A review on underwater beamforming: Techniques, challenges, and future directions

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ABSTRACT

This paper comprehensively reviews recent advancements in Underwater Beamforming (UWB) systems, highlighting its pivotal role in underwater communication, sensing, and environmental monitoring. It explores the various beamforming applications, ranging from maritime surveillance to marine life monitoring, and indicates its significance in enhancing signal clarity, spatial resolution, and noise suppression in underwater acoustic environments. The unique challenges posed by the underwater environment that introduce complexities into the beamforming process such as non-stationary noise interference, severe signal attenuation, multipath propagation, and dynamic environmental variability are thoroughly discussed. The review systematically discusses and examines conventional, adaptive, and learning-based beamforming techniques, analyzing their strengths, limitations, and suitability for various underwater conditions. A detailed analysis of Direction of Arrival (DOA) estimation methods is provided. Furthermore, the review surveys the metrics commonly used to assess the performance of beamforming algorithms and compares their performance numerically. Emerging trends in beamforming, particularly the integration of data-driven machine learning approaches with traditional signal processing methods, are also discussed. The paper concludes by highlighting critical gaps in existing research and proposing future directions.

1. Introduction

Underwater Beamforming (UWB) is a critical technique with extensive applications in marine exploration [1], military surveillance [2], and underwater communication systems [3]. Underwater acoustic data carry valuable insights that must be interpreted for various purposes, such as environmental monitoring, marine biology research, underwater navigation, marine vehicle detection, and communication in remote underwater environments [4]. The intelligent and efficient processing of acoustic signals through beamforming is essential for advancing modern underwater technologies. Unlike terrestrial and aerial environments, the underwater domain presents unique challenges, where electromagnetic waves are significantly attenuated, making acoustic waves the preferred medium for long-range communication and sensing [5], [6]. However, the propagation of acoustic signals in underwater environments is heavily influenced by factors such as multipath effects, scattering, and Doppler shifts that lead to complex and unpredictable transmission channels [7]. These challenges are further exacerbated by intense ambient noise and interference, which can severely degrade signal quality and hinder the accurate detection and localization of targets [8], [9].

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List of Important Abbreviations

AG	Array Gain
BER	Bit Error Rate
CBF	Conventional Beamforming
CS	Compressive Sensing
CNN	Convolutional Neural Network
DOA	Direction of Arrival
DAS	Delay-and-Sum
DL	Deep Learning
ESPRIT	Estimation of Signal Parameters via Rotational Invariance Technique
EMD	Empirical Mode Decomposition
FFNN	Feed Forward Neural Network
GSC	Generalize Sidelobe Cancellation
LCMV	Linearly Constrained Minimum Variance
MAE	Mean Absolute Error
MMPE	Monterrey–Miami Parabolic Equation
ML	Machine Learning
MSE	Mean Square Error
MUSIC	Multiple Signal Classification
MVDR	Minimum Variance Distortionless Response
NF	Normalization Factor
RCB	Robust Capon Beamforming
SBL	Sparse Bayesian Learning
SNR	Signal-to-Noise Ratio
SINR	Signal-to-Interference-plus-Noise Ratio
SSP	Sound Speed Profile
STFT	Short-Time Fourier Transform
TL	Transmission Loss
ULA	Uniform Linear Array
UWSN	Underwater Wireless Sensor Network
UWA	Underwater Acoustic Communications
UWB	Underwater Beamforming

Beamforming is a spatial filtering operation typically using an array of radiators/receivers to radiate or capture energy in a specific direction over its aperture. It is a key method for enhancing signal strength, improving signal-to-noise ratio (SNR) and reducing interference by focusing the energy in desired directions and suppressing it in others [10]. Early experiments with beamforming could be traced back to the 1950s-1960s, with the development of phased array Sound Detection and Ranging (SONAR), the steering of signals with antenna arrays was no longer restricted to electromagnetic waves [11]. The improvement achieved over omnidirectional transmission/reception is the transmit/receive gain. Over the years, the research community has made significant advancements in the development of beamforming techniques, evolving from basic to more sophisticated methods to address the challenges of underwater environments. Initially, conventional beamforming methods such as the Delay-and-Sum (DAS) technique were widely adopted due to their simplicity and ease of implementation [12]. However, these methods often struggled with resolving closely spaced sources and were highly sensitive to varying statistics of noise and interference. To overcome these limitations, adaptive beamforming techniques such as Minimum Variance Distortionless Response (MVDR) and the Capon beamformer were introduced [13]. These methods offered enhanced resolution and better interference suppression by adapting the beam pattern based on the received signals. Despite these improvements, adaptive methods still face challenges in highly dynamic underwater environments due to the sensitivity to the estimation error corresponding to the desired acoustic signal [14].

Moreover, researchers also turned to model-based and data-driven approaches after recognizing the need for robust solutions. The robust adaptive beamforming techniques emerged to incorporate regularization and covariance matrix reconstruction methods to mitigate the effects of signal model mismatch and uncertainties in the underwater environment. These methods marked a significant step forward in scenarios with limited snapshots or strong interference [15].

In recent years, beamforming techniques have taken a significant leap forward. Machine learning (ML) and Deep learning (DL) based beamforming methods have shown great promise in learning complex patterns from vast amounts of data and their minimal reliance on predefined assumptions about the acoustic environment. ML and DL-based beamforming frameworks typically leverage various neural network architectures to optimize beam patterns and improve the detection and localization of underwater targets [16, 17].

The field of beamforming has been well-reviewed in terrestrial domains, such as radar and wireless communications. However, there has been no comprehensive review dedicated to the underwater context, which requires different strategies due to the different physical properties of water. In RF beamforming, several features and assumptions, such as narrowband signal processing, far-field plane wavefronts, and static propagation environments, are commonly exploited to simplify design and enable high-resolution spatial filtering. For instance, antenna arrays in RF systems typically assume uniform wavefront arrival and exploit phase coherence across elements for direction-of-arrival (DoA) estimation and beam steering [18]. Moreover, techniques like angle-of-arrival-based tracking and planar array calibration rely on the relatively constant dielectric properties of the RF propagation medium [19]. However, these features do not directly apply to underwater acoustics, where spatial coherence is rapidly degraded by multipath propagation, wavefronts may be spherical or distorted due to sound speed variability with depth and temperature, and wideband modeling becomes essential due to frequency-dependent attenuation and dispersion [20]. As a result, conventional RF-based features such as narrowband phase-only steering, array manifold interpolation, or static calibration techniques often fail in underwater contexts [21]. Another critical distinction between RF and underwater environments lies in the statistical nature of noise. In terrestrial RF systems, noise is typically modeled as additive white Gaussian noise (AWGN) due to the thermal origin of most interference sources. However, underwater noise exhibits fundamentally different characteristics: it is often non-Gaussian, non-stationary, and highly impulsive, in shallow water and near coastal environments [8, 22].

While the work by Madhusoodanan et al. [23] provides a comparative performance evaluation of several beamforming algorithms for underwater 2D acoustic imaging, it primarily focuses on simulation-based assessments of DAS, MVDR, and MUSIC. Their study emphasizes comparative imaging performance and computational complexity in the context of a specific imaging configuration. In contrast, this review offers a comprehensive and structured examination of the full spectrum of underwater beamforming systems and covering conventional, adaptive, learning-based and data-driven approaches in two distinct yet related areas: Underwater sensing and communication. Given that no previous work has reviewed the entire scope of underwater beamforming in this manner, this study represents a pioneering effort. It serves as a valuable resource for future research and development for setting the foundation for innovations that can further improve underwater acoustic systems for a wide range of applications. Therefore, this study contributes to the existing literature in the following ways:

- This paper provides a detailed review of recent developments in UWB and focuses on its importance in underwater communication and sensing. Then, the unique challenges posed by underwater acoustic environments that affect the beamforming process are discussed.

- Moreover, it systematically categorizes and evaluates conventional, adaptive and learning-driven beamforming techniques, analyzing their strengths and limitations under dynamic underwater conditions. The effectiveness of the DOA estimation methods in underwater settings is also assessed. Moreover, it provides

Underwater Beamforming (UWB)			
Application category	**Transmit beamforming**	**Receive beamforming**	**Key application**
Underwater Communications	Long range communications	Enhanced signal reception	Acoustic modems and Underwater wireless sensor networks (UWSNs)
	Energy efficiency	Interference/noise suppression	
Underwater Sensing/tracking	Target detection	Accurate target localization and tracking	Active SONAR and Passive SONAR
	Environmental monitoring	Echo processing	

Table 1
Summary of transmit and receive beamforming in underwater applications.

an analysis of the computational complexity of baseline beamforming and DOA estimation algorithms.

- Subsequently, it discusses the most widely used evaluation metrics for assessing the performance of beamforming algorithms and compares their performance numerically across different underwater applications.
- This paper concludes by identifying gaps in the existing research and proposing future research directions for improving underwater beamforming techniques.

The rest of the article is organized into the following sections. Section 2 outlines the applications of beamforming in different underwater scenarios. Section 3 discusses the challenges posed by the underwater environment for beamforming processes. Section 4 comprehensively reviews the relevant state-of-the-art UWB techniques in the literature for sensing and communication applications. Section 5 surveys the widely used evaluation metrics in the DOA estimation and UWB. Section 6 compares the performance of several beamforming methods numerically. Section 7 highlights the gap in the current literature and suggests future research directions for the next-generation technologies. Section 8 concludes our paper with some insights gained from this review.

2. Application highlights

Beamforming is an important component in Underwater Communication and SONAR systems that play a critical role in various underwater applications. These applications leverage beamforming to enhance signal transmission and reception performance, reliability, and accuracy in the challenging underwater environment. Table 1 highlights the key applications of beamforming in underwater communications and sensing/tracking which are further categorized into transmit and receive beamforming.

2.1. Underwater Communication

- **Transmit beamforming**: In underwater communication, transmit beamforming is used to direct the transmitted acoustic signals toward a specific receiver. It ensures that the signal travels over long distances with minimal loss. This is especially important in applications like underwater data transmission between submarines or underwater vehicles and surface stations. Beamforming reduces power consumption by focusing transmission energy in the desired direction, making it crucial for battery-operated underwater communication devices [24, 25].
- **Receive beamforming**: The beamforming in the receiver improves the reception of weak signals that have traveled long distances underwater. The beamformer can enhance the SNR by focusing on the direction from which the signal is expected. Thus, receive beamforming helps in clearer and more reliable communication [26]. Moreover, in environments with multiple communication sources or high levels of ambient noise, receive beamforming helps in distinguishing the desired signal from interference and noise [27], [28].

Recent advancements in distributed acoustic applications have led to increasing interest in single-input multiple-output (SIMO) and multiple-input single-output (MISO) beamforming architectures for underwater communications. Unlike conventional centralized arrays, distributed SIMO/MI -SO systems employ spatially separated transmitters or receivers, such as underwater sensor nodes or AUVs, to form cooperative beams across extended spatial domains. This strategy is well-suited to UWSNs, where energy efficiency, spatial scalability, and wide-area coverage are critical. Such systems improve signal gain, interference suppression, and link robustness by synchronizing distributed nodes through coordinated control and phase alignment for larger areas. However, the implementation of distributed beamforming in underwater environments is challenged by synchronization requirements, node mobility, and time-varying acoustic channels.

