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Regional Astrogeodetic Validation of GPS/Levelling Data and Quasigeoid Models

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Abstract. In the context of a GOCE regional validation and combination experiment in Germany, a work package within the framework of the GOCE-GRAND II project, gravity observations, vertical deflections and GPS/levelling data are collected as independent data sets. The observation of absolute gravity values is carried out by the Bundesamt für Kartographie und Geodäsie (BKG), while the vertical deflections are observed by the Institut für Erdmessung (IfE) using the Hannover digital transportable zenith camera system TZK2-D. The vertical deflections have an accuracy of approx. 0.1 arc seconds and are arranged along a north-south and east-west profile. The two profiles have a length of about 500 km each with a spacing of 2.5 – 5 km between adjacent stations. Furthermore, a national GPS and levelling data set of about 900 stations with an accuracy of approx. 1 cm is available for Germany.

The analysis of the vertical deflections is carried out by the astronomical levelling method, resulting in two (quasi)geoid profiles. The accuracy of the profiles is expected to be at the cm level. A cross-validation of both the vertical deflection and GPS/-levelling data is realised by traversing the profiles through all nearby GPS/levelling stations (approx. 40 in total). In addition, comparisons are performed with the German Combined QuasiGeoid 2005 (GCG05) and the purely gravimetric solution EGG07 (European Gravimetric Quasigeoid 2007).

Keywords. Digital zenith camera system, deflections of the vertical, astrogeodetic quasigeoid profile, GPS/levelling data, quasigeoid models

1 Introduction

Within the next few years, improved high-resolution global gravity field models are anticipated from the GOCE mission. The expected accuracies are about 1 – 2 cm in terms of geoid undulations and 1 mgal for gravity, both at a resolution of about 100 km (see, for example, ESA 1999). Then, from a combination of the GOCE based global gravity field models (expected to be available up to spherical harmonic degree and order 250) with regional terrestrial data sets, an accuracy of about 1 cm is expected for the complete geoid spectrum. In this context, accurate and independent data sets are essential for the combination process as well as for the validation of the results. Therefore, a regional validation and combination experiment is carried out in Germany as a work package within the framework of the GOCE-GRAND II project. In the first stage, new absolute gravity observations, GPS/levelling data, as well as astronomic vertical deflections are collected.

While the validation of the terrestrial gravity data base is carried out by the Bundesamt für Kartographie (BKG) using absolute gravity observations at field stations as spot-checks, a new regional data set of astronomically determined vertical deflections is collected by the Institut für Erdmessung (IfE) as a completely independent observation type. The vertical deflections are also employed for a cross-validation of the existing GPS/levelling data.

During recent years, the economic and precise determination of vertical deflections with an accuracy of approx. 0.1" was realised by the development of digital transportable zenith camera systems such as the TZK2-D (IfE; Hirt 2004) and DIADEM (Geodesy and Geodynamics Laboratory GGL, ETH Zurich; Bürki et al. 2004). Both zenith camera systems were extensively employed for local and regional gravity field determinations in Germany (Hirt and Flury 2007), Switzerland (Müller et al. 2004, Bürki et al. 2005) and Greece (Müller et al. 2006).

In this study, the TZK2-D is employed for the determination of vertical deflections along a north-south and east-west profile with a spacing of 2.5 to 5 km between adjacent stations. For the cross-validation with the GPS/levelling data, each profile traverses through approx. 20 nearby GPS/levelling control points. At present, a few vertical deflection observations are still missing in the east-west profile, while the north-south profile is completely finished. Hence, this publication discusses mainly the results from the north-south profile.

Finally, the astrogeodetic north-south profile is compared with the existing German Combined QuasiGeoid 2005 (GCG05; Liebsch et al. 2006) as well as the purely gravimetric quasigeoid EGG07 (European Gravimetric Quasigeoid 2007; Denker et al. 2007). In addition, the gravimetric quasigeoid models (GCG05 and EGG07) are evaluated by GPS/levelling control points along the profile; corresponding evaluations of gravimetric quasi-geoid models by GPS/levelling data can be found, for example, in Lysaker et al. (2007) and Krynski and Lyszkowicz (2006).

