# A Polarization MDCSK Modulation Without PDL Over Multipath Rayleigh Fading Channels 

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#### Abstract

Traditional polarization modulations suffer from polarization dependent loss (PDL) which affects the the bit error rate (BER) performance. To solve this problem, a polarization $M$-ary differential chaos shift keying ( P -MDCSK) modulation is proposed rather than the conventional MDCSK in this paper. In the proposed scheme, the reference signals are transmitted with polarization states first, and then the information bearing signals are modulated by selecting both phase and polarization states. The union bound of BER expression of the proposed scheme is derived and verified by simulations over both additive white Gaussian noise (AWGN) and multipath Rayleigh fading channels. Results show that the BER performance of the proposed scheme is better than that of other corresponding traditional schemes with a low computational complexity.


Index Terms-Polarization modulation, $M$-ary differential chaos shift keying modulation(MDCSK), bit error rate (BER).

## I. Introduction

Chaotic communications have attracted much attention in the spread spectrum communication community due to the low cross correlation values, excellent auto-correlation and wideband spectrum of the chaotic signals. A non-coherent modulation called differential chaos shift keying (DCSK) is proposed to avoid chaotic sequence recovery [1]. DCSK also owns advantage against severe multipath fading as spreading spectrum systems [2]. Furthermore, frequency-modulated DCSK (FM-DCSK) is proposed with a constant power in one signal period [3]. However, DCSK suffers from some drawbacks such as high energy consumption and low spectral efficiency. To improve DCSK spectral efficiency with a single carrier, quadrature chaos shift keying (QCSK) is proposed in [4]. An M-ary DCSK generalized from QCSK with M-ary phase-shift-keying (MPSK) constellation is proposed in [5]. Furthermore, a square constellation based M-ary DCSK is proposed in [6]. In [5] multiresolution M-ary differential chaos shift keying (MRMDCSK) using non-uniform plane phase constellation is a promising technique that can satisfy the different BER requirements within one symbol.

Polarization modulations suffer from polarization dependent loss (PDL) which is caused by frequency selective fading channels [7]. Although some works [8] have been done to solve this problem to some extent, they increase the system hardware/software complexity. As a good candidate for low-cost spread-spectrum communications, DCSK has a good performance over frequency selective fading channels without channel estimation [9], and MDCSK inherits this characteristic.

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Fig. 1. Transmitter of P-MDCSK.


Fig. 2. The $M$-ary polarization constellation (a) $M=4$, (b) $M=8$, (c) $M=16$.

Inspired by this, we propose a polarization MDCSK as a simple and low-cost scheme.

In the proposed scheme, the constellation mapping is implemented by polarization states and constellation phases. The polarized reference chaotic signals are transmitted first, then the information bearing chaotic signals are modulated by selecting both phase and polarization state. Furthermore, the theoretical bounds of BER performance of the proposed system are analyzed over both additive white Gaussian noise (AWGN) and multipath Rayleigh fading channels. The results reveal that the polarization $M$-ary differential chaos shift keying (P-MDCSK) system outperforms the other corresponding systems.

The remainder of this paper is organized as follows. Section II presents the model of a polarization MDCSK system, and explains the relevant parameters. The energy efficiency and the union bounds of BER expressions of the proposed system are derived in Section III. Simulation results are presented and discussed in Section IV. Section V concludes the paper.

## II. System Model

A P-MDCSK constellation is characterized by the horizontal polarized state, vertical polarized state and phase. The block diagram of the transmitter of a P-MDCSK system is shown in Fig. 1, where $\mathbf{c}_{\mathrm{x}}$ is the copy of reference chaos signal, and the quadrature signal $\mathbf{c}_{\mathbf{y}}$ is the Hilbert transform of $\mathbf{c}_{\mathbf{x}}$. The information bit sequence consists of polarization component and phase component. We are able to convey $m_{c}=l_{b}+n_{b}$ bits in total by packing $l_{b}$ bits on the sphere surface and $n_{b}$ bits on the MDCSK phase. Thus, $L_{b}=2^{l_{b}}$ symbols are located on the sphere and $N_{b}=2^{n_{b}}$ symbols are with the MDCSK constellation which are shown on Fig. 2 for $M=4,8,16$-ary P-MDCSK, where $M=2^{m_{c}}$.

