

# Joint Transceiver Optimization for Multiuser MIMO Relay Communication Systems

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**Abstract**—In this paper, we address the optimal source, relay, and receive matrices design for linear non-regenerative uplink multiuser multiple-input multiple-output (MIMO) relay communication systems. The minimum mean-squared error (MMSE) of the signal waveform estimation at the destination node is adopted as our design criterion. We develop two iterative methods to solve the highly nonconvex joint source, relay, and receiver optimization problem. In particular, we show that for given source precoding matrices, the optimal relay amplifying matrix diagonalizes the source-relay-destination channel. While for fixed relay matrix and source matrices of all other users, the source matrix of each user has a general beamforming structure. Simulation results demonstrate that the proposed iterative source and relay optimization algorithms perform much better than existing techniques in terms of both MSE and bit-error-rate.

**Index Terms**—MIMO relay, MMSE, multiuser, two-hop relay.

## I. INTRODUCTION

IN next generation wireless systems, multiple users equipped with multiple antennas will transmit simultaneously to the base station with multiple receive antennas and vice versa [1], [2]. Transceiver design for multiuser MIMO systems has been studied in [1]. The capacity of multiuser MIMO systems was investigated for flat fading channels in [2] using real channel measurement data.

Incorporating relays in a MIMO network can significantly extend the coverage and improve the link reliability of the network [3], [4]. MIMO relaying schemes can be categorized into two general groups: non-regenerative and regenerative schemes [3]. Compared with regenerative schemes, non-regenerative scheme has lower computational complexity and shorter delay, thus it has attracted much research interest [4]–[11]. The capacity of a single-user non-regenerative MIMO relay channel has been studied in [4]. In [5] and [6], the optimal relay amplifying matrix maximizing the mutual information (MI) between source and destination was derived assuming that the source covariance matrix is an identity matrix. In [7]

and [8], minimum mean-squared error (MMSE)-based approaches for MIMO relay systems have been studied. In [9], an iterative tri-step source precoder, relay amplifying matrix and destination equalizer design algorithm has been proposed for a single-user MIMO relay system with channel uncertainties. A unified framework was developed in [10] and [11] to jointly optimize the source precoding matrix and the relay amplifying matrices for a broad class of frequently used objective functions in MIMO relay system design.

For a multiuser MIMO relay system, the achievable sum rate has been derived in [12] using non-regenerative relaying scheme. In [13], both non-regenerative and regenerative relays have been considered in a multiuser MIMO network without optimizing the power loading schemes at the relay and the source nodes. An adaptive relay power allocation algorithm has been developed in [14] to mitigate the self-interference. An MMSE-based joint filter design has been proposed for a multiuser non-regenerative MIMO relay system in [15]. All these works [12]–[15] assume that each user is equipped with a single antenna. Several recent works have addressed multiuser MIMO relay systems where users also have multiple antennas. In [16], the optimal source and relay matrices were developed to maximize the source-destination MI. The non-regenerative MIMO relay technique has been applied to multi-cellular (interference) systems in [17]. The joint source and relay optimization problem has been addressed in [18] for multiple-antenna users using the MMSE criterion. The authors in [19] addressed the joint transceiver and relay design problem in a downlink (broadcast) multiuser system.

The main contribution of this paper is the joint source, relay, and receiver optimization for multiuser MIMO relay communication systems under the MMSE criterion where all nodes (users, relay, and destination) are equipped with possibly different number of multiple antennas. In contrast to [19], we consider an uplink (multiaccess) multiuser MIMO relay system. Note that although we consider the joint transceiver design problem for an uplink system, transceivers in a downlink system can be obtained by exploiting the uplink-downlink duality of MIMO relay channel [20], [21]. This problem has not been addressed in [3]–[17]. In particular, [3]–[11] considered the transceiver and/or relay design problems for single-user MIMO relay systems whereas [12]–[15] considered multiuser MIMO relay design problems with single-antenna transmitters. The problems addressed in [16] and [17] are also different from our problem. In this paper, we derive the optimal structure of the source precoding matrix of each user and the relay amplifying matrix to jointly minimize the MSE of the signal waveform estimation at the destination node in a multiuser MIMO relay

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system. The original optimization problem is highly nonconvex and a closed-form solution is intractable. To overcome this difficulty, we develop a Tri-Step iterative algorithm to jointly optimize the source, relay, and receive matrices through solving convex subproblems. It is shown that this algorithm is guaranteed to converge to (at least) a locally optimal solution. Note that the Tri-Step algorithm is not presented in [18].