In this contribution, the three basic data sets, namely astronomically determined vertical deflections, GPS/levelling data and gravimetric quasigeoid models, are described in section 2. The astro-geodetic quasigeoid computation is described in section 3, while sections 4 and 5 deal with the comparison of the north-south astrogeodetic quasi-geoid profile with corresponding values from GPS/levelling and gravimetric quasigeoid models. Emphasis is put on the analysis of the differences between the relevant data sets and their possible origins.

2 Data Sets

In this section, the main features and accuracies of all relevant data sets are described.

2.1 Vertical Deflections

Since September 2006, vertical deflections have been determined astronomically along two profiles across Germany using the Hannover digital transportable zenith camera system TZK2-D. The determination of the vertical deflections is based on the automatic determination of the direction of the plumb line in combination with the determination of ellipsoidal coordinates by GPS measurements. For further details on the TZK2-D system see Hirt (2004).

The vertical deflection stations are arranged along a north-south and east-west profile (see Fig. 1). The north-south profile has a length of 540 km and runs from the Harz Mountains in the north to the Bavarian Alps in the south. In total, 137

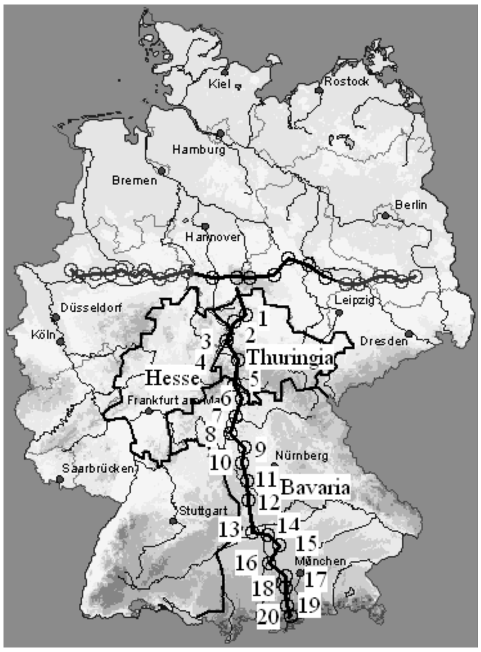

Fig. 1. Astrogeodetic profile lines and GPS/levelling points.

vertical deflection stations are observed, having an average spacing of 3.9 km (from 2.5 to 5 km). The station spacing is reduced to approx. 2.5 km in regions where rapid gravity field changes occur, while in homogeneous areas, a spacing of nearly 5 km is selected. The vertical deflection observations along the north-south profile have been completely finished. Along the east-west profile with a length of 518 km, running from Lusatia in the east to the Münsterland region in the west, approx. 80 % of the 133 vertical deflection stations have been determined up to now. Therefore, the cross-validation with the GPS/levelling data and the gravimetric quasigeoid models is only carried out with the vertical deflections along the north-south profile. For a statistics of the profiles see Table 1.

Because of the high degree of automation of the zenith camera system, it was possible to determine vertical deflections on up to 12 stations in a single night. The accuracy of these vertical deflections is stated to be approx. 0.1" (see, for example, Hirt and Flury 2007). This figure is in close agreement with the statistics of the differences between 23 double observations on identical points in different nights along the north-south profile (see Table 2). For further details and accuracy analyses of the digital zenith camera system TZK2-D see, for example, Hirt (2004) and Hirt and Flury (2007).

Table 1. Characteristics of the astrogeodetic profiles.

	north-south	east-west
Length	540 km	518 km
Number of stations	137	61/133 (observed/planned)
Average spacing	3.9 km	3.9 km
Number of GPS/ levelling points	20	23

Table 2. Statistics of 23 double observations in different nights on stations along the north-south profile ["].