At the transmitter, linear polarization is selected to transmit the signal. The polarization states are represented in the form of Jones


Fig. 3. Receiver of P-MDCSK.
vector based on Stokes parameters [10]

$$
\mathbf{E}_{\mathbf{0}}=\left[\begin{array}{c}
E_{h}  \tag{1}\\
E_{v}
\end{array}\right]=\left[\begin{array}{c}
\cos \varphi \cdot e^{j \epsilon_{h}} \\
\sin \varphi \cdot e^{j \epsilon_{v}}
\end{array}\right]
$$

where $\varphi$ denotes the polarized angle, $\epsilon_{h}$ and $\epsilon_{v}$ are the phase of the signal transmitted with horizontal and vertical polarized state, respectively. Linear polarization is used in the paper, i.e., $\vartheta=\epsilon_{h}-\epsilon_{v}=0$ and $\varphi=0, \pi / 2[11]$, thus $\left|E_{h}\right|^{2}+\left|E_{v}\right|^{2}=1$. Assuming that $L_{b}=2$ symbols are located on the sphere and $N_{b}$ symbols are with the MDCSK constellation, the information-bearing transmitted signal as a function of Stokes parameters is described as

$$
\mathbf{S}_{\mathbf{0}}=\mathbf{E}_{\mathbf{0}} \mathbf{s}_{\mathbf{x}}=\left[\begin{array}{c}
E_{h}  \tag{2}\\
E_{v}
\end{array}\right] \mathbf{s}_{\mathbf{x}}=\left[\begin{array}{c}
\cos \varphi \cdot e^{j \epsilon_{h}} \\
\sin \varphi \cdot e^{j \epsilon_{v}}
\end{array}\right] \mathbf{s}_{\mathbf{x}}
$$

where $\mathbf{s}_{\mathbf{x}}$ is the symbol with MDCSK constellation modulated in phase, which can be expressed as

$$
\begin{equation*}
\mathbf{s}_{\mathbf{x}}=\left[\mathbf{s}_{\mathbf{r e f}}, \mathbf{s}_{\mathbf{i n f}}\right]=[\underbrace{\mathbf{c}_{\mathbf{x}}}_{\text {reference }}, \underbrace{\cos \theta \mathbf{c}_{\mathbf{x}}+\sin \theta \mathbf{c}_{\mathbf{y}}}_{\text {information-bearing }}] \tag{3}
\end{equation*}
$$

where $\mathbf{c}_{\mathbf{x}}=\left[c_{x, 1}, c_{x, 2}, \ldots, c_{x, i}, \ldots, c_{x, \beta}\right]$ is $\beta$-length chaotic signal, $\mathbf{c}_{\mathbf{y}}$ is the Hilbert transform of $\mathbf{c}_{\mathbf{x}}$, and $\theta$ is the phase of MDCSK modulation. It is worth mentioning that the $\mathbf{s}_{\mathbf{x}}$ component does not affect the computation of Stokes parameters, as it is independent of the polarization modulation.

Assuming the modulated symbols are transmitted over multipath Rayleigh fading channel, thus the received signal is written as

$$
\begin{align*}
& \mathbf{r}_{\mathbf{h}}=E_{h} \mathbf{s}_{\mathbf{x}} \otimes \mathbf{h}_{\mathbf{h}, \mathbf{h}}+E_{v} \mathbf{s}_{\mathbf{x}} \otimes \mathbf{h}_{\mathbf{h}, \mathbf{v}}+\mathbf{n}_{\mathbf{h}} \\
& \mathbf{r}_{\mathbf{v}}=E_{v} \mathbf{s}_{\mathbf{x}} \otimes \mathbf{h}_{\mathbf{v}, \mathbf{v}}+E_{h} \mathbf{s}_{\mathbf{x}} \otimes \mathbf{h}_{\mathbf{v}, \mathbf{h}}+\mathbf{n}_{\mathbf{v}} \tag{4}
\end{align*}
$$