To reduce the computational complexity of the Tri-Step algorithm, we develop a simplified Bi-Step algorithm, where the source and relay matrices are optimized in an alternating fashion. The receive matrix is not updated in each iteration, and instead, it is obtained as an MMSE receiver after the convergence of the Bi-Step algorithm. We show that for given source precoding matrices, the optimal relay amplifying matrix diagonalizes the source-relay-destination channel. While for fixed relay matrix and source matrices of all other users, the source matrix of each user has a beamforming structure. Simulation results demonstrate that both the proposed Tri-Step and Bi-Step iterative algorithms perform much better than existing techniques in terms of both MSE and bit-error-rate (BER). Moreover, it is shown that compared with the Tri-Step algorithm, the Bi-Step algorithm requires less number of iterations till convergence with only a small degradation in MSE and BER. Such performance-complexity tradeoff is very important for practical multiuser MIMO relay communication systems. We would like to mention that such Bi-Step algorithm is not considered in [19].

The rest of this paper is organized as follows. In Section II, the system model of a multiuser MIMO relay network is introduced. The iterative source, relay, and receive matrices optimization algorithms are developed in Section III. Section IV shows the simulation results which justify the significance of the proposed algorithms under various scenarios. Conclusions are drawn in Section V.

## II. SYSTEM MODEL

We consider a two-hop multiuser MIMO relay communication system as illustrated in Fig. 1 where  $K$  users transmit information to the same destination node with the aid of one relay node. The  $i$ th user,  $i = 1, \dots, K$ , the relay and the destination nodes are equipped with  $N_i$ ,  $N_r$ , and  $N_d$  antennas, respectively. We denote  $N_b = \sum_{i=1}^K N_i$  as the total number of independent data streams from all users, and assume that  $N_b \leq \min(N_r, N_d)$ , since otherwise the system cannot support  $N_b$  independent data streams simultaneously. For simplicity, as in [5]–[16], a linear non-regenerative strategy is applied at the relay node to process and forward the received signal.

We assume that the relay node works in the practical half-duplex mode. Thus, the communication between the users and the destination is completed in two time slots. In the first time slot, the  $N_i \times 1$  modulated signal vector  $\mathbf{s}_i$  is linearly precoded at the  $i$ th user by the  $N_i \times N_i$  source precoding matrix  $\mathbf{B}_i$ . The precoded signal vector

$$\mathbf{x}_i = \mathbf{B}_i \mathbf{s}_i \quad (1)$$

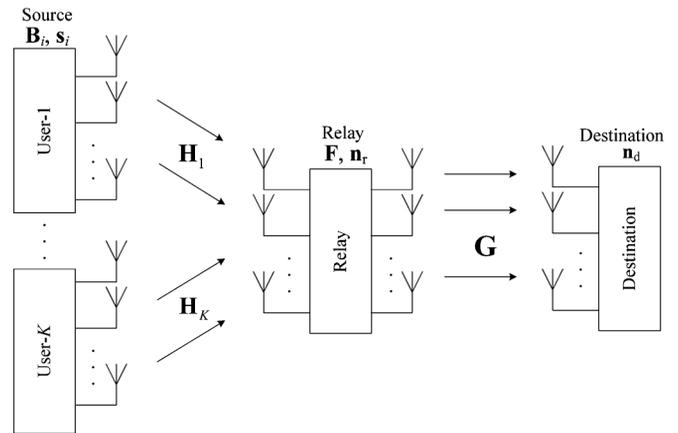


Fig. 1. Block diagram of a  $K$ -user linear non-regenerative MIMO relay communication system.