	Mean	RMS	Min	Max
$\Delta\xi$	-0.021	0.097	-0.210	0.172
$\Delta\eta$	0.025	0.121	-0.268	0.231

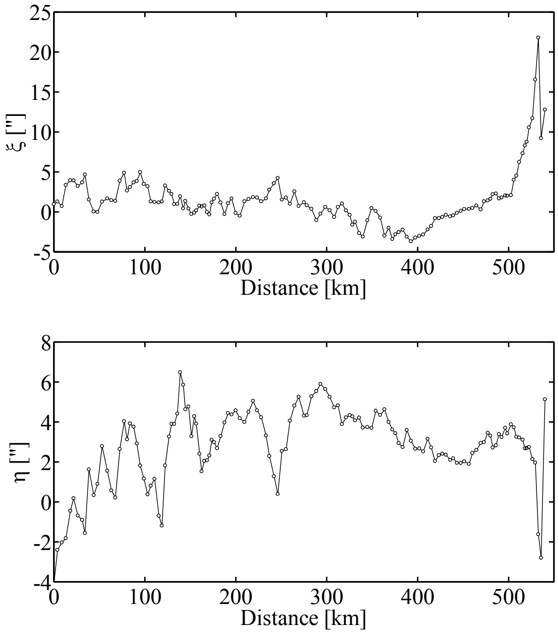

Fig. 2. Observed vertical deflections on stations along the north-south profile (from north to south).

Fig. 2 shows the vertical deflection components ξ (north-south) and η (east-west) along the north-south profile.

2.2 GPS/Levelling Data

Fig. 1 shows that the north-south profile was set up to traverse through 20 nearby GPS/levelling points, which has an effect on the azimuths of the corresponding profile sections. The GPS/levelling points no. 1, 2 and 4 are located in Thuringia, while point no. 3 and points 5 to 20 are located in Hesse and Bavaria, respectively (Thuringia, Hesse and Bavaria are federal states in Germany).

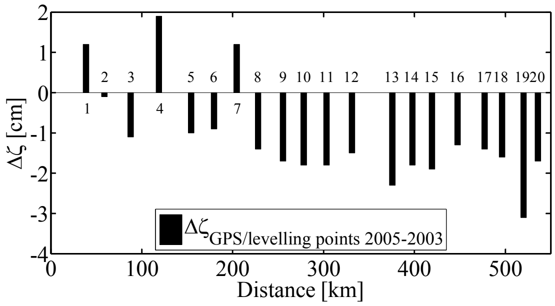

Fig. 3. Differences between GPS/levelling data sets 2005 and 2003.

All GPS/levelling points are extracted from a national data set of approx. 900 points with a spacing of 25 to 30 km. The national data set was collected by the BKG. Two versions of this data set exist, one from 2003 and one from 2005. The major difference between the two data sets is a re-adjustment of the SAPOS GPS reference stations, which was introduced at the beginning of 2004. However, the federal states of Germany used individual procedures to implement the new GPS reference frame, ranging from re-observation and re-adjustment to different transformation procedures, which may introduce some inhomogeneities in the GPS coordinates. It should be noted that the 2005 GPS/-levelling data set was also used for the computation of the German Combined QuasiGeoid 2005 (GCG2005). For further details on the GPS/levelling data sets see, for example, Liebsch et al. (2005).

The differences between the height anomalies of the 2005 and 2003 GPS/levelling points along the north-south profile are shown in Fig. 3, ranging from about -3 to +2 cm. While the differences between the two data sets represent mainly a bias in Bavaria (pts. no. 5-20, except point no. 7), a more irregular pattern is found at the beginning of the profile (pts. no. 1-4). The exact reason for this behaviour is presently unknown.

The accuracy of the quasigeoid heights derived from GPS/levelling data depends on the one hand on the accuracy of the ellipsoidal heights determined with GPS (1-2 cm) and on the other hand on the accuracy of the precise geometric levelling, where an accuracy of approx. 1 mm / $\sqrt{}$ km yields 23 mm over the whole profile length of 540 km.