where $\mathbf{r}_{\mathbf{h}}$ and $\mathbf{r}_{\mathbf{v}}$ are the received signals in the horizontal and vertical polarized states, respectively, $\mathbf{r}_{\mathbf{h}}=\left[\mathbf{r}_{\mathbf{h}_{\mathbf{r e f}}}, \mathbf{r}_{\mathbf{h}_{\mathbf{i n f}}}\right], \mathbf{r}_{\mathbf{v}}=\left[\mathbf{r}_{\mathbf{v}_{\mathbf{r e f}}}, \mathbf{r}_{\mathbf{v}_{\mathbf{i n f}}}\right]$, $\mathbf{n}_{\mathbf{h}}$ and $\mathbf{n}_{\mathbf{v}}$ are the additive white Gaussian noise with zero mean and variance $N_{0} / 2, \otimes$ denotes the convolution operator.

Here $\mathbf{h}_{\mathbf{h}, \mathbf{v}}$ and $\mathbf{h}_{\mathbf{v}, \mathbf{h}}$ are the complex gain between input $h / v$ polarized component and the output $v / h$-polarized component. In order to minimize the complexity of the system model without loss of accuracy, we assume $\mathbf{h}_{\mathbf{h}, \mathbf{v}}=\mathbf{h}_{\mathbf{v}, \mathbf{h}}=0$ [11]. We also assume that $\mathbf{h}_{\mathbf{h}, \mathbf{h}}$ and $\mathbf{h}_{\mathbf{v}, \mathbf{v}}$ have the same parameters as $\mathbf{h}_{\mathbf{h}, \mathbf{h}}=\mathbf{h}_{\mathbf{v}, \mathbf{v}}=\sum_{l=1}^{L} \alpha_{l} \delta\left(t-\tau_{l}\right)$, with $L$ paths, are modeled by zero-mean complex-valued Gaussian processes, and the envelope of $\alpha_{l}$, i.e., $\left|\alpha_{l}\right|$, which is a Rayleigh distributed random variable. the $\tau_{l}$ is the path delay of the $l$ th path. Note that (4) becomes an AWGN channel if $L=1, \alpha_{l}=1$. It is noteworthy that an advantage of using MDCSK is that it does not affect the Stokes parameter. It can be seen from Fig. 3 that, the receiver decouples the signal into two parts: MDCSK and polarization states. Each part
is decoded by an independent receiver. For the polarization states, considering the character of differential modulation and polarization modulation, the maximum energy comparator is expressed as below, where the $l$ th polarization state on sphere is estimated by.

$$
\begin{equation*}
\hat{l}=\underset{l \in(h, v)}{\arg \max }\left(\left|D_{h}\right|,\left|D_{v}\right|\right) \tag{5}
\end{equation*}
$$

where $|\cdot|$ denotes the absolute value, $D_{h}$ and $D_{v}$ can be expressed as

$$
\begin{align*}
& D_{h}=\mathbf{r}_{\mathbf{h}_{\mathbf{r e f}}} \mathbf{r}_{\mathbf{h}_{\mathrm{inf}}}^{\mathbf{T}}+j \cdot \mathrm{H}\left(\mathbf{r}_{\mathbf{h}_{\mathbf{r e f}}}\right) \mathbf{r}_{\mathbf{h}_{\mathbf{i n f}}}^{\mathbf{T}} \\
& D_{v}=\mathbf{r}_{\mathbf{v}_{\mathbf{r e f}}} \mathbf{r}_{\mathbf{v}_{\mathbf{i n f}}}^{\mathbf{T}}+j \cdot \mathrm{H}\left(\mathbf{r}_{\mathbf{v}_{\mathbf{r e f}}}\right) \mathbf{r}_{\mathbf{v}_{\mathbf{i n f}}}^{\mathbf{T}} \tag{6}
\end{align*}
$$

where $H(\cdot)$ is the Hilbert transform. After $\hat{l}$ is determined, for the MDCSK part, the decision variables $z_{a}$ and $z_{b}$ are obtained as