is transmitted to the relay node from the  $i$ th user. The received signal vector at the relay node can be written as

$$\mathbf{y}_r = \sum_{i=1}^K \mathbf{H}_i \mathbf{x}_i + \mathbf{n}_r \quad (2)$$

where  $\mathbf{H}_i$  is the  $N_r \times N_i$  MIMO channel matrix between the  $i$ th user and the relay,  $\mathbf{y}_r$  and  $\mathbf{n}_r$  are the received signal and the additive Gaussian noise vectors at the relay node, respectively. Substituting (1) into (2), we have

$$\mathbf{y}_r = \sum_{i=1}^K \mathbf{H}_i \mathbf{B}_i \mathbf{s}_i + \mathbf{n}_r = \bar{\mathbf{H}} \mathbf{s} + \mathbf{n}_r \quad (3)$$

where  $\bar{\mathbf{H}} \triangleq [\mathbf{H}_1 \mathbf{B}_1, \dots, \mathbf{H}_K \mathbf{B}_K]$  is the equivalent multiaccess MIMO channel matrix of the source-relay link,  $\mathbf{s} \triangleq [\mathbf{s}_1^T, \dots, \mathbf{s}_K^T]^T$  is the equivalent transmitted signal vector, and  $(\cdot)^T$  denotes matrix (vector) transpose. We assume that  $E[\mathbf{s} \mathbf{s}^H] = \mathbf{I}_{N_b}$ , where  $\mathbf{I}_n$  is an  $n \times n$  identity matrix,  $(\cdot)^H$  denotes matrix (vector) Hermitian transpose, and  $E[\cdot]$  stands for the statistical expectation.

In the second time slot, the users remain silent and the relay node multiplies (linearly precodes) the received signal vector  $\mathbf{y}_r$  by an  $N_r \times N_r$  relay amplifying matrix  $\mathbf{F}$  and transmits the signal vector

$$\mathbf{x}_r = \mathbf{F} \mathbf{y}_r \quad (4)$$

to the destination node. The received signal vector at the destination node can be written as

$$\mathbf{y}_d = \mathbf{G} \mathbf{x}_r + \mathbf{n}_d \quad (5)$$

where  $\mathbf{G}$  is the  $N_d \times N_r$  MIMO channel matrix between the relay and the destination nodes,  $\mathbf{y}_d$  and  $\mathbf{n}_d$  are the received signal and the additive Gaussian noise vectors at the destination node, respectively.

Substituting (3) and (4) into (5), we obtain

$$\begin{aligned} \mathbf{y}_d &= \mathbf{G} \mathbf{F} \sum_{i=1}^K \mathbf{H}_i \mathbf{B}_i \mathbf{s}_i + \mathbf{G} \mathbf{F} \mathbf{n}_r + \mathbf{n}_d \\ &= [\mathbf{G} \mathbf{F} \mathbf{H}_1 \mathbf{B}_1, \dots, \mathbf{G} \mathbf{F} \mathbf{H}_K \mathbf{B}_K] \mathbf{s} + \mathbf{G} \mathbf{F} \mathbf{n}_r + \mathbf{n}_d \\ &= \mathbf{H} \mathbf{s} + \mathbf{n} \end{aligned} \quad (6)$$