The accuracy issue is especially important in the context of the cross-validation of the astronomic and GPS/levelling data as well as the detection of possibly existing systematic errors in geometric levelling and GPS ellipsoidal heights.

2.3 Gravimetric Quasigeoid Models

The gravimetric quasigeoid models used in this work are all based on the combination of global gravity field models with terrestrial gravity anomalies and terrain data.

In addition to this, the German Combined Quasi-Geoid 2005 (GCG05) includes also GPS and levelling data from the above mentioned 2005 national data set. This model was calculated by BKG and IfE using two different approaches, the final model being derived simply by averaging both individual solutions. The accuracy of the GCG05 model is stated to be 1-2 cm, for further details see Liebsch et al. (2005).

As a second quasigeoid model, the purely gravimetric solution (without GPS/levelling data) EGG07 (European Gravimetric Quasigeoid 2007) is employed. This solution was computed at IfE using the spectral combination method with integral formulas within the framework of the European Gravity and Geoid Project (EGGP), see, for example, Denker (2006) and Denker et al. (2007).

The differences between the 2005 GPS/levelling data set and the GCG05 and EGG07 quasigeoid models are depicted in Fig. 4. The figure shows that the GCG05 model, including the GPS/levelling data, closely resembles the GPS/levelling control points (introduced in GCG05 with a standard deviation of about 1 cm). The EGG07 differences exhibit mainly long wavelength structures, except at pt. no. 4 where a peak of about 3 cm appears to be present.

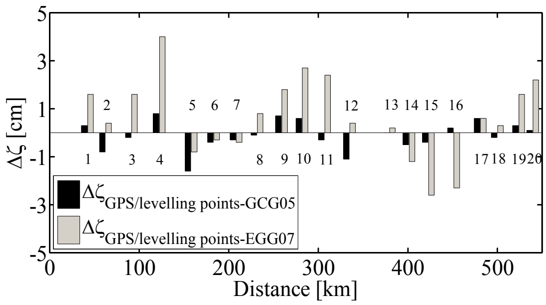

Fig. 4. Comparison of GPS/levelling data with the quasigeoid models GCG05 and EGG07.

3 Astrogeodetic Quasigeoid Profile

The method of astronomical levelling is documented very well (see, for example, Heiskanen and Moritz 1967, Torge 2001). Hence, only a summary of the most important equations is given in this section.

In this study, vertical deflections ε at the surface of the Earth according to the definition of Helmert are utilized to compute quasigeoid height differences. The basic equation is

$$\Delta\zeta_{1,2} = -\int_1^2 \varepsilon\, ds - E_{1,2}^N , \qquad (1)$$

where $\Delta\zeta$ is the height anomaly difference between points 1 and 2, ds is the line element, and E^N is the normal height reduction. The negative sign results from the definition of the vertical deflections.

The vertical deflection ε is the deflection component in path direction, specified by the azimuth α. It can be computed from the observed north-south and east-west component ξ and η:

$$\varepsilon = \xi \cos\alpha + \eta \sin\alpha \, . \qquad (2)$$

Furthermore, the normal height reduction E^N can be computed by

$$E_{1,2}^N = \int_1^2 \frac{g-\gamma_0^{45}}{\gamma_0^{45}} dn + \frac{\bar{\gamma}_1 - \gamma_0^{45}}{\gamma_0^{45}} H_1^N - \frac{\bar{\gamma}_2 - \gamma_0^{45}}{\gamma_0^{45}} H_2^N \, . \qquad (3)$$

In the above equation, g is the surface gravity which is available with an accuracy of about 1 mgal, $\bar{\gamma}_1, \bar{\gamma}_2$ are the mean normal gravity values at stations 1 and 2, H_1^N, H_2^N are the corresponding normal heights, γ_0^{45} is the constant normal gravity value at latitude φ=45°, and dn is the levelling increment. The normal gravity values are computed following the standard equations as described, for example, in Heiskanen and Moritz (1967). The heights are extracted from a digital terrain model (DTM) named DGM50/M745, which was supplied by the BKG. The DTM has a grid spacing of 1" x 1" (approx. 30 m) and an accuracy of about ± 1 – 8 m. For a study of DTM resolutions recommended for the analysis of high precision vertical deflections (accuracy about 0.1") see Voigt and Denker (2006).