$$
\begin{align*}
& z_{a}=\mathbf{r}_{\hat{1}_{\mathbf{r e f}}} \mathbf{r}_{\hat{\mathrm{I}}_{\mathrm{inf}}}^{\mathbf{T}} \\
& z_{b}=\mathrm{H}\left(\mathbf{r}_{\hat{1}_{\mathbf{r e f}}}\right) \mathbf{r}_{\hat{\mathrm{i}}_{\mathrm{inf}}}^{\mathbf{T}} \tag{7}
\end{align*}
$$

where $\hat{l}$ is determined from $h$ and $v, \mathbf{r}_{\hat{1}_{\mathbf{r e f}}}$ is either $\mathbf{r}_{\mathbf{h}_{\mathbf{r e f}}}$ or $\mathbf{r}_{\mathbf{v}_{\mathbf{r e f}}}$, and $\mathbf{r}_{\hat{1}_{\mathbf{i n f}}}$ is either $\mathbf{r}_{\mathbf{h}_{\mathbf{i n f}}}$ or $\mathbf{r}_{\mathbf{v}_{\mathrm{inf}}}$, depending on $\mathbf{r}_{\hat{1}}$. Then, the phase of MDCSK is decided by $z_{a}$ and $z_{b}$. The corresponding phase $\operatorname{arccot}\left(z_{a} / z_{b}\right)$ and the decision boundaries are used for recovering the corresponding phase parts of information bits [5]. It is important to remark that the MDCSK estimation depends on the estimation of $\hat{l}$.

## III. PERFORMANCE ANALYSIS OF P-MDCSK

In this section, union bounds of expressions of P-MDCSK system are derived over both AWGN and multipath fading channels. We assume that the fading is slow fading with constant coefficients, and the largest multipath delay is much shorter than a symbol period so that the inter symbol interference is negligible [5].

## A. Decision of MDCSK

We convey $l_{b}+n_{b}$ bits in total, $n_{b}$ bits are mapped to constellation in phase which can be expressed as

$$
\begin{align*}
z_{a}=\Re & {\left[\sum_{i=1}^{\beta}\left(\sum_{l=1}^{L} \alpha_{l} E_{\hat{l}} c_{x, i-\tau_{l}}+n_{\hat{l}, i}\right)^{*}\right.} \\
& \left.\times\left(\sum_{l=1}^{L} \alpha_{l} E_{\hat{l}}\left(\cos \theta c_{x, i-\tau_{l}}+\sin \theta c_{y, i-\tau_{l}}\right)+n_{\hat{l}, i+\beta}\right)\right]  \tag{8}\\
z_{b}=\Re[ & \sum_{i=1}^{\beta}\left(\sum_{l=1}^{L} \alpha_{l} E_{\hat{l}} c_{y, i-\tau_{l}}+\widetilde{n_{\hat{l}, i}}\right)^{*} \\
& \left.\times\left(\sum_{l=1}^{L} \alpha_{l} E_{\hat{l}}\left(\cos \theta c_{x, i-\tau_{l}}+\sin \theta c_{y, i-\tau_{l}}\right)+n_{\hat{l}, i+\beta}\right)\right] \tag{9}
\end{align*}
$$

where $E_{\hat{l}}=\cos 0=\sin \pi / 2=1$ is the estimated polarization state selected from $E_{h}$ and $E_{v}$ as linear polarization. The $\Re$ is the real part, the $(\cdot)^{*}$ denotes the conjugate operator. $n_{\hat{l}, i}, n_{\hat{l}, i+\beta}$ and $\widetilde{n_{\hat{l}, i}}$ are independent complex Gaussian noises with zero mean and $N_{0} / 2$ power spectral density corresponding to estimated polarization state, here $n_{\hat{l}, i}$ is caused by reference signal, $\widetilde{n} \hat{l}, i^{\text {is caused by Hilbert transform of } n_{\hat{l}, i} \text {. For large }{ }^{\text {. }} \text {. }}$ spreading factor, the following approximated expression is used as [5]

$$
\begin{equation*}
\sum_{i=1}^{\beta} c_{y, i-\tau_{p}} c_{y, i-\tau_{q}} \approx 0(p \neq q) \tag{10}
\end{equation*}
$$

