_d = 6, L = 3$ . For both configurations, we consider  $\sigma_h^2 = 15$ ,  $\sigma_q^2 = 10$  and K = 20. We plot the total power consumed by all source nodes and relay nodes versus the target SINR threshold  $\gamma$  (dB). Fig. 2 shows the performance of both algorithms. It can be seen from Fig. 2 that for both network setups the proposed jointly optimal algorithm requires much less total power compared with the relay only optimal algorithm proposed in [10].

In the next example, we study the performance of the proposed algorithm for different number of relays K with  $N_d = 4, L = 3, \sigma_h^2 = 15$ , and  $\sigma_g^2 = 10$ . The total power required for K = 10, 20, and 30 versus  $\gamma$  (dB) are displayed



Fig. 3. Total power versus target SINR. L = 3,  $N_d = 4$ ,  $\sigma_h^2 = 15$ ,  $\sigma_g^2 = 10$ .



Fig. 4. Total power versus target SINR for different channel conditions.  $L=3,\ N_d=4$  and K=20.

in Fig. 3. As is expected, if we increase the number of relays the proposed algorithm requires less power since more relays provide more spatial diversity.

In the third example, we study the impact of channel quality on the proposed algorithm. We assume that a higher variance of channel coefficients indicates a better channel. The impact of different  $\sigma_h^2$  and  $\sigma_g^2$  on the proposed algorithm is shown in Fig. 4 for L = 3,  $N_d = 4$ , and K = 20. The results clearly demonstrate that the proposed algorithm performs better as the channel quality improves.

In the last example, we study the effect of channel interferences on the proposed algorithm. By increasing the number of source-destination pairs L, the interfering signal received at each destination node is also increased. The performance of the algorithm for different L is illustrated in Fig. 5. From this figure it is clear that if there are more active users communicating simultaneously in the system, we need more power to achieve the same target SINR threshold,  $\gamma$ .



Fig. 5. Total power versus target SINR for different number of users.  $\sigma_h^2 = 15, \sigma_q^2 = 20, N_d = 4.$ 

#### V. CONCLUSIONS

We considered a two-hop interference MIMO relay system with distributed relay nodes and developed an iterative technique to minimize the total transmit power consumed by all source and relay nodes such that a minimum SINR threshold is maintained at each receiver. The proposed algorithm exploits beamforming techniques at the relay nodes and the destination nodes in conjunction with transmit power control. Simulation results demonstrate that the jointly optimal power control and beamforming algorithm outperforms the existing techniques.

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