Theoretically, a requirement for the evaluation of the integral in Eq. (1) is the continuous availability of vertical deflections along the integration path. In

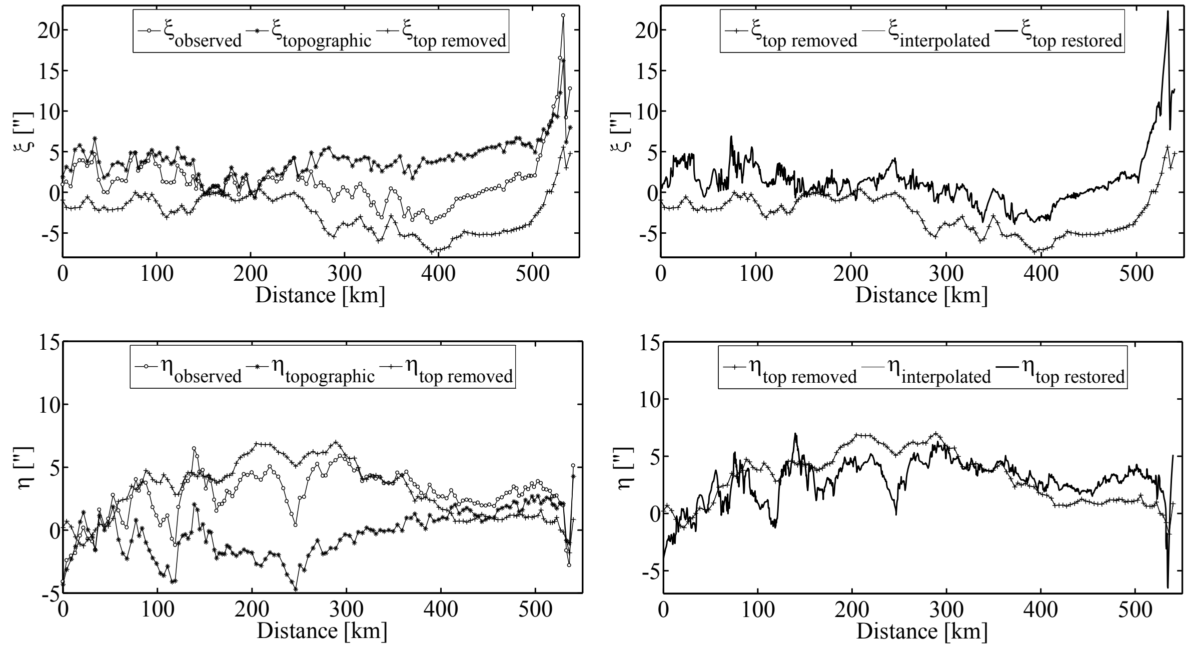

Fig. 5. Steps of the remove-restore-procedure for the vertical deflections.

this context, a remove-restore procedure, taking into account the effect of the topography (DTM, see above), is applied. Fig. 5 shows the observed and the terrain reduced vertical deflections, demonstrating that the terrain reductions lead to a significant smoothing of the deflection signals. Then, for the evaluation of the integral according to Eq. (1), always 10 intermediate points are arranged between neighbouring observation stations and linear interpolation is done between corresponding stations, having an average distance of about 400 m.

The normal height reductions along the profile are illustrated in Fig. 6, and the computed astrogeodetic quasigeoid heights are shown in Fig. 7. Incidentally, the astrogeodetic solution provides only differences of height anomalies without any absolute datum.

Before comparing the astrogeodetic results with the GPS/levelling data sets and the gravimetric quasigeoid models, the effect of different error sources in the vertical deflection data is discussed.