From (8) and (9), the means and the variances of $z_{a}$ and $z_{b}$ are approximated as following

$$
\begin{align*}
\mathrm{E}\left[z_{a}\right] & =\cos \theta \sum_{i=1}^{\beta} \sum_{l=1}^{L}\left|\alpha_{l}\right|^{2} E_{\hat{l}}^{2} c_{x, i}^{2}=\sum_{l=1}^{L}\left|\alpha_{l}\right|^{2} \cos \theta E_{c}  \tag{11}\\
\mathrm{E}\left[z_{b}\right] & =\sin \theta \sum_{i=1}^{\beta} \sum_{l=1}^{L}\left|\alpha_{l}\right|^{2} E_{\hat{\imath}}^{2} c_{x, i}^{2}=\sum_{l=1}^{L}\left|\alpha_{l}\right|^{2} \sin \theta E_{c}  \tag{12}\\
\delta^{2} & =\delta^{2}\left[z_{a}\right]=\delta^{2}\left[z_{b}\right]=\sum_{l=1}^{L}\left|\alpha_{l}\right|^{2} E_{c} N_{0}+\frac{N_{0}^{2} \beta}{4} \tag{13}
\end{align*}
$$

where $\quad E_{c}=E_{\hat{l}}^{2}\left(\sin ^{2} \theta \sum_{i=1}^{\beta} c_{x, i}^{2}+\cos ^{2} \theta \sum_{i=1}^{\beta} c_{y, i}^{2}\right)=\sum_{i=1}^{\beta} c_{x, i}^{2}$ $=\sum_{i=1}^{\beta} c_{y, i}^{2}=\frac{E_{k}}{2}$, and $E_{k}=2 E_{\hat{l}}^{2} \sum_{i=1}^{\beta} c_{x, i}^{2}=2\left(\left|E_{h}\right|^{2}+\left|E_{v}\right|^{2}\right)$ $\sum_{i=1}^{\beta} c_{x, i}^{2}$ is the total energy of one symbol. We can see that $z_{a}$ and $z_{b}$ are two independent normal random variables with two different means.

## B. System BER Analysis

The instantaneous symbol error rate (SER) is described in [8]. Gray mapping is used because it produces the best BER performance, which maximizes the Hamming distance. Thus, the BER and the union bound can be described as following which is based on [10]

$$
\begin{equation*}
P_{P-M D C S K} \leq P_{\text {signal }}+P_{\text {polarization }}+P_{\text {joint }} \tag{14}
\end{equation*}
$$

where
$P_{\text {signal }}=\frac{n_{b}}{l_{b}+n_{b}} P_{M D C S K}$,
$P_{\text {polarization }}=\frac{1}{L_{b}\left(l_{b}+n_{b}\right)} \sum_{l_{s}=1}^{L_{b}} \sum_{l_{n}=1}^{L_{b}} D\left(l_{n} \rightarrow l_{s}\right) Q\left(\sqrt{\frac{d_{l_{n}, l_{s}}^{2}}{2 N_{0}}}\right)$,
$P_{\text {joint }}=\frac{1}{L_{b} N_{b}\left(l_{b}+n_{b}\right)}$
$\times \sum_{l_{s}=1}^{L_{b}} \sum_{n_{s}=1}^{N_{b}} \sum_{l_{n}=1}^{L_{b}} \sum_{n_{n}=1}^{N_{b}}\left(D\left(l_{n} \rightarrow l_{s}\right)+D\left(n_{n} \rightarrow n_{s}\right)\right) Q$

$$
\begin{equation*}
\times\left(\sqrt{\frac{d_{l_{n}, l_{s}, n_{n}, n_{s}}^{2}}{2 N_{0}}}\right) \tag{15}
\end{equation*}
$$

where $D\left(l_{n} \rightarrow l_{s}\right)$ is the Hamming distance,i.e., the number of different bits between symbols defined by $l_{n}$ and $l_{s}$, the same with $D\left(n_{n} \rightarrow n_{s}\right)$. $l_{n}$ and $n_{n}$ denote the wrong symbols defined. The average BER of MDCSK can be calculated by union probability density function (PDF) with means and variance in (11), (12) and (13),

$$
\begin{equation*}
p\left(z_{a}, z_{b}\right)=\frac{1}{\sqrt{2 \pi \delta^{2}}} \exp \left(-\frac{\left(z_{a}-E\left[z_{a}\right]\right)^{2}+\left(z_{b}-E\left[z_{b}\right]\right)^{2}}{2 \delta^{2}}\right) \tag{16}
\end{equation*}
$$