Few researchers have investigated these applications, such as Huang et al. [29] conducted foundational sea trials using a single transmitter and a 12-element hydrophone array to emulate a distributed SIMO system. Their work has demonstrated that iterative OFDM receivers can maintain robust performance under significant Doppler shifts and channel dynamics. Similarly, authors in [30] developed a MISO system of two distributed transmitters and a single receiver and tested it through lake experiments. Moreover, Enhos et al. [31] developed a software-defined distributed SIMO framework that leverages maximal-ratio combining (MRC) and cloud-based signal fusion to achieve substantial improvements in Bit Error Rate (BER) for UWSNs.

These studies establish the feasibility of distributed acoustic beamforming, yet they also highlight critical limitations such as the need for robust synchronization protocols, Doppler-resilient receiver design, and efficient coordination mechanisms across nodes in real-time underwater operations.

2.2. Underwater Sensing/tracking

- **Transmit beamforming**: In active SONAR, transmit beamforming is used to direct a pulse of sound toward a target. The beamforming process focuses the acoustic energy in the direction of the target, increasing the chances of detecting the reflected signal [32]. SONAR systems used for mapping the seafloor or monitoring underwater environments use transmit beamforming to cover specific areas, ensuring that the emitted sound waves efficiently illuminate the area of interest [33].

- **Receive beamforming**: Receive beamforming is crucial in passive SONAR systems, where the goal is to detect and localize sound sources by listening to the sounds they emit. Beamforming helps in identifying the direction of the incoming sound and enhancing the SNR, which is vital for accurate localization. In active SONAR, receive beamforming is used to process the echoes returning from the transmitted pulse. By focusing on the direction from which the echo is expected, beamforming improves the detection and analysis of targets, even in the presence of noise and reverberation [34].

3. Challenges of the underwater environment for beamforming

The underwater environment is challenging for beamforming due to several factors. Multipath propagation, caused by reflections off surfaces and objects leads to signal interference and distortion. Additionally, acoustic signals experience absorption and attenuation at higher frequencies that reduce their strength over distance. The presence of ambient noise from marine life, waves, and human activities further complicates signal detection. Accurate direction estimation becomes very difficult due to Doppler shifts as a result of movement. Moreover, the underwater environment's spatial and temporal variability, along with non-stationary noise, disrupts the consistent performance of traditional beamforming techniques and requires robust and improved techniques to handle these complexities [35].

3.1. Underwater acoustic propagation losses

Sound propagates underwater as longitudinal waves; these waves are generated by the sources in the form of vibration in the surrounding medium. The vibration propagates away from particle to particle at the speed of sound. The propagation of the sound wave refracts upwards or downwards. Seawater's sound speed is affected by salinity, temperature, and pressure which ranges between 1450 m/s to 1540 m/s [36]. The underwater acoustic sound speed profile (SSP) for depths less than 1000 meters is given by [37], [38]:

$$\begin{aligned} c = 1449.2 + 4.6T - 0.055T^2 + 0.00029T^3 + \\ (1.34 - 0.01T)(S - 35) + 0.016z \end{aligned} \tag{1}$$

where T represents the water temperature in degrees Celsius, S indicates water salinity in parts per thousand and z corresponds to the depth of the water in meters. The oceanic temperature decreases with increasing depth. Additionally, the speed of sound initially decreases to a minimum at a certain depth before rising again. Also, higher salinity levels result in increased sound speed [38].

The underwater channel is highly complex and presents numerous challenges for sound propagation from the transmitter to the receiver. Propagation loss occurs due to factors such as geometric spreading, scattering, and signal energy absorption (attenuation). Attenuation specifically results from the conversion of signal energy into heat energy, and it is directly proportional to both the sound frequency and the transmission range [39]. Scattering, on the other hand, occurs when the acoustic wave is altered due to obstacles in the propagation medium, such as the seabed, marine animals, and other objects in the water. The effects of spreading and absorption can be mathematically incorporated to express the path loss or attenuation of underwater acoustic waves over a distance d and a signal frequency f [20].

$$A(d, f) = N_0 \times d^l \times \beta(f)^d \tag{2}$$

where the term N_0 represents a normalization factor (NF) that is inversely related to the transmitted power. The term d^l accounts for the spreading loss over a distance d, where l is the path loss exponent. This exponent varies depending on the wave's propagation surface. For short distances, the wavefront can be approximated as spherical, while for longer distances, the wavefront is considered cylindrical due to the constraint of the seabed and the water surface. The value of l is 1 for cylindrical spreading, 2 for spherical spreading, and 1.5 for practical spreading scenarios. Propagation losses increase with increasing frequency largely due to the effects of absorption. Additionally, $\beta(f)^d$ represents absorption loss over a distance d. By taking the logarithm of both sides of equation 3.2, the total path loss can be expressed as:

$$10logA(d, f) = 10logN_0 + 10llogd + 10dlog\beta(f) \tag{3}$$

The absorption coefficient in (3) can be defined more accurately in terms of frequency f as [20]:

$$\beta(f) = \frac{0.11f^2}{1+f^2} + \frac{44f^2}{4100+f^2} + 2.75 + 10^{-4}f^2 + 0.003 \quad [dB/km] \tag{4}$$

Authors in [37] argue that the absorption coefficient $\beta(f)$ is a function of the signal frequency f, water salinity S, water temperature T, speed of acoustic wave propagation c, water pH and water depth z. Taking into account this claim, (2) can be re-written as

$$10logA(d,f) = 10logN_0 + 10llogd + 10dlog\beta(f,S,T,c,pH,z) \quad (5)$$

The Monterrey–Miami Parabolic Equation (MMPE) includes the effect of activity on the surface, shapes of the sea floor and changes in the salinity of the water. Hence, the model gives a realistic approach to transmission loss prediction and better underwater acoustic propagation prediction. According to the MMPE model, the propagation loss can be written as [40], [41].

$$L_p(t) = \gamma(f,d,z_S,z_Q) + w(t) + n_e(t) \quad (6)$$

where $L_p(t)$ is the propagation loss of signal traveling from source S to destination Q on the depth z_S and z_Q (in meters) respectively. $\gamma()$ is a function of propagation loss with parameters, and $w(t)$ denotes a periodic function for signal loss due to wave movement at time t. Additional random noise is denoted by $n_e(t)$.

Signal loss during transmission (TL) is the reduction in the sound intensity while traveling from the source to the destination node. The signal transmission loss is formulated in (7) [42], [43].

$$TL = s_f + \beta R \times 10^{-2} \quad (7)$$

where the frequency is represented by f in kHz, the range is denoted by R in meters and s_f is the Spherical Spreading factor.

3.2. Underwater Noise/Interference

Underwater noise/interference types can be broadly categorized into ambient noise and site-specific noise [37]. The ambient noise is always present in the background, while the site-specific noise is intermittent [44]. The nature of noise can be non-stationary, colored, and often deviates from the standard Gaussian distribution. Therefore, it is very important to understand the nature of noise and the challenges posed by different kinds of noise when developing approaches in underwater beamforming. In underwater settings, the noise and interference sources vary and include natural and anthropogenic elements. Natural sources such as marine life activity, wind-driven waves, and precipitation contribute significantly to the background noise. On the other hand, anthropogenic sources include ship traffic, industrial activities, sonar systems, and other man activities, all adding complexity to the noise environment [45], [20]. Figure 1 shows the challenging underwater environment for various applications due to various noise contributors.

- **Physical Noise Sources**: Underwater acoustic environments are formed by various natural noise sources, which pose significant challenges for underwater beamforming. Rainfall generates broadband noise from infrasonic to audible frequencies while wind-driven surface waves and wave breaking against coastlines introduce noise across a wide frequency spectrum [46, 47, 48]. Additionally, bubble formation, underwater seismic activity, and tectonic movements contribute to low-frequency rumblings and impulsive signals which further complicate underwater signal processing [49, 50, 22].

- **Biological Noise Sources**: Marine life, including fish, cetaceans, snapping shrimps, and pinnipeds produces a wide range of vocalizations for communication, navigation, and echolocation. These biological sounds range from low-frequency grunts to high-frequency clicks that add random transients and interference to the desired acoustic signals [51, 52, 53].

- **Anthropogenic Noise Sources**: Human activities, such as shipping, sonar operations, offshore wind farm construction and dredging introduce significant noise into the underwater environment. These activities generate continuous low-frequency noise, intense sonar pulses, and broadband noise from pile driving and drilling [54, 55, 56]. The cumulative effect of this can interfere with signals of interest and degrade the performance of underwater beamforming algorithms.

- **System Noise Sources**: Data quality can be degraded by the noise produced by recording devices, electronic interference from sensors and recording equipment [57].

3.3. Multipath Delay Spread

The underwater acoustic channel is inherently multipath-rich due to signal reflections from the sea surface, seabed, and submerged objects. These multipath components arrive at the receiver with varying delays and angles, resulting in both temporal and spatial smearing of the received waveforms. This leads to a loss of spatial coherence across the array and introduces ambiguity in direction-of-arrival (DoA) estimation [21]. Furthermore, the combined effect of multipath propagation and transmission loss gives rise to significant delay spread, which poses additional challenges for beamforming applications by distorting the temporal structure of the signal and reducing the SNR.