If only random errors with a magnitude of 0.1" exist in the vertical deflection data, an accuracy of 2.2 cm can be expected for the quasigeoid profile over the full length of 540 km; this figure is resulting from simple error propagation with uncorrelated observation errors, neglecting any signal omission errors and systematic error components. Furthermore, from Eq. (2) it is obvious that in our case (north-south profile with azimuths α

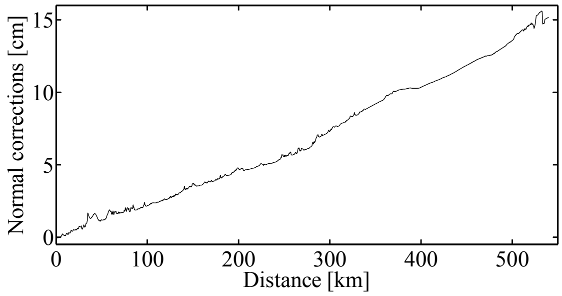

Fig. 6. Normal corrections along the north-south profile.

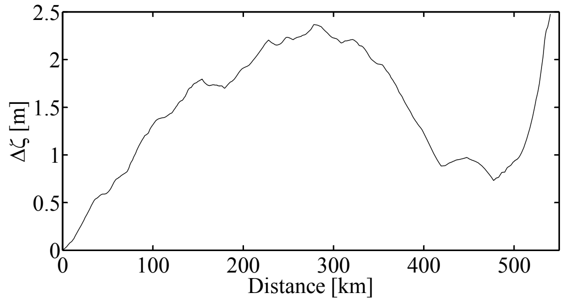

Fig. 7. Astrogeodetic quasigeoid profile.

near 180°) the north-south vertical deflection component ξ is more critical than the east-west component η.

In order to detect systematic and gross errors, double observations have been carried out on 23 stations in different nights. From Table 2 it can be seen that the mean values of the differences are not significantly different from zero, and secondly, the maximum difference of 0.27" is lying within the 3σ tolerance. Hence, no systematic difference could be detected between observations on different days.

Furthermore, a gross error in the vertical deflection data has a bias-like impact on the height anomalies for the remaining part of the profile. The magnitude of this bias naturally depends on the magnitude of the gross error in the vertical deflections and on the station separation. For example, a gross error of 1" can already cause a bias of up to 2 cm over a 4 km long path.

4 Results

Since the method of astronomical levelling only provides differences of height anomalies, an absolute level has to be defined. In the comparison between the astrogeodetic quasigeoid profile (shown in Fig. 7) with the GPS-levelling data and the gravimetric quasigeoid models, the absolute level is taken from the GCG05 model at the first station. The height anomalies are shown in Fig. 8, while the corresponding differences are illustrated in Fig. 9.

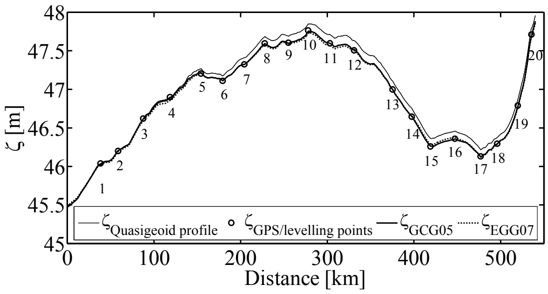

Fig. 8. Astrogeodetic quasigeoid profile, GPS/levelling data set 2005, quasigeoid models GCG05 and EGG07.