Then it can be deduced detailed from [5] as

$$
\begin{align*}
P_{M D C S K}=\frac{2}{n_{b}} \int_{\pi / N_{b}}^{\pi} & \left(\frac{\exp \left(-\frac{\rho^{2}}{8}\right)}{2 \pi}+\exp \left(-\frac{\rho^{2} \sin ^{2} \psi}{8}\right)\right. \\
& \left.\times \frac{\rho \cos \frac{\psi}{2}}{\sqrt{2 \pi}} Q\left(-\frac{\rho \cos \psi}{2}\right)\right) \mathrm{d} \psi \tag{17}
\end{align*}
$$

where $\rho=\sum_{l=1}^{L}\left|\alpha_{l}\right|^{2} 2 E_{c} / \delta=\sum_{l=1}^{L}\left|\alpha_{l}\right|^{2} E_{k} / \delta=2 \gamma_{s} / \sqrt{2 \gamma_{s}+\beta}$, $\psi$ is the phase error between the transmitted and received signals,
$\gamma_{s}=\sum_{l=1}^{L}\left|\alpha_{l}\right|^{2} E_{k} / N_{0}$ and $Q(x)=\frac{1}{\sqrt{2 \pi}} \int_{x}^{\infty} e^{-\frac{t^{2}}{2}} d t$, for $x \geq 0$. The generic distance $d_{l_{n}, l_{s}, n_{n}, n_{s}}$ in the Euclidean space is expressed with (2) as

$$
\begin{align*}
d_{l_{n}, l_{s}, n_{n}, n_{s}}^{2}=2 E_{k}( & -\left(\cos \left(\Delta \varepsilon-\frac{\Delta \vartheta}{2}\right) \cos \frac{\varphi_{l_{n}}}{2} \cos \frac{\varphi_{l_{s}}}{2}\right. \\
& \left.\left.+\cos \left(\Delta \varepsilon+\frac{\Delta \vartheta}{2}\right) \sin \frac{\varphi_{l_{n}}}{2} \sin \frac{\varphi_{l_{s}}}{2}\right)\right), \tag{18}
\end{align*}
$$

where $\Delta \varepsilon=\varepsilon_{n_{n}}-\varepsilon_{n_{s}}, \varepsilon_{n_{s}}$ is the MDCSK component of symbol $n$, $\Delta \vartheta=\vartheta_{l_{n}}-\vartheta_{l_{s}},\left(\vartheta_{l_{s}}, \varphi_{l_{s}}\right)$ is the polarization of the $l$ symbol. Note that if both symbols have the same MDCSK component, i.e., $\Delta \varepsilon=0$, then, (18) is reduced to

$$
\begin{equation*}
d_{l_{n}, l_{s}}^{2}=2 E_{k}\left(1-\left(\cos \left(\frac{\Delta \vartheta}{2}\right) \cos \left(\frac{\Delta \varphi}{2}\right)\right)\right) \tag{19}
\end{equation*}
$$

where $\Delta \varphi=\varphi_{l_{n}}-\varphi_{l_{s}}$. Finally, substituting (15)- (19) into (14), we can get the total system BER of P-MDCSK over AWGN channel at $L=1, \alpha_{l}=1$.

For Rayleigh multipath fading channel case, $\alpha_{l}$ and $\tau_{l}$ are the channel coefficients and the path delay of the $l$ th path. $\alpha_{l}$ are assumed to be independent Rayleigh distributed random variables. The total system BER of the proposed scheme over Rayleigh fading channel is given by

$$
\begin{equation*}
P_{\text {multi }} \approx \int_{0}^{\infty} P_{P-M D C S K} f\left(\gamma_{b}\right) \mathrm{d} \gamma_{b} . \tag{20}
\end{equation*}
$$

where $f\left(\gamma_{b}\right)$ is the PDF of $\gamma_{b}$ which can be found in [5].