Delay spread happens due to the temporal dispersion of a signal caused by differences in the propagation paths between the source and receiver. In wideband underwater communication and localization systems, this phenomenon induces inter-symbol interference (ISI) and causes misalignment between the actual signal and the assumed steering vectors. As a result, conventional narrowband beamformers become suboptimal under high-delay-spread conditions [58, 59]. To address these problems, beamforming strategies must adopt robust multipath-resilient techniques, such as matched field processing, coherence-restoration algorithms,

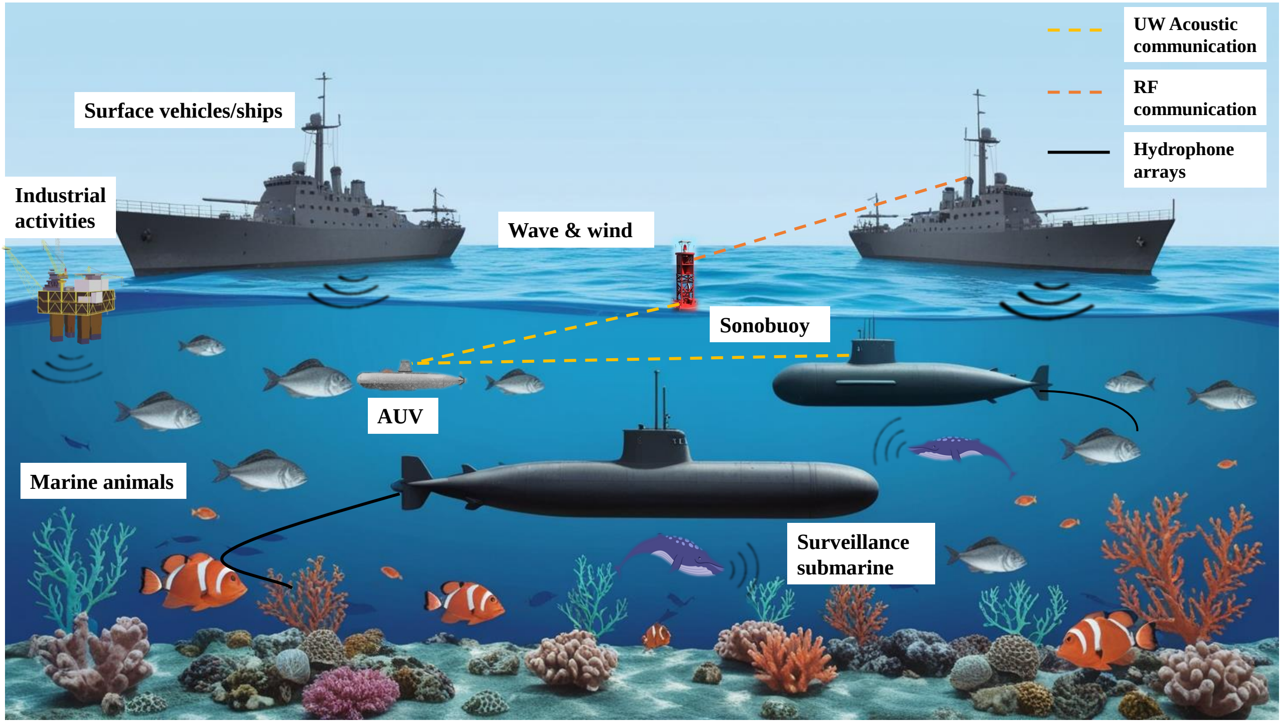

Figure 1: Illustration of underwater communication and sensing with various noise and interference.

or wideband space-time adaptive processing (STAP) frameworks that can model and compensate for the effects of propagation-induced dispersion.

3.4. Doppler Spread

Motion-induced Doppler effects arise from the relative velocity between the transmitter, receiver, and moving water masses. These effects introduce frequency shifts and spreads that distort phase alignment across array elements to disrupt the coherent beamforming. Doppler spread particularly affects moving platform scenarios such as autonomous underwater vehicles (AUVs) or drifting arrays [60, 25]. To mitigate these distortions, beamforming frameworks must incorporate Doppler-resilient designs, such as frequency tracking filters, motion-compensated delay alignment, or time-frequency adaptive beamformers that dynamically adjust to spectral variations.

In summary, while beamforming is a powerful mechanism for suppressing noise and interference, its effectiveness in underwater applications is fundamentally constrained by propagation-induced channel impairments. Robust underwater beamforming demands joint consideration of these physical-layer phenomena, along with signal processing solutions tailored to the highly dispersive and non-stationary characteristics of the ocean environment.

4. Underwater Beamforming methods: Sensing and Communication

This section provides a comprehensive evaluation of various underwater beamforming (UWB) methods organized into conventional beamforming, adaptive beamforming, learning-based beamforming, and some distinct novel approaches. These methods, originally developed for SONAR applications, are now increasingly adapted in underwater communication systems to enhance directionality, reduce BER, and improve network reliability. Each subsection discusses the core concept, application areas and performance of these beamforming techniques based on advantages and limitations in various underwater applications. Moreover, some DOA methods have also been discussed. In addition, we analyze the computational complexities associated with model-based baseline UWB approaches. Through this structured examination, we aim to highlight the strengths and weaknesses of various beamforming strategies and guide future research in underwater communication and sensing systems.

4.1. Conventional beamforming

The most common beamforming method is Conventional Beamforming (CBF), where the phase of the signals received by a specific array element is used as a reference. Through a process known as "phase alignment and accumulation," CBF converts the signal from the time domain to the spatial domain, much like the Fourier transform is applied in time-domain signal processing [12, 61]. Although CBF is highly adaptable, it has a limited array gain because of its simple summation technique. One of the fundamental forms of spatial filtering, called delay-and-sum beamforming, involves delaying the outputs from multiple sensors by specific amounts to align the spatial components of the signals coming from the target's direction, followed by summation, as shown in Figure 2. This method maximizes the average output power when the beamformer is steered toward a single target.

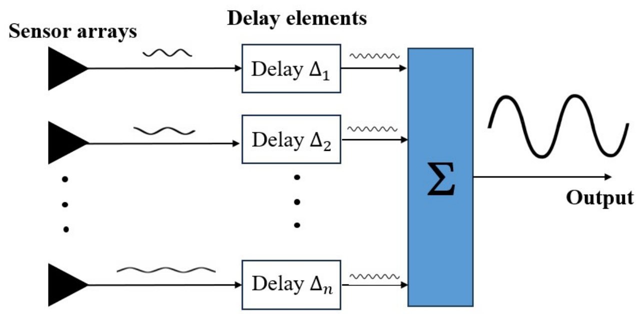

Figure 2: A conventional delay and sum beamformer.

However, the delay-and-sum beamformer has a key limitation that it cannot manage directional interferences. One study highlights that DAS beamforming provides noise suppression gains in underwater channels [62]. However, its effectiveness is limited by signal coherence length and inability to resolve closely spaced multipath arrivals. To address these limitations, the study suggests combining DAS with multichannel equalization, such as beam-domain decision feedback equalizers, to improve performance under low SNR and multipath conditions.

4.2. Adaptive underwater beamforming

As the adaptive beamformer places nulls in the direction of the interfering source, in this way, the output signal-to-noise ratio of the system is increased, and the directional response of the system is thereby improved. In this section, the mostly used adaptive beamforming methods have been discussed comprehensively. Moreover, different variants of these adaptive beamforming methods have been studied and summarized based on their application areas, advantages and limitations. Table 2 summarizes the strengths and limitations of various adaptive beamforming methods along with their applications. Additionally, based on the type of adaptive technique, we have organized the works into the following main categories:

- Minimum Variance Distortionless Response (MVDR) beamforming
- Linearly Constrained Minimum Variance (LCMV) beamforming
- Robust adaptive beamforming

4.2.1. Minimum Variance Distortionless Response (MVDR) beamforming

The Minimum Variance Distortionless Response (MVDR) beamforming, also known as Capon beamforming, is a popular adaptive beamforming technique that aims to minimize the power from interference and noise and maintain a distortionless response to the signal arriving from the desired direction. This means the beamformer maximizes the Signal-to-Interference-plus-Noise Ratio (SINR) by adjusting the weights applied to each sensor in the array.

Consider an adaptive beamformer that utilizes a linear array of M identical sensors, as depicted in Figure 3. The outputs from each sensor, assumed to be in baseband form, are weighted and summed. The beamformer must meet two key criteria: (1) the ability to steer the target signal to ensure its consistent protection, and (2) the minimization of interference effects. One way to meet these requirements is by minimizing the variance (or average power) of the beamformer's output and ensuring that the weights used during adaptation adhere to a specified constraint which is a distortionless response constraint in the case of MVDR.

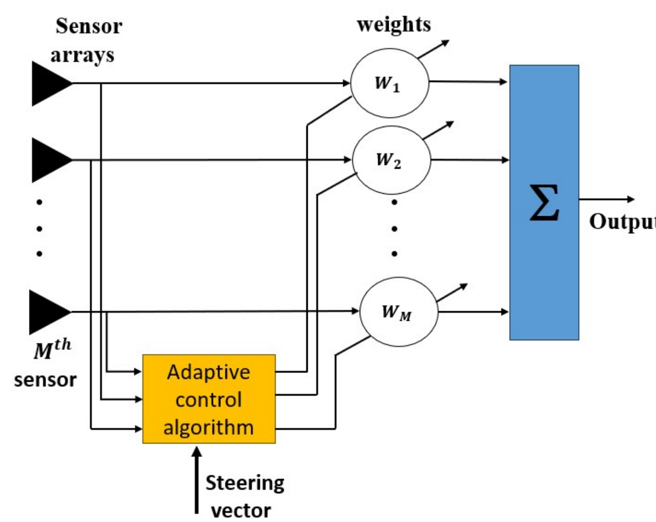

Figure 3: An adaptive beamformer with M number of hydrophone sensors.

$$W^H(n)V(\psi) = 1 \quad for\ all\ n \quad and \quad \psi = \psi^t \tag{8}$$

Here, W represents the $M \times 1$ weight vector, and $V(\psi)$ is the $M \times 1$ steering vector. The superscript H indicates the Hermitian transpose, which involves transposition combined with complex conjugation. Since the baseband data in this application are complex-valued, complex conjugation is necessary. The electrical angle ψ is determined by the direction of the target, with sensor 1 (located at the top of the array) serving as the reference point for measuring ψ. The relationship between the steering vector $V(\psi)$ and the angle ψ is defined as follows.

$$V(\psi) = [1, e^{-j\psi}, ..., e^{-j(M-1)\psi}]^T \tag{9}$$

The angle ψ is itself related to the incidence angle θ of a plane wave, measured for the normal to the linear array, as follows:

$$\psi = \frac{2\pi d}{\lambda} sin\theta \tag{10}$$

where, d refers to the spacing between neighboring sensors in the array, and λ is the wavelength. If $\mathbf{C_x} = E[\mathbf{x(t)x^H(t)}]$ is the covariance matrix of the received signal $x(t)$, then the weight vector for the MVDR beamformer can be written as

$$\mathbf{W}_{\text{MVDR}} = \frac{\mathbf{C_x}^{-1} V(\psi)}{V^H(\psi)\mathbf{C_x}^{-1} V(\psi)} \tag{11}$$

While the MVDR beamformer adapts in the data space, the process of interference cancellation can also take place in the beam space. To do this, the input data from the sensor array are transformed into beam space via an orthogonal multiple-beamforming network. The output is subsequently processed by a multiple sidelobe canceler to suppress interference from unknown directions. If the sensor outputs are equally weighted and exhibit a uniform phase, the array's response to an incoming plane wave at an angle θ, measured for the array's normal, can be represented by the following equation.

$$P(\psi, \alpha) = \sum_{n=-M}^{M} e^{jn\psi} e^{-jn\alpha} \tag{12}$$

In this case, $M = (2N + 1)$ denotes the total number of sensors in the array, with the center sensor acting as the reference point. The electrical angle ψ is linked to θ as detailed in (10), and α is a constant known as the uniform phase factor. The term $P(\psi, \alpha)$ represents the array pattern, which characterizes the array's response to variations in direction.