Fig. 9 shows that long wavelength differences with a range of up to 14 cm occur between the astrogeodetic quasigeoid profile and the purely gravimetric solution EGG07. For the combined GCG05 model, the differences decrease to a range of about 12 cm. Regarding the differences between the astrogeodetic profile and the GPS/levelling control points (2005 data set), two station groups can be distinguished, namely points no. 1-4 and 5-20. When looking only at the Bavarian GPS/levelling points (pts. no. 5-20), a small long-wavelength difference exists, while the differences in the northern section (pts. no. 1-4) appear to be quite random. Between the northern and Bavarian section, a jump of almost 5-6 cm shows up (between pts. no. 4 and 5). It should be pointed out, that this is exactly at the border between the two federal states Thuringia and Bavaria, and this is also the region where the 2003 and 2005 GPS/levelling data show larger irregular discrepancies. When comparing with the 2003 GPS/levelling data, the

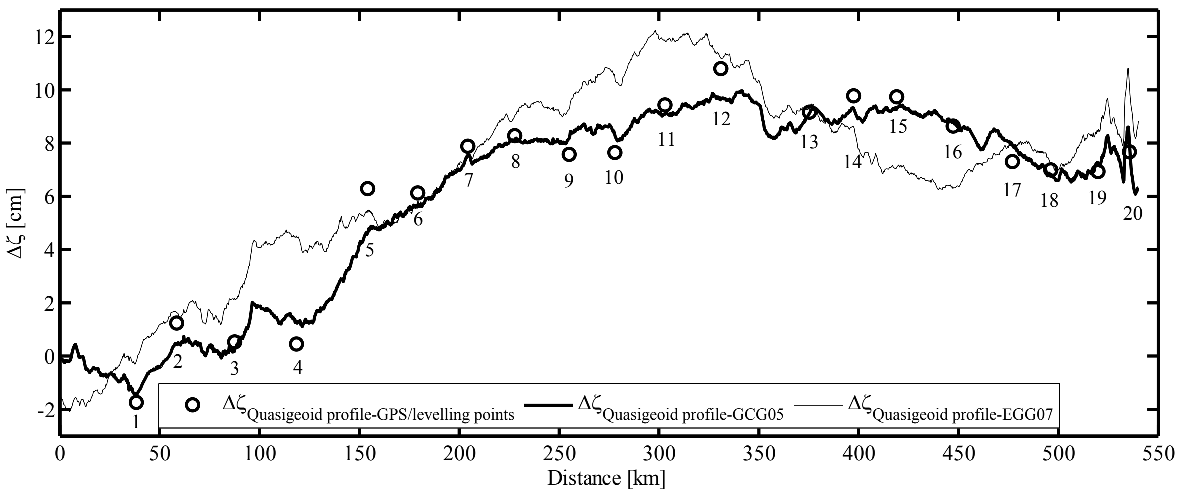

Fig. 9. Comparison of the astrogeodetic quasigeoid profile with GPS/levelling data set 2005 and the quasigeoid models GCG05 and EGG07.

jump between the northern and southern section reduces to about 3 cm (see also section 2.2 and particularly Fig. 3). Hence, in this region, some inconsistencies may exist in the GPS/levelling data sets. In order to prove that the jump in the differences between the 4th and 5th point is not caused by signal omission errors or systematic errors in the astrogeodetic solution, a densification of the astrogeodetic stations has been carried out in this region lately, but with no significant change in the results.

Finally, the differences between the astrogeodetic profile and the GPS/levelling points are within ±1-2 cm for each of the two station groups. This reveals a quite good consistency, considering the predicted accuracy for the astrogeodetic quasigeoid profile (2.2 cm), the selected station spacing of 3.9 km on average, and the stated accuracy of the GPS/levelling points (1-2 cm).

5 Summary and Conclusions

In this contribution, a regional astrogeodetic quasigeoid profile has been computed by applying the method of astronomical levelling, using high-precision vertical deflections determined with the Hannover digital zenith camera system TZK2-D. With an accuracy of the vertical deflections of 0.1", and an average station spacing of 3.9 km, a more than 500 km long profile has been attained with an accuracy at the cm level.

In comparison with a GPS/levelling data set from 2005, a jump of nearly 6 cm was found between Thuringia (4th point) and Bavaria (5th point), and a small long-wavelength difference exists for the Bavarian GPS/levelling stations (points no. 5-20). Hence, at the border between Thuringia and Bavaria some inconsistencies may exist in the GPS/levelling data sets which have to be further investigated.

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