## C. Energy Efficiency (EE)

We assume that all systems use the same $\beta$-length chaotic signal with the same energy as a reference signal. In the proposed scheme, the total symbol energy without polarization $E_{s}$ is expressed as [12]

$$
\begin{equation*}
E_{s}=E_{\text {sdata }}+E_{\text {ref }} \tag{21}
\end{equation*}
$$

where $E_{\text {ref }}$ and $E_{\text {sdata }}$ are the energies of the reference sequence and data sequence without polarization, respectively. They are equal to

$$
\begin{align*}
E_{\text {sdata }} & =\cos ^{2} \theta \sum_{i=1}^{\beta} c_{x, i}^{2}+\sin ^{2} \theta \sum_{i=1}^{\beta} c_{y, i}^{2} \\
& =E_{\text {ref }}=\sum_{i=1}^{\beta} c_{x, i}^{2} \tag{22}
\end{align*}
$$

Then the P-MDCSK symbol energy is expressed as

$$
\begin{equation*}
E_{k}=\left(\left|E_{h}^{2}\right|+\left|E_{v}^{2}\right|\right) E_{s}=2 \sum_{i=1}^{\beta} c_{x, i}^{2} . \tag{23}
\end{equation*}
$$

It is obvious to see from (23) that $E_{k}$ is used to transmit $\log _{2} M=$ $l_{b}+n_{b}$ bits. Thus the energy required for transmitting one bit is

$$
\begin{equation*}
E_{b}=\frac{E_{k}}{\log _{2} M} \tag{24}
\end{equation*}
$$

Subsequently, the EE (Data-energy to Bit-energy [12]) can be written as

$$
\begin{equation*}
E E=\frac{E_{k}}{E_{b}}=\frac{\log _{2} M E_{k}}{E_{k}}=\frac{\log _{2} M}{2}=\frac{l_{b}+n_{b}}{2} \tag{25}
\end{equation*}
$$

Table I shows the EE of different chaotic modulations. The EE can be simply denoted as the ratio of the number of bits per symbol to

TABLE I
EE Comparison of Modulations

| Modu | DCSK | MDCSK | MR-MDCSK | S-MDCSK | P-MDCSK |
| :---: | :---: | :---: | :---: | :---: | :---: |
| EE | $1 / 2$ | $\frac{\log _{2} M-1}{2}$ | $\frac{\log _{2} M-1}{2}$ | $\frac{\log _{2} M-1}{2}$ | $\frac{\log _{2} M}{2}$ |



Fig. 4. Simulation (sim) and analytical upper bound (ub) performance comparisons of P-MDCSK over AWGN and multipath Rayleigh fading channels with $S F=128$.
the number of chaotic sequences needed to transmit one symbol from (25). As shown in Table I DCSK only transmits one bit within two chaotic sequences, while the conventional MDCSK and MR-MDCSK transmit $\log _{2} M-1$ bits ( $m_{c}=n_{b}=\log _{2} M-1$ ) because $m_{c}$ bits are mapped in the phases. Thus, their EEs are higher than DCSK. It can be seen in Table I that the EE values of MDCSK, MR-MDCSK are identical and little lower than P-MDCSK, and their EE all increases with $M$. It means P-MDCSK has a higher data rate.

## IV. Simulation Results and Discussion

In this section, simulations are carried out to evaluate the performances of the P-MDCSK system over AWGN and multipath Rayleigh fading channels, and verify the theoretical analysis results over the two channels. In all figures, SF represents the spreading factor. Three paths $L=3$, are considered with equal channel average power gain $E\left[\left|\alpha_{1}\right|^{2}\right]=E\left[\left|\alpha_{2}\right|^{2}\right]=E\left[\left|\alpha_{3}\right|^{2}\right]=1 / 3$, and time delays $\tau_{1}=0, \tau_{2}=$ $2 T_{c}, \tau_{3}=5 T_{c}\left(T_{c}\right.$ is the sampling period of the chaotic signal $\mathbf{c}_{\mathbf{x}}$ ) over multipath Rayleigh fading channel.