The Minimum Variance Distortionless Response (MVDR) beamformer is a powerful and widely used technique for enhancing signals from a specific direction while minimizing interference and noise. However, like all techniques, it has several limitations that affect its performance in practical application. MVDR assumes perfect knowledge of the steering vector, factors such as sensor positioning errors, environmental changes, or incorrect estimation of the DOA can lead to steering vector mismatches, which degrade the performance of the MVDR beamformer. Secondly, MVDR relies on an accurate estimation of the covariance matrix of the received signals. In practice, this matrix is estimated from a finite number of data snapshots. When the number of snapshots is small, the estimated covariance matrix can be inaccurate or ill-conditioned that leads to poor beamforming performance.

4.2.2. Linearly Constrained Minimum Variance (LCMV) beamforming

The Linearly Constrained Minimum Variance (LCMV) beamformer is a generalized form of the Minimum Variance Distortionless Response (MVDR) beamformer. While MVDR beamforming places single constraint to preserve the signal coming from a single direction, LCMV beamformer imposes multiple linear constraints to preserve the desired signals coming from different directions, and suppressing noise and interference from others. The linearly constrained minimization problem may be defined as that of finding the weight vector W which satisfies

$$\min_{W} W^H \mathbf{C_x} W \tag{13}$$

$$\text{subject to:} \quad \mathbf{Q}^H \mathbf{W} = R \tag{14}$$

where $\mathbf{W}$ is the beamforming weight vector $\mathbf{C_x}$ is the covariance matrix of the received signals. $\mathbf{Q}$ is the constraint matrix. R is the desired response vector. W^H is the conjugate transpose of the weight vector. The solution to the LCMV optimization problem, i.e., the weight vector W_{LCMV} can be computed as:

$$W_{\text{LCMV}} = \mathbf{C_x}^{-1}\mathbf{Q}\left(\mathbf{Q}^H\mathbf{C}^{-1}\mathbf{Q}\right)^{-1} R \tag{15}$$

LCMV is used to null interference sources by adding constraints that enforce nulls in the direction of interference while maintaining a distortionless response for the desired signal. It can be extended to wideband signals, where constraints are applied across multiple frequencies. LCMV is a remarkably flexible beamforming algorithm, however, it has some limitations. Like MVDR, LCMV requires an accurate estimate of the covariance matrix. Also, if the number of snapshots is small, LCMV may overfit to the noise which leads to poor generalization.

4.2.3. Robust adaptive beamforming

Robust adaptive beamforming is developed to improve the performance of traditional adaptive beamformers such as MVDR when faced with real-world issues like model mismatches, uncertainties, sensor errors, interference, and dynamic environments. In classical adaptive beamforming, the performance can degrade significantly when there are inaccuracies in the signal model, such as errors in the DOA estimation or noise covariance matrix. Robust beamforming methods introduce various techniques to make the system more resilient to these imperfections. Some of these techniques include diagonal loading, spatial averaging, generalized Sidelobe cancellation (GSC), and Robust Capon Beamforming (RCB).

- Diagonal loading is a technique introduced to improve the robustness of adaptive beamforming by modifying the covariance matrix used in the beamforming algorithm. It works by adding a small constant (called a loading factor) to the diagonal elements of the covariance matrix. This technique helps to stabilize the covariance matrix inversion and makes the beamformer more resistant to errors and noise. By introducing diagonal loading in the covariance matrix, the covariance matrix gets regularized even when the

BF Method	Variant method	Application	Advantages	Limitations
MVDR variants	Fast Broadband MVDR [63]	Target detection in SONAR systems	Narrower beam compared to Conventional Beamforming (CBF) in high SNR conditions. Reduced computational complexity.	Similar Performance in Low SNR. The algorithm's beam width is highly dependent on SNR
	Subarray MVDR beamformer [64].	Active Littoral Sonar Systems	More robust to errors in the signal model than CBF. Improved Bearing Resolution.	Using only two subarrays limits the ability to suppress multiple interference sources.
	MVDR and Low complexity adaptive (LCA) beamformer [65].	Active sonar for seafloor imaging in shallow-water environments	LCA offers improved performance over MVDR with fewer receiver elements. Improvements in SNR with small vertical arrays.	MVDR beamformer does not perform well in cases where there are very few receiver elements.
	Zero-Order MVDR (ZMVDR) beamformer [66].	Towed linear array sonar in antisubmarine warfare	It performs better than the MVDR beamformer in both Gaussian and non-Gaussian environments.	Computational complexity hasn't been discussed. Further generalization might be necessary for a wider range of conditions.
	MVDR beamformer with signal self-nulling [67].	Long distance passive SONAR	Improved DOA estimation under low SNR conditions. Superior performance in achieving sharper spectrum peaks.	Assumes prior knowledge of the angular sector where the desired signal is located. The complexity of the algorithm may require further work.
	MVDR focused beamformer [68].	Passive underwater acoustic localization	Significantly reducing pseudo-peaks and improving noise immunity The method enhances the accuracy of localization.	The MVDR-FB method, while superior, may still produce pseudo-peaks at low SNR.
	Pilot-based Frequency-Difference MVDR (P-FD MVDR) [21].	Underwater acoustic (UWA) communications	Addresses bearing ambiguity caused by the large element spacing in the receiving array. Improved resolution of multipath DOAs from different transmitters.	The complexity of algorithm increases with the number of pilots and directional signals, which might limit its practical implementation in scenarios with limited computational resources.
LCMV variants	LCMV with Frost space-time wideband beamforming [69].	Underwater acoustic communication	Integrates sensor patterns into the LCMV beamforming algorithm, making it adaptable to conformal arrays with arbitrary shapes. Creates frequency-invariant beam patterns that are important when dealing with wideband signals.	Integration of each sensor's individual response into algorithm can increase the complexity of the system therefore, significant resources are required for implementation.
	DAS beamforming LCMV with PID control optimization and Kalman filtering [70]	Hydroacoustic communication in underwater environments	Improves the accuracy of DOA estimation by iteratively correcting the direction. Robust in high SNR environments due to its ability to reduce the effect of small eigenvalue perturbations.	The combination of techniques may increase the complexity of implementation. Limited discussion on real-world implementation.
	LCMV with joint polarization-space matched filtering [71]	Underwater sonar system	Enhances signal-to-noise ratio (SNR) by adjusting the polarization of the elements of the conformal array to match the incident signal. The algorithm improves robustness in scenarios where traditional methods fail due to polarization mismatch.	The method is complex due to the curvature of the conformal array and requires sophisticated modelling. High computational complexity arises in implementing the algorithm for large array practical applications.
Robust adaptive beam-formers	MPD with diagonal loading beamformer [72].	Sonar system in shallow water	Provides an asymptotic characterization of the power estimators for snapshot-deficient conditions, which can be useful in real-world scenarios with limited data availability.	Limited snapshots are assumed for analysis, which might limit its applicability in environments where an abundance of snapshots is available.
	DR-MVDR (Diagonal Reduction) and spatial resampling [73]	underwater acoustic environments for DOA estimation	Enhances the azimuth resolution and output SINR by applying diagonal reduction to the covariance matrix. Spatial resampling helps in focusing wideband signals.	Requires a stable covariance matrix estimation, which is challenging when the number of snapshots is small. Due to eigenvalue decomposition for each sub-band, the approach is computationally complex.

(Continued on the next page)

Table 2
Summary of underwater adaptive beamforming in literature.

(Continued from the previous page)			
MVDR with diagonal loading and spatial averaging beamformer [74]	Active sonar imaging and underwater acoustic communications	Improves the computational performance of the beamformer using GPU implementation. The optimized GPU implementation allows for real-time processing of sonar data.	Due to the matrix inversion step in MVDR remains a bottleneck that limits speed improvements.
Wideband Robust Capon Beamformers (WBRCBs) and Wideband Subarray RCBs (WBSARCBs) [75]	Passive sonar system	The use of ellipsoidal steering vector uncertainty sets in the RCB framework improves robustness against Array Steering Vector (ASV) errors. Reduces the dimensionality and computational cost for large aperture arrays.	Higher sidelobes than WBRCB, particularly in scenarios with missing sensor elements. May have resolution loss due to ASV uncertainty sets which are also important for handling errors.
Robust Capon Beamforming (RCB) and Worst-Case Robust Adaptive Beamforming (WC-RAB) [76]	Passive sonar system	The proposed methods are designed to be computationally efficient with low complexity using reduced-dimension Krylov subspace, Kalman filtering, and gradient-based methods. The algorithms also show real-time adaptability.	The method still requires careful parameter selection which can be complex. The performance of WC-IG (iterative gradient minimization-based WC-RAB) may degrade in rapidly changing environments due to the slow convergence of the iterative scheme.

number of snapshots is small. Also, it makes the beamformer less sensitive to inaccuracies in steering vector or covariance matrix.

- Spatial avergaing or spatial smoothing is a technique used to improve the performance of beamformers in environments where there are coherent sources e.g., multipath signals or reflections. Traditional beamformers like MVDR can struggle when dealing with coherent signals, as they are highly correlated. Spatial averaging helps to decorrelate these sources and improves the beamformer's ability to reject interference without cancelling the desired signal. Thus, it offers improved resolution in highly correlated signal environments.
- The GSC is a robust beamforming architecture that divides the beamforming process into a blocking matrix, which blocks the desired signal and an adaptive filter that cancels interference. This structure makes it more resilient to changes in the signal environment.
- RCB includes uncertainty models in the desired signal's steering vector to improve robustness. It introduces bounds on the mismatch between the true steering vector and the assumed one. This makes the system resistant to DOA errors.

4.3. State-of-the-art DOA estimation methods

Direction of Arrival (DOA) estimation plays an important role in underwater beamforming applications [77]. Accurate DOA estimation is essential for determining the direction from which signals arrive at a sensor array. This helps the systems to isolate desired signals, filter out interference, and focus on communication beams. This has become critical in underwater environments where signals are often distorted by multipath propagation, interference, and noise, making robust DOA estimation techniques vital for enhancing the clarity and reliability of underwater surveillance and communication.

DOA estimation is a core component of array signal processing that relies on aligning signal phases received by multiple sensors to maximize signal strength from a desired direction along with minimizing interference from others [78]. This subsection explores various DOA estimation methods used in beamforming and underwater applications. Table 3 analyzes the model-based DOA estimation methods in terms of their computational complexity, robustness to noise and estimation accuracy. These methods can be broadly categorized as:

4.3.1. Subspace-based methods

Techniques such as MUSIC (Multiple Signal Classification) and its variants that exploit the signal and noise subspace separation for high-resolution DOA estimation are often used in large arrays with multiple closely spaced sources. These methods are highly accurate and provide high-resolution DOA estimation where the number of signals is known. However, they may be computationally expensive due to their reliance on eigenvalue decomposition and spectral search. MUSIC requires a large number of snapshots to achieve their high resolution and their performance may degrade in low SNR environments or when the array elements are insufficient to cover multiple sources.

4.3.2. Parametric methods

Parametric approaches include ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques) and its variants, which offer computational efficiency and real-time applicability by leveraging array structure information to model signal's parameters [93, 94]. The key assumption that makes it parametric is the rotational invariance property that arises from the sensor geometry. ESPRIT is computationally efficient because it doesn't perform spectral search like MUSIC. Parametric methods tend to perform better in real-time systems due to their efficiency but may suffer from model mismatch if the underlying assumptions

Method	Type	Computational Complexity	Robustness to Noise	Accuracy in DOA Estimation	Key Applications
			MUSIC		
Generalized MUSIC [79]	Subspace-Based	Moderate to High	Very High	High	Spatially correlated noise, underwater environments
GA-MUSIC [80]	Subspace-Based	Moderate	High	High	Real-time DOA estimation, optimized for large arrays
ANE-MUSIC [81, 82]	Subspace-Based	Moderate	Very high	Moderate	Noisy environments, limited snapshots
STFD-MUSIC [83]	Subspace-Based	High	High	High	Non-stationary, multipath signals
			ESPRIT		
ESPRIT [84]	Parametric	Moderate	Moderate	High	Real-time DOA estimation, underwater applications with moving targets
EA-ESPRIT [85]	Parametric	Moderate	High	High	Multi-source azimuth and elevation DOA estimation with sparse vector hydrophones
TVI-ESPRIT [86]	Parametric	High	Moderate	High	Velocity-independent underwater environments
TLS-ESPRIT [87]	Parametric	High	high	High	Model uncertainty scenarios, polarization diversity
			Sparsity Methods		
Compressive sensing [88]	Sparsity-based	Low	High	High	Sparse signal environments, coherent sources
CSS-CS (Coherent Signal Subspace and Compressed Sensing) [89]	Signal subspace and sparsity	Moderate	High	Very high	Wideband signals, weak targets, sparse environments
TPD-compressive sensing [90]	Sparsity-based	Moderate	High	High	Few element arrays, low-SNR environments
Sparse Spectrum Estimation [91]	Sparsity-based	Moderate	Very High	High	Strong interference, wideband underwater environments
CS based sparse reconstruction [92]	Sparsity-based	Moderate	Moderate	High	Faulty sensors, joint model-order estimation

Table 3
Comparison of model-based DOA estimation methods

about the sensor geometry or signal model are violated due to high power noise or interference.

4.3.3. Sparsity-based methods

Sparsity methods in DOA estimation leverage the concept that only a few signal sources are active in a spatial domain which makes the signal representation sparse. These methods are based on compressive sensing (CS) theory which exploits the sparse nature of the signal in a high-dimensional space to achieve accurate DOA estimation with fewer sensor measurements or lower computational resources [95, 96]. These approaches are particularly useful in scenarios with limited sensors, non-uniform arrays, or when dealing with underdetermined systems. Sparsity-based methods have been shown to outperform traditional beamforming algorithms in terms of resolution and robustness under challenging conditions like multipath propagation and coherent sources. In underwater acoustic environments, these methods improve the precision of source localization along with minimizing the computational load.

4.4. Computational complexity analysis

The computational complexity of various model-based beamforming is influenced by multiple factors, including the number of hydrophones, the number of sources, the number of scanning angles, and the number of snapshots

Algorithm	Hydrophones (N)	Snapshots (S)	Scanning angles θ, Sources (K)
DAS	$O(N)$	–	–
MVDR	$O(N^3)$	$O(N^2S)$	$O(N\theta)$
MPDR	$O(N^3)$	$O(N^2S)$	$O(N\theta)$
LCMV	$O(N^3)$	$O(N^2S)$	$O(N\theta)$
MUSIC	$O(N^3)$	$O(N^2S)$	$O(N\theta K)$
ESPRIT	$O(N^3)$	$O(N^2S)$	$O(N^2K)$

Table 4
Computational complexity of various beamforming algorithms.

used for covariance matrix estimation. Conventional DAS is the simplest algorithm where each sensor's signal is delayed and summed which involves straightforward arithmetic operations corresponding to number of hydrophones only. It doesn't depend heavily on the number of sources or snapshots as it simply sums the delayed signals. The number of scanning angles may increase the complexity slightly but remains low overall. Therefore, the above mentioned parameters significantly impact the overall processing time and resource requirements of each algorithm. In case of MVDR and MPDR, the number of hydrophones and snapshots affects matrix dimensions requiring covariance matrix inversion and estimation. The complexity of LCMV is dominated by covariance matrix inversion. The number of sources affects the constraints applied, slightly increasing the complexity. More constraints lead to an increase in matrix inversion complexity.

Moreover the number of scanning angles and snapshots influences the complexity of search-based methods like MUSIC, where grid searches over possible angles increase computation. ESPRIT avoids the grid search over scanning angles, making it more efficient compared to MUSIC when estimating multiple sources. The complexity grows with the number of sources but not with the number of scanning angles. Similarly, the number of sources adds complexity in methods that need to differentiate and manage multiple signal components. Thus, the computational burden of beamforming techniques is not fixed but scales based on these key parameters. Table 4 summarizes the computational complexities of different model-based algorithms based on number of hydrophones, snapshots, scanning angles and sources.

4.5. Learning-based beamforming

In recent years, machine learning (ML) techniques have made significant progress into beamforming, as other areas of signal processing. ML is subset of AI that enables Neural Networks (NNs) to learn from data, examples, and past experiences without the need for explicit programming. Unlike traditional model-based approaches, learning-based hybrid beamforming adopts a model-free perspective, establishing a nonlinear relationship between the input data such as the channel matrix and array outputs and the resulting beamformers. Neural networks are especially advantageous in this scenario as they excel at approximating complex nonlinear functions or identifying patterns when the solution space is divided by nonlinear boundaries.

This learning-based approach offers several key advantages over traditional model-based techniques. One significant benefit is its robustness, particularly when dealing with errors like mismatched received paths or inaccurately estimated channel gains and directions [97]. Learning algorithms can quickly adapt to new or incoming data by identifying patterns in the data, making them highly responsive to environmental changes. On the other hand, model-based beamformers often rely on statistical predictions and lack the adaptability provided by ML-based techniques. Additionally, after the training phase, ML methods typically involve lower computational complexity and a faster design process, further enhancing their efficiency [98].

In this section, an extensive literature review is conducted on data-driven learning techniques for beamforming in underwater environments. Table ?? compares various data-driven learning approaches in underwater applications, highlighting their advantages and limitations. Furthermore, based on the type of learning technique used in conjunction with traditional signal processing, the works are categorized into the following main categories:

- Sparse Bayesian Learning (SBL)
- FeedForward and unrolling Neural Networks
- Convolutional Neural Networks (CNNs) and ImageNet models
- Deep reinforcement Learning (DRL)

4.5.1. *Input formulation*

Properly formulating the input for UWB is essential to the effectiveness of data-driven learning techniques. While processing raw beamforming data directly is a simple approach, creating a more structured and meaningful input representation can greatly enhance the training of ML models. UWB community have explored various features depending on the nature of the problem and area of application. Some studies have employed time-domain features as input to their algorithms [99]. Then, SBL based algorithms are mostly fed with sparse representations or sparsity priors of the data [100]. Frequency-domain features have been extensively investigated to extract spectral peaks, energy distributions, and Doppler shifts caused by target motion. One study has demonstrated the usefulness of higher order statistics in the bispectrum domain to improve the recognition accuracy and resistance to noise further [101]. Researchers have also extracted phase and amplitude information of the waves from covariance and channel matrices which also helped in training of learning algorithms [102]. Various studies are compiled in Table 6 indicate different input features for the learning algorithms reported in the literature.

Learning method	Beamforming	Application	Advantages	Limitations
Sparse Bayesian Learning (SBL)	Online Bayesian compressive beamformer based on Kalman filtering (online-KSBL)[103].	Underwater acoustic imaging systems	Computationally efficient for long-term underwater imaging tasks. Effectively exploits the temporal correlation between snapshots that improves image quality.	The approach relies on a simplified model for the transition matrix which might not capture all the complexities of real-world underwater environments.
	Compressive beamforming and Kalman filtering combined with Rauch-Tung-Striebel (RTS) smoothing [104].	Underwater sonar systems	The non-iterative nature of CSBL significantly reduces the computational load compared to traditional iterative methods like MSBL. Theoretical proof of the algorithm's convergence ensures robustness even in noisy environments.	While the method performs well in high SNR conditions (≥30 dB), its recovery performance deteriorates significantly in lower SNR scenarios. Compromised performance occurs when the sources are not sparse.
	Super-resolution beamforming method [105].	AUV sonar systems	The method achieves better angular resolution compared to CBF. Reduces sidelobe levels and improves the accuracy of DOA estimation.	The performance is sensitive to hyperparameter selection, and incorrect parameters may lead to deteriorated performance. Extremely strong interferences could still pose challenges.
	Spectrum reconstruction of DOAs [100]	Underwater acoustic target classification	The method achieves better resolution in identifying targets' directions. Suppresses interference effectively and preserves detailed spectral structures.	The method relies on well-defined priors and inaccuracies here could degrade performance. Offers higher computational complexity.
FeedForward Neural Network (FNN) and Unrolling Neural Network (UNN)	Subarray beamforming with DAS [106].	Underwater source localization	The work effectively localizes multiple sources by employing a two-stage deep learning approach. Reduces reliance on environmental data due to the dependence on beamformed data only.	The method reduces dependence on environmental data, its performance relies heavily on the quality and variety of the training data. Therefore, generalization problems may occur.
	Deconvolution Beamforming[107].	Underwater source localization	Suppresses sidelobes and pseudo-peaks and improves SNR gain by about 10 to 20 dB compared to traditional methods like CBF, MVDR, and MUSIC. The Deconvolution-UNN leverages prior knowledge from the R-L algorithm that improves the generalization ability of the model.	The method is tested on a single-frequency dataset. Performance depends on the design of the unrolling structure, which may limit flexibility in other contexts.
Convolutional Neural Network (CNN) and ImageNet models	Auto-Correlation data matrix [108].	Underwater sonar arrays	The proposed CNN model shows higher accuracy and shorter estimation time. The method is scalable.	Overfitting is observed in some low-SNR conditions when using a small number of array elements. The model relies heavily on large datasets for training, which can be computationally intensive.
	DAS beamforming [109].	Underwater acoustic target detection in shallow water	Demonstrate strong anti-interference capabilities. Low computational complexity as compared to CNN.	The performance deteriorates with increasing distance between the source and the ULA. The method fails where the signal characteristics begin to resemble noise.
	CNN-based beamforming [110]	shallow-water underwater acoustic communications	Improves spectral efficiency. Performs well with a larger number of subcarriers. BFNet shows a better tradeoff between performance and complexity	Unsupervised learning is used which may have limitations compared to supervised techniques in some cases. Real-world validation is not presented.
	Short-time conventional beamforming (STCBF) combined with modal dispersion ranging [99].	Source localization and ranging in shallow-water environments	The proposed method is robust to fluctuations in shallow-sea environmental parameters. The attention mechanism in the ResNet model enhances the ability to extract crucial features from the beam-time domain	The model's effectiveness depends on the quality and quantity of the training data. where mode separation is insufficient, the method may not perform well.