## A. Verification of the Theoretical BER With Simulation Results

Fig 4 shows the comparisons between the analytical upper bound of BER with the simulated BER of P-MDCSK system over both AWGN and multipath fading channels with spreading factor $S F=128$. The simulations are tightly matched with the BER results obtained by the analytical upper bound expression of (14) and (20) over both channels. The BER performance of the P-MDCSK is not always deteriorating with $m_{c}$ increased, and the best BER performance is obtained at 8 -P-MDCSK. The reason is that the decision is separated to two parts, where one part is for the plane constellation and the other part is for the polarization state. While the BER performance of QCSK is best with a large decision distance on the plane constellation, the decision of the polarization state has little effect on the change of $M$. This behavior is just the characteristic of the MDCSK, as the BER of QCSK is lower than that of DCSK at high spread factor. This has been analyzed in


Fig. 5. BER performance comparisons among P-MDCSK, MDCSK, MRMDCSK, S-MDCSK, DS-P-MPSK, and P-MDCSK over (a) AWGN and (b) Rayleigh multipath fading channels with $S F=128, L=3$.
detail in [4] and confirmed in [5]. The tendency behavior about $M$ is the same with BER performance over AWGN channel, the loss between optimal BER $(M=8)$ and $M=4$ is about 0.5 dB while the energy efficiency is increased predictably with increasing $M$.

## B. Performance Comparisons

Fig. 5 shows the comparisons among P-MDCSK, MDCSK, SMDCSK (square-constellation MDCSK in [6]), DS-P-MPSK (direct spread-polarization-MPSK), MR-MDCSK (multiresolution-MDCSK) and P-MDCSK with ML (maximum Likelihood) receiver. Fig. 5(a) shows the comparisons of these schemes over AWGN channel. It can be seen that the P-MDCSK achieves lower BER than other corresponding schemes (MR-MDCSK with priority vectors $\Theta=\pi / 3 \pi / 15 \pi / 40$ in [5]. The parameters of S-MDCSK are $d=1 / \sqrt{2}$ for $M=4$, and $d=1 / \sqrt{10}$ for $M=16$, respectively). They have the best BER performance at $M=8$ for P-MDCSK, P-MDCSK with the ML receiver, and at $M=4$ for other corresponding schemes. The gains of P-MDCSK over MDCSK and MR-MDCSK are about 0.4 dB and 3.8 dB at $M=4$ when BER is $10^{-4}$. Comparing the BERs of the proposed scheme and P-MDCSK with the ML receiver, it can be seen that they all
have the best BER performance at $M=8$, which depends on the charactor of MDCSK [5]. The BERs of P-MDCSK with the ML receiver are slightly lower than that with maximum energy comparator. However, the latter has a lower computational complexity compared with the former. For example, in traditional optimum ML receiver, the search space is $L_{b} \times N_{b}$ and, hence, its computational complexity is $O\left(L_{b} \times N_{b}\right)$, while the computational complexity of the maximum energy comparator receiver is $O\left(L_{b}\right)+O\left(N_{b}\right)$ [10].

As shown in Fig. 5(b), for multipath fading channel, the P-MDCSK schemes (with both two receivers) have better BER performances than that of other corresponding modulations at $S F=128$. They all have the same decreasing tendency of BER. More specifically, the gain of the proposed scheme over other chaotic modulation or polarization modulation schemes is $2-8 \mathrm{~dB}$ at BER level $10^{-4}$. In particular, the superiority of 8-P-MDCSK can be found in the high SNR region.

## V. CONCLUSION

In this letter, a P-MDCSK modulation scheme without PDL effect is proposed to obtain an improved BER performance with a low computational complexity. The P-MDCSK can transmit information bits by selecting both phase and polarization states. The upper bounds of the BER expression of the proposed P-MDCSK are derived and verified by simulation over both AWGN and multipath Rayleigh fading channels. The BER simulation results show that the proposed scheme outperforms the other corresponding MDCSK and traditional polarization modulations over both AWGN and multipath Rayleigh fading channels. Furthermore, the proposed scheme can eliminate the PDL problems since it does not need channel estimation, and thus, performs well in fading channel. Therefore, the proposed P-MDCSK scheme has a simple, high data rate and excellent BER performance in the fading channel environment.

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