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Table 5
Summary of learning-based hybrid UWB methods in literature.

(Continued from the previous page)

Learning method	Beamforming	Application	Advantages	Limitations
	Diagonal Beam Spectrum (DBS) feature (SA-U-NET)[101].	Hydro-acoustic detection and positioning.	Effective under non-Gaussian noise conditions and achieves high-resolution performance even at low SNR. The method surpasses the theoretical $10logM$ array gain by achieving up to $25logM$ array gain.	The method has some computational complexity due to high-order spectral calculations. The model's performance may degrade when there is a large discrepancy between training data and real-world noise data.
	Conventional Beamforming (CBF) (U-Net) [111].	Underwater acoustic imaging	This method provides a significant improvement in noise suppression and resolution of adjacent features. Computationally efficient.	The assumption of circular complex Gaussian random (CCGR) statistics may limit the generalizability to certain ocean environments.
	Swin Transformer + YOLOv5 + Ex-DeepSORT [112]	Underwater target tracking using Forward-Looking Sonar (FLS)	The method captures global and local features for improved object tracking and reduces identity switching. Adapts well to cluttered sonar environments.	Evaluation is limited to controlled lake/tank data and is not validated in open-sea or deep-water scenarios.
	Convolutional Block Attention Module (CBAM) with U-Net [113]	Shallow-water source localization	The proposed method improves robustness to SSP mismatches due to internal waves. CBAM effectively captures both spatial and channel-wise features with improved range estimation accuracy.	Depth estimation performance is still limited. The performance degrades under severe noise and real-world mismatches not covered by training.
Deep Reinforcement Learning (DRL)	Cooperative beamforming (CB) [114]	Underwater wireless sensor networks (UWSNs)	The paper develops a secure localization model for underwater environments, which is the first to apply CB-based physical layer security (PLS) in UWSNs. Use DRL to optimize both security performance and energy consumption.	The method requires offline training for the neural network, which could be computationally intensive. The system's performance is dependent on the accuracy of the AUV positioning and may be impacted by multipath propagation.
Markov Decision Process(MDP-DRL)	Cooperative beamforming with AUV anchors [115]	Localization in UWSNs	The proposed method enhances security against eavesdropping, reduces energy consumption via joint optimization and supports multi-agent coordination in multipath environments.	The method requires significant offline training time; performance may degrade under severe navigation/time-sync errors

4.5.2. Experimental datasets

The evaluation of beamforming algorithms has often relied on various datasets due to the complexities of the underwater environment and the challenges in collecting real-world data. As a result, numerous studies have used simulated/synthetic datasets and artificially generated noise in controlled conditions to assess their algorithms [116]. These datasets typically involve transmitting signals through water tanks using hydrophone arrays to study beamforming in highly controlled conditions. Simulated datasets are generated using software that models underwater acoustic propagation and array responses. These datasets allow researchers to test beamforming algorithms under different controlled scenarios, where factors such as SNR, interference, and array geometry can be precisely manipulated [117]. In contrast, only a limited number of studies have employed actual beamforming datasets from real-world experiments. The popular real-world datasets are listed below that can be used to validate the beamforming algorithms in different underwater environments.

- **SWellEx-96 dataset**: SWellEx-96 is a well-known open source dataset used extensively in underwater acoustics. It was collected during an experiment in shallow water off the coast of San Diego. The experiment involved acoustic sources transmitting signals through the water, which were recorded by a vertical and horizontal array of hydrophones. It is widely used in literature to evaluate beamforming and acoustic source localization algorithms. Data provides both single, multi-tone signals and ambient noise data, which can be used to test algorithms under realistic underwater conditions [118].
- **MakaiEx dataset**: MakaiEx is another well-known dataset that was collected in water depths ranging from 104 meters to 265 meters over frequencies from 500Hz to 50kHz using vector sensor arrays (VSA). The VSA measured both acoustic pressure and particle velocity, making it capable of detecting both vertical and horizontal directions of incoming sound waves. The primary goal of the experiment was to gather a wide range of acoustic data for applications such as high-frequency tomography, sonar systems, and underwater acoustic communications [119].

- **LOAPEX (Long-Range Ocean Acoustic Propagation Experiment)**: The experiment took place in 2004 in the northeast Pacific Ocean at various depths in deep waters. The study demonstrated understanding of coherence over long ranges, tidal influences on travel times, and shadow zone phenomena. This understanding can be useful for further research in ocean acoustic modeling and understanding of oceanic sound propagation for beamforming applications in deep shadow zones [120].

4.6. Other works on UWB

In addition to the above-mentioned UWB approaches, some distinct methods are being investigated by researchers in the field of communications, SONAR and underwater acoustic imaging [121]. Table 7 summarizes these works based on their advantages and limitations. In addition, authors in [122] proposed an Andrew's sine estimation (ASE) algorithm that adapts well with impulsive noise. The study demonstrated superior SINR in low Generalized SNR environments. However, there is a mismatch between the assumed steering vector and the true steering vector of the desired signal. The paper does not extensively discuss the computational overhead introduced by the ASE function or how scalable the method is for larger arrays. One study [123] proposed an adaptive beamforming based on oblique projection (OP-ABF) to address the problem of steering vector mismatch in sonar systems. The OP-ABF can accurately remove steering vector mismatches in the received data and relies on little prior information. However, the method offers a trade-off between interference suppression and mismatch compensation.

Advances in 2D and 3D underwater array configurations have enabled volumetric beamforming for high-resolution acoustic imaging. Rypkema et al. [124] introduced a memory-efficient approximate 3D beamforming method that decomposes the volumetric steering process into sequential azimuth and elevation operations using low-rank approximations of the spatial Green's function. This approach significantly reduces memory requirements while maintaining comparable image quality to full-resolution 3D processing, though its accuracy may diminish in scenes with strong vertical features. Alternatively, authors in [125] proposed spatial matched filtering for 2D arrays to perform 3D beamforming by analytically modeling the array's spatial impulse response. While this method enhances spatial resolution and SNR across both azimuth and elevation, its performance is susceptible to sensor placement errors and environmental complexity. These contributions demonstrate the growing feasibility of real-time underwater volumetric imaging using compact array platforms.

5. Evaluation metrics

Performance evaluation is important for determining the effectiveness of beamforming techniques. The literature uses various metrics to assess UWB performance with a predominant focus on Signal-to-Noise Ratio (SNR) and Signal-to-Interference-and-Noise Ratio (SINR). These metrics quantify the level of the desired signal relative to background noise and interference and provide insights into signal clarity and quality. It is important to note that the relevance of evaluation metrics often depends on the specific application domain within underwater acoustics. Bit Error Rate (BER) and SNR are critical in underwater communication scenarios due to bandwidth and energy limitations. Beamforming directly impacts these metrics by improving link robustness and reducing packet errors. SINR is used in sensing applications where it is necessary to isolate weak target signals from strong background interference. In contrast, beamwidth is highly relevant in imaging tasks, such as seafloor mapping or object localization, where spatial resolution is a key requirement. MSE and RMSE are commonly employed in DOA estimation algorithms. Table 8 highlights the most frequently used evaluation metrics in the literature, along with their corresponding application domains.

Input	Feature domain	Architecture	Literature
FFT coefficients	Frequency	FFNNs and LSTM	[106]
Sparse representations	Spatial	Dictionary learning	[126]
Channel matrix	Frequency	CNN (BFNet)	[110]
Auto correlation matrix	Frequency	CNN	[108]
Estimated DOA	Spatial	CNN	[109]
Sound intensity maps	Time	Attention ResNet	[99]
Frequency shifts invariant	Frequency	Bayesian learning	[100]
Diagonal beam spectrum	Frequency	Self attention UNet	[101]
Phase components	STFT	CRNN	[102]

Table 6
Summary of input features in learning-based beamforming approaches.

Therefore, this section provides a summary of the evaluation metrics frequently used in model-based and data-driven approaches in UWB research. The metrics listed here are often used in SONAR systems and communication systems to analyze the performance of beamforming methods.

5.1. Signal-to-Noise Ratio (SNR)

SNR measures the power of the desired signal relative to the background noise. It is an important metric for evaluating how well a beamformer can extract the signal of interest from the noisy environment [68, 127]. If P_{signal} and P_{noise} are the power of the signal of interest and noise, respectively, then

Application	Beamforming technique	Advantages	Limitations
Underwater acoustic communication	Ultra-Wideband (UWB) beamforming [127]	SBL is sensitive to array position errors. The Acoustic RIS significantly improves the SNR. Achieves the same data rate as traditional systems with much lower power consumption.	Introduces significant complexity. Narrowband assumptions are made for some parts of the system, which may hinder its applicability in real-world conditions.
	Suppressing Multiple Unsteady Sub-Gaussian Interferers (SMUSGI) beamformer [128]	Improved suppression of unsteady interferers. The proposed method converges faster with fewer samples.	Its performance drops when the input SNR is extremely low, as it focuses more on suppressing interference than noise. The method involves complex optimization techniques.
	Time-Frequency-Time with Cross Power Spectral Density (TFT-CPSD) beamforming [129]	The method outperforms existing methods in terms of DOA accuracy under low SNR conditions. Offers a lower complexity solution without sacrificing too much accuracy. Minimizes energy leakage between overlapped signal segments.	The DOA estimation fluctuates over time due to environmental changes such as turbulence. The method can't perform well under shallow water conditions due to signal multipaths.
	Deconvolved Conventional Beamforming (DCB) [58]	The DCB method provides superior angular resolution. Improves the SNR of received signals and lowers the BER.	The DCB method involves greater computational complexity. Effectiveness in more turbulent or rapidly changing underwater environments needs to be tested.
	Angle-based beamforming with null steering [25]	Beamforming for transmitter with low feedback overhead. The method effectively suppresses the interference from multipath components and improves BER.	Performance depends on the reliable estimation of the principal path angle. Highly reverberant or multipath-dense environments may degrade this estimation.
	Transmit beamforming using Zero-Forcing (ZF) linear precoding [130]	Instead of transmitting full CSI, receivers send back only grid indices, reducing overhead by $4000\times$. Accurate grid-based localization improves beamforming performance.	Higher grid resolution improves accuracy but increases computational complexity.
	Signal-Space-Frequency Beamforming (SSFB) [131]	The method enables transmission of compressed video over short/medium-range underwater channels and is robust to mobility.	At low SNR, the performance degrades under high Doppler conditions. The beamforming is dependent on the accuracy of localization.
Active sonar systems	Striation-based beamforming [132]	A significant increase in signal gain. Tested with real-world underwater data. Improved multipath handling.	If the target range or waveguide conditions are not well-known, there can be striation pattern estimation errors.
	Optimal adaptive transmit beamforming for cognitive MIMO [133]	The method achieves better localization performance compared to traditional non-adaptive beamformers.	The model may be sensitive to strong environmental mismatch.
Passive sonar systems	Modified Differential Beamforming (MDBF) [134]	The proposed method offers significantly better robustness than DBF, especially in noisy underwater environments. Improves DOA estimation accuracy at low frequencies compared to CBF and DBF.	The robustness improvement comes at the cost of some directivity loss. MDBF introduces additional complexity due to the optimization problem and the need to solve for the Lagrange multipliers.
	Near-Field Interference Mitigation (NFIM) beamformer [135]	The beamformer enhances the DOA estimation of far-field sources by mitigating near-field interference. Reduced computational complexity. Reduced sensitivity to localization errors of near-field interferences.	The optimal configuration of subarrays requires careful tuning, which might be challenging for practical implementations in varying underwater environments. Focuses on interferences rather than noise.
Underwater acoustic imaging	Iterative-Convex Optimization method for beamforming [33]	The iterative-convex method requires fewer iterations to converge to the desired beam pattern. Offers robustness to quantization errors.	The current implementation of the method is limited to narrowband signals. The method offers offline optimization and may not be suitable for real-time applications.
	Feature-Enhanced Beamforming (FEBF) with L1-total variation (L1-TV) mixed norm regularization [136]	The FEBF method improves the imaging quality significantly by offering higher resolution. Offers adaptability to clearer target contours and more accurate imaging outputs.	Despite the acceleration strategies, the FEBF method still involves significant computational complexity. The method relies on the availability of prior knowledge about the sparsity and local density characteristics of the scene.

(Continued on the next page)

Table 7
Summary of other distinct UWB approaches in literature for various UWB applications.

Application	Beamforming technique	Advantages	Limitations
	Sparse Synthetic Aperture Beamforming (SSAB) [137]	Offers high frame rate 3D imaging.Utilizes a 1D array with mechanical scanning, which is more feasible than fabricating and processing data from a full 2D array.	Reduced number of transmit events inherently lowers signal strength. The system's resilience to phase aberrations from overlying tissues was not established.
	Generalized wave-number domain unified beamforming [138]	Provides a single theoretical framework encompassing various sonar modalities. Introduces a polar wave number spectrum perspective to compare the resolution and spatial coverage of each method.	Requires high computational power and memory due to tomographic and SAS methods that involve interpolation in wave-number space and inverse transforms. High sensitivity to the platform movements.

Table 8
Evaluation metrics in different underwater applications.

Metric	**Application Domain**
SNR	Underwater communication systems
SINR	Passive sonar, surveillance systems
Beamwidth	Seafloor imaging, object localization
Array Gain (AG)	Sparse/hybrid arrays, low-power monitoring
MSE / RMSE	DOA estimation, AUV tracking
Classification Accuracy	Target recognition, signal classification
MAE	Robust DOA regression, angle tracking
BER	Underwater digital communications
Convergence Rate	Iterative/adaptive beamforming algorithms

SNR is given by:

$$\mathrm{SNR} = \frac{P_{\text{signal}}}{P_{\text{noise}}} \tag{16}$$

$$\mathrm{SNR}_{\text{dB}} = 10 \log_{10} \left(\frac{P_{\text{signal}}}{P_{\text{noise}}} \right) \tag{17}$$

5.2. Signal-to-Interference-and-Noise Ratio (SINR)

The Signal-to-Interference-and-Noise Ratio (SINR) is a key metric used in communication systems and underwater beamforming, to evaluate the quality of the signal received in the presence of both interference and noise [76, 69, 122]. It represents the ratio of the power of the desired signal to the combined power of interference and noise.

The SINR is mathematically expressed as:

$$\mathrm{SINR} = \frac{P_{\text{signal}}}{P_{\text{interference}} + P_{\text{noise}}} \tag{18}$$

$$\mathrm{SINR}_{\text{dB}} = 10 \log_{10} \left(\frac{P_{\text{signal}}}{P_{\text{interference}} + P_{\text{noise}}} \right) \tag{19}$$

5.3. Beamwidth

Beamwidth is the angular width of the main lobe of a beamformer's output, measured at a specific point where the power drops to half of its maximum value (commonly at -3 dB) [105, 100, 33]. It reflects the spatial resolution of the beamformer. If θ_1 and θ_2 are the angles at the half points, the beamwidth is written as:

$$\mathrm{BW} = \theta_2 - \theta_1 \tag{20}$$

5.4. Array gain (AG)

AG measures the improvement in SNR when using an array of sensors compared to a single sensor. It is defined as the ratio of the output SNR of the beamformer to the input SNR [101, 132]. It can be expressed as follows:

$$\mathrm{AG} = \frac{\mathrm{SNR}_{\text{out}}}{\mathrm{SNR}_{\text{in}}} \tag{21}$$

5.5. Mean Square Error (MSE)

The MSE measures the average squared difference between the estimated and actual values. It is commonly used to evaluate the performance of estimation algorithms, such as DOA estimation in beamforming [72, 139, 107]. If N denotes the total number of samples, $\hat{x}_i$ is the estimated value and x_i is the actual value then the MSE can be computed by the following expression:

$$\mathrm{MSE} = \frac{1}{N} \sum_{i=1}^{N} (\hat{\theta}_i - \theta_i)^2 \tag{22}$$

Root Mean Square Error (RMSE) is often used instead of MSE to evaluate the accuracy of DOA estimation algorithms and can be written as:

$$RMSE = \sqrt{MSE} \tag{23}$$

5.6. Classification Accuracy

Classification accuracy is a metric used in machine learning or decision-making beamforming systems to evaluate how well the system classifies signals (e.g., target vs. interference) [108, 102, 109]. It is defined as the ratio of correctly classified instances to the total number of instances.

$$\text{Accuracy} = \frac{\text{Number of Correct Predictions}}{\text{Total Number of Predictions}} \tag{24}$$

$$= \frac{TP + TN}{TP + TN + FP + FN} \tag{25}$$

where TP and TN denote the number of true positives and true negatives respectively, FP and FN represent false positives and false negatives respectively.

5.7. Mean Absolute error (MAE)

The MAE is a metric that measures the average magnitude of errors between predicted and actual values. Unlike MSE, it does not square the differences, making it less sensitive to outliers [106, 33].

$$\text{MAE} = \frac{1}{N} \sum_{i=1}^{N} \left| \hat{x}_i - x_i \right| \tag{26}$$

5.8. Bit Error Rate (BER)

BER measures the ratio of incorrectly received bits to the total number of transmitted bits in a communication system. It evaluates the system's ability to transmit data accurately. It is often used as a metric for assessing beamforming performance in communication systems [129, 21]. If the number of bit errors is denoted by Bit_e and the total number of transmitted bits is represented by Bit_{tx}, the BER can be calculated by the following expression:

$$\text{BER} = \frac{\text{Bit}_\text{e}}{\text{Bit}_\text{tx}} \tag{27}$$

5.9. Convergence rate

The convergence rate refers to how quickly an algorithm converges to the optimal solution. For iterative algorithms like those used in adaptive beamforming, the convergence rate is usually evaluated by measuring how fast the error decreases with iterations [128, 108]. If the error $E(k)$ after the $k^t h$ iteration is

$$E(k) = f(\theta_k - \theta') \tag{28}$$

Then the convergence rate can be represented as the rate at which $E(k)$ decreases as k increases is given by:

$$\text{Convergence Rate} = \lim_{k \to \infty} \frac{E(k+1)}{E(k)} \tag{29}$$

Convergence is usually considered successful when $E(k)$ reaches below a predefined threshold.

6. Performance comparison of UWB methods

Figure 4 provides a comparative analysis of various underwater beamforming techniques based on two performance metrics: BER in underwater communication systems and output SINR in passive SONAR-based target detection.

In Figure 4a, the BER performance of seven beamforming methods: PFD-MVDR [21], MB-WCPO-MRC [31], MB-dCv-MRC [58], Trans-BF-OFDM [25], TFT-CSDM [129], TFT-CPSD [130], and SSFB [131], are evaluated against varying input SNR levels. The MB-dCv-MRC and TFT-CPSD methods demonstrate superior robustness, achieving BERs below 10^{-4} at SNRs as low as 10 dB. PFD-MVDR and Trans-BF-OFDM also perform well, maintaining BERs in the range of 10^{-3} to 10^{-4} under moderate SNRs. In contrast, SSFB, while effective at low SNRs, shows a slower rate of improvement as SNR increases.

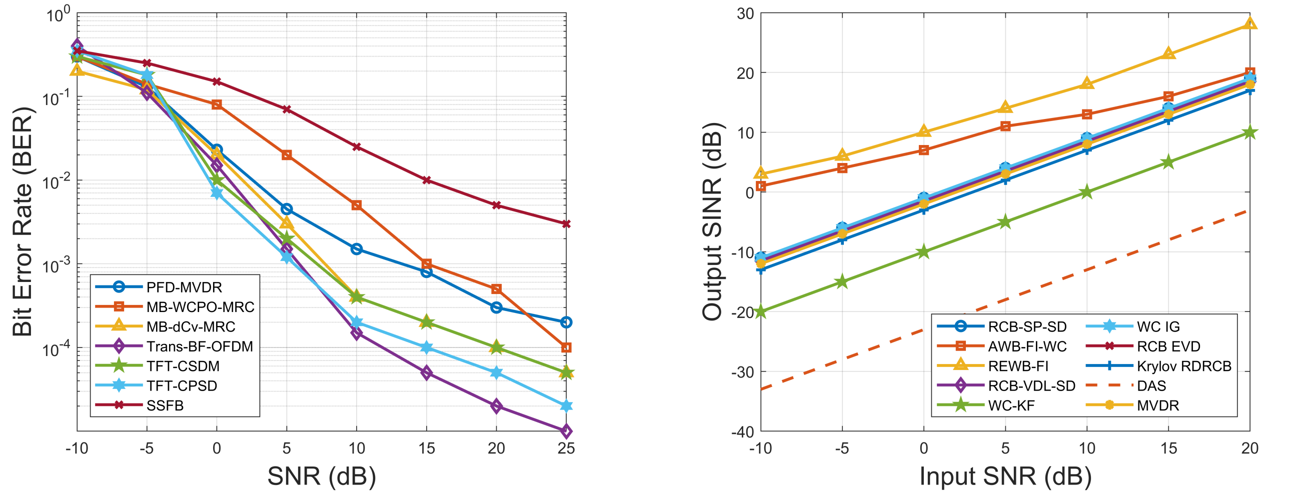
In Figure 4b, which highlights target detection performance in passive SONAR, the Output SINR is plotted against Input SNR for ten beamforming techniques. REWB-FI [140] and RCB-EVD [76] exhibit the highest SINR gains, approaching 30 dB at 20 dB input SNR. RCB-SP-SD, RCB-VDL-SD, and Krylov-RDRCB methods deliver consistent and reliable performance across all SNR levels. Meanwhile, WC-KF lags, due to sensitivity to environmental changes and mismatch errors. The DAS method consistently underperforms due to its limitations in complex acoustic environments.

Moreover, we have organized the numerical results of various state-of-the-art learning based beamforming approaches in Table 9. This table reports the numerical performance of learning-based UWB beamforming methods across a range of underwater acoustic applications. The performance metrics include root mean square error (RMSE), classification accuracy, spectral efficiency, and localization precision, evaluated over synthetic, simulated, and real-world datasets. Several studies demonstrate high localization fidelity, with RMSE values as low as 6.06e-5. Moreover, classification accuracy exceeding 99% in real experimental conditions demonstrates the efficacy of deep learning frameworks in discriminative tasks involving underwater targets.

Communication-oriented metrics further highlight significant advancements. For instance, one study reports spectral efficiency values of up to 190 bit/s/Hz. Additionally, methods integrating hybrid datasets that combine simulation (e.g., BELLHOP, Kraken) with empirical measurements and achieve robust performance under environmental uncertainty, with prediction accuracies reaching over 95%.

7. Future Research Directions

Underwater beamforming (UWB) research remains a highly dynamic field, driven by the demands of emerging applications and rapid theoretical advancements. Despite the progress, UWB still faces numerous challenges and limitations that present significant opportunities for future exploration. In this section, several recommendations for the research community are outlined based on the insights gained in this review. These suggestions aim to address both general methodological issues and specific aspects of UWB technology. We believe that pursuing these directions will not only enhance the performance of existing UWB



(a) (b)

Figure 4: Performance of beamforming approaches in (a) Underwater acoustic communications, (b) SONAR systems for target detection.

Table 9
Performance of learning-based UWB approaches included in this review.

Research	Data Set	Numerical Results
[103]	Synthetic data and own experimental data	Recovery performance over measurements: Avg RMSE is 0.057
[105]	Synthetic data	Recovery performance is reported as avg RMSE: 4.8e-1 over multiple measurements.
[104]	SWellEx-96 experiment	Recovery performance is demonstrated in terms of avg RMSE: 6.06e-5 over multiple measurements.
[100]	Real own experiment data	Mean classification accuracy over feature dimensions: 92.86%
[106]	Simulated and SWellEx-96 experiment	Positive Detection Rate is 100%, False Detection Rate is 0%, MAE (Azimuth): 2.74°, MAE (Range): 0.11 km
[108]	Simulated data	At SNR 10 dB, DOA accuracy for 10 and 20 sensors: 97.65% and 98.75% respectively
[109]	Real own experiment data	Detection Accuracy: 93.81% (2016O1), 97.94% (2016O2), 97.32% (2018O); False Alarm < 7%; Detection Speed: 14.5 ms/sample
[110]	Simulated data	Spectral efficiency: 110 bit/s/Hz (64 subcarriers), 190 bit/s/Hz (512 subcarriers)
[99]	Kraken mode simulated data + own field experiment data	MRE: 0.07, RMSE: 0.90 km, Prediction Accuracy is 85%
[101]	Bellhop + SWellEx-96 experiment	Classification accuracy: up to 99.5%, SNR gain: up to 47.3 dB output SINR
[111]	Own simulated and real experiment data	3 dB beamwidth reduction, Max RMSE (DOA): 0.09°
[112]	Custom experimental data	State-of-the-art mAP (94.4%), robust multi-target tracking
[113]	RAM model + own experimental data	>70% range accuracy with ±20 m thermocline shift; MAE (range):6.91 km, MAE (depth): 30.52 m
[114]	Own field experimental data	Max SLL: 11.46 dB, Min SLL:-20 dB, Localization rate: 0.43%
[115]	BELLHOP + Qingshitan Reservoir field data	Security capacity: 1.9e4 bps, BERs: 0%, 1.88%, 3.75%

systems but also contribute to a deeper understanding of the underlying concepts.

7.1. Expansion of real-world datasets

Many existing UWB studies rely on simulated datasets due to the difficulty of collecting real-world underwater data. However, real-world environmental variability and noise conditions are crucial for robust algorithm testing. The creation and expansion of real-world underwater beamforming datasets, which include long-term recordings under varying oceanographic conditions is essential. This would facilitate the development of more generalizable and resilient algorithms. Access to more diverse datasets would enable better benchmarking of algorithms and improve machine learning-based beamformers.

7.2. Model-Driven networks. for UWB

Pure data-driven models such as deep learning lack interpretability and are computationally expensive. In underwater environments, model-based approaches are more reliable but can be slow. Model-driven networks need to be explored that integrate the theoretical understanding of underwater signal propagation into deep learning models [141]. This could enhance interpretability while bounding the computational complexity.

7.3. Deep Unfolding (Deep Unrolling) for iterative beamforming

UWB algorithms such as MVDR, and LCMV rely on iterative methods, which are computationally intensive for large arrays and long-duration signals. Deep unfolding techniques need to be explored for UWB which will significantly reduce the number of iterations required for convergence. Researchers can combine the advantages of traditional optimization and machine learning by converting iterative algorithms into fixed-layer neural networks [142].

7.4. Joint sonar communications (JSC)

For several decades, underwater communication and sensing systems have traditionally operated in separate frequency bands to avoid interference and ensure reliable performance. However, due to limited bandwidth and the growing demand for improved performance in both domains, this conservative approach to spectrum access is becoming increasingly unsustainable. Recently, there has been growing interest in designing Joint Sonar and Communication (JSC) systems, where both functionalities share the same spectrum [143, 144]. From a beamformer design perspective, JSC integrates the distinct problem settings of communication and sensing that enable spectrum sharing and more efficient use of bandwidth without compromising the performance of either system. Thus, advanced beamforming designs must be investigated that are essential for realizing the full potential of JSC systems. This research will facilitate more efficient spectrum sharing, reduced interference, and enhanced functionality and reliability of underwater sensing and communication systems.

7.5. Near-field beamforming cases

Almost all studies in UWB have predominantly assumed far-field conditions and overlooked the complexities of near-field scenarios. Therefore, the exploration of near-field beamforming in underwater environments presents a significant area of potential. In near-field UWB, the principles of near-field beamforming require adaptations due to the unique propagation characteristics of sound in water. In underwater environments, the wavefront of the transmitted acoustic signal behaves differently depending on the distance to the receiver. In the near-field region, where the transmission range is shorter than the Fraunhofer distance, the wavefront exhibits a spherical shape as opposed to the plane wavefront seen in the far-field region. This near-field behavior leads to range-dependent variations in the beam pattern and impacts both the directionality and the resolution of the beamformer. Mathematically, the near-field condition for an underwater array can be described by a similar relationship, where the Fraunhofer distance D_{NF} is given by:

$$D_{NF} = \frac{2a^2 f_s}{c} \tag{30}$$

where a is the array aperture, f_s is the signal center frequency and c is the speed of sound. In this near-field region, the beam pattern is influenced by the range and direction. The array response vector for a uniform linear array (ULA) in the presence of near-field interference is a function of both direction θ and range d, which modifies the behavior of the beamformer:

$$a(\theta, d) = \frac{1}{\sqrt{M}} \left[e^{\frac{-j2\pi d_1}{\lambda}}, e^{\frac{-j2\pi d_2}{\lambda}}, \dots, e^{\frac{-j2\pi d_M}{\lambda}} \right]^T \tag{31}$$

Here, the distance d_M for each sensor element in the array depends on both the range and the direction of the transmitted signal. In the near-field, this variation introduces range-dependent interference that complicates the beamforming process, as each sensor receives a signal with a slightly different phase and amplitude due to its proximity to the source.

7.6. Real-time speed of neural networks

Although various advanced DL architectures have been explored, UWB applications demand high computational speed due to the dynamic and challenging nature of underwater environments. In such scenarios, the beamforming models must quickly adapt to changing contexts and noise conditions. Gradient-based DL models may struggle to meet these real-time adaptation and speed requirements. In contrast, deep randomized neural networks offer a promising alternative as they combine strong non-linear feature extraction capabilities with rapid training speed [145].

7.7. Self-supervised beamforming

While many underwater beamforming algorithms rely on supervised learning approaches, a few pioneering studies have begun to explore the potential of self-supervised learning for other underwater applications [146, 147]. However, a wide range of advanced self-supervised learning techniques remain largely unexplored and under-researched in this domain. Developing suitable pretext tasks is critical for the success of self-supervised learning algorithms, given the unique challenges posed by underwater acoustic data. Additionally, there is significant potential to design pretext tasks that are specifically tailored to the characteristics of underwater environments and acoustic propagation. Furthermore, integrating self-supervised learning with existing model-based and adaptive beamforming techniques offers a promising direction.

8. Concluding Remarks

Underwater beamforming is essential for improving communication, navigation, and surveillance in underwater environments, where single sensor-based methods struggle due to the unique propagation characteristics of sound waves in water. Beamforming facilitates precise localization of sound sources, interference mitigation and improved signal clarity. These attributes are crucial for the success of underwater sensor networks, marine environmental monitoring, and naval operations. However, the inherently complex nature of the underwater environment—characterized by non-stationary noise, multipath propagation and significant signal attenuation poses serious challenges to beamforming processes and applications.

This review has comprehensively analyzed various beamforming techniques, their associated challenges, and future research directions in the context of underwater environments. Conventional, adaptive and learning-based methods each demonstrate unique capabilities in addressing the complexities posed by underwater noise and interference. Despite progress, challenges such as the high computational complexity of advanced algorithms, limited generalization to non-stationary environments, and robustness against strong interference remain critical barriers to widespread adoption and real-world application. Furthermore, the survey of DOA estimation techniques and their computational costs highlights the growing need for real-time adaptability in beamforming that prioritize both speed and accuracy to meet the demands of dynamic underwater scenarios.

Recent advancements in deep learning (DL) have significantly impacted the field of underwater beamforming. DL-based methods offer the ability to optimize both signal processing and DOA estimation by learning complex patterns from noisy underwater environments. By leveraging data-driven models, these approaches are capable of handling diverse and challenging acoustic conditions, including non-Gaussian noise and dynamic interference. Moreover, DL models can integrate beamforming with recognition objectives using multi-term loss functions and enable task-oriented solutions that are more robust and adaptable. This capability extends beyond conventional methods, allowing simultaneous enhancement of SNR and target identification.

Insights from this review suggest that future research should focus on improving computational efficiency, developing generalizable models for non-stationary environments, and designing hybrid approaches that integrate conventional signal processing principles with deep learning techniques. Real-time implementation and scalability across diverse underwater scenarios remain key goals. The integration of multi-modal data, continual learning paradigms, and domain-specific transfer learning are promising avenues for further enhancing the robustness and adaptability of underwater beamforming systems.

CRediT authorship contribution statement

Ruba Zaheer: Conceptualization of this study, Methodology, Software, Original draft preparation. **Viet Quoc Phung:** Conceptualization of this study, Methodology, Software, Original draft preparation. **Iftekhar Ahmad:** Data curation, Methodology, Writing - Original draft preparation. **Asma Aziz:** Conceptualization of this study, Original draft preparation, Writing. **Daryoush Habibi:** Conceptualization of this study, Writing - Original draft preparation. **Yue Rong:** Conceptualization of this study, Writing - Original draft preparation. **Walid K Hasan:** Writing - Original draft preparation.

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