where  $\mathbf{H} \triangleq [\mathbf{G}\mathbf{F}\mathbf{H}_1\mathbf{B}_1, \dots, \mathbf{G}\mathbf{F}\mathbf{H}_K\mathbf{B}_K] = \mathbf{G}\mathbf{F}\bar{\mathbf{H}}$  is the equivalent MIMO channel matrix of the source-relay-destination link, and  $\mathbf{n} \triangleq \mathbf{G}\mathbf{F}\mathbf{n}_r + \mathbf{n}_d$  is the equivalent noise vector at the destination. We assume that the channel matrices  $\mathbf{H}_i$ ,  $i = 1, \dots, K$ , and  $\mathbf{G}$  are all quasi-static, i.e., the channel matrices are constant throughout a block of transmission and known to the relay and the destination nodes. In practice, the channel state information (CSI) of  $\mathbf{G}$  can be obtained at the destination node through standard training method. The relay node can have the CSI of  $\mathbf{H}_i$ ,  $i = 1, \dots, K$ , through channel training, and obtain the CSI of  $\mathbf{G}$  by a feedback from the destination node. The quasi-static channel model is valid in practice when the mobility of all communicating nodes is relatively slow. As a result, we can obtain the necessary CSI with a reasonably high precision during the channel training period. The relay node calculates the optimal source matrices  $\{\mathbf{B}_i\} \triangleq \{\mathbf{B}_i, i = 1, \dots, K\}$ , and the relay matrix  $\mathbf{F}$ , and forwards  $\mathbf{B}_i$  to the  $i$ th source node and forwards  $\mathbf{F}$  and  $\mathbf{H}_i$ ,  $i = 1, \dots, K$  to the destination node. Note that individual users do not require any channel knowledge. This is a very important assumption for multiuser communication since in a multiuser scenario the users are distributed and cannot cooperate. We assume that all noises are independent and identically distributed (i.i.d.) complex circularly symmetric Gaussian noise with zero mean and unit variance.

Due to its simplicity, a linear receiver is used at the destination node to retrieve the transmitted signals. Denoting  $\mathbf{W}$  as an  $N_d \times N_b$  weight matrix, the estimated signal vector  $\hat{\mathbf{s}}$  is given by

$$\hat{\mathbf{s}} = \mathbf{W}^H \mathbf{y}_d. \quad (7)$$

### III. PROPOSED SOURCE, RELAY, AND RECEIVE MATRICES DESIGN ALGORITHM

In this section we develop the optimal source precoding matrices  $\{\mathbf{B}_i\}$ , the relay amplifying matrix  $\mathbf{F}$ , and the destination receive matrix  $\mathbf{W}$  to minimize the MSE of the signal waveform estimation. Using (6) and (7), the MSE of the signal waveform estimation at the destination is given by

$$\begin{aligned} \text{MSE} &= \text{tr} \left\{ E [(\hat{\mathbf{s}} - \mathbf{s})(\hat{\mathbf{s}} - \mathbf{s})^H] \right\} \\ &= \text{tr} \left\{ (\mathbf{W}^H \mathbf{H} - \mathbf{I}_{N_b})(\mathbf{W}^H \mathbf{H} - \mathbf{I}_{N_b})^H + \mathbf{W}^H \mathbf{C}_n \mathbf{W} \right\} \end{aligned} \quad (8)$$

where  $\text{tr}\{\cdot\}$  is the trace of a matrix, and  $\mathbf{C}_n$  is the equivalent noise covariance matrix given by

$$\begin{aligned} \mathbf{C}_n &= E [\mathbf{n}\mathbf{n}^H] \\ &= E \left[ (\mathbf{G}\mathbf{F}\mathbf{n}_r + \mathbf{n}_d)(\mathbf{G}\mathbf{F}\mathbf{n}_r + \mathbf{n}_d)^H \right] \\ &= \mathbf{G}\mathbf{F}\mathbf{F}^H \mathbf{G}^H + \mathbf{I}_{N_d}. \end{aligned}$$

From (4), the power of the signal transmitted by the relay node can be expressed as

$$\text{tr} \left\{ E [\mathbf{x}_r \mathbf{x}_r^H] \right\} = \text{tr} \left\{ \mathbf{F} \left( \sum_{i=1}^K \mathbf{H}_i \mathbf{B}_i \mathbf{B}_i^H \mathbf{H}_i^H + \mathbf{I}_{N_r} \right) \mathbf{F}^H \right\}. \quad (9)$$

From (8) and (9), the joint source, relay, and receive matrices optimization problem for the linear non-regenerative multiuser MIMO relay system can be formulated as

$$\min_{\{\mathbf{B}_i\}, \mathbf{F}, \mathbf{W}} \text{tr} \left\{ (\mathbf{W}^H \mathbf{H} - \mathbf{I}_{N_b})(\mathbf{W}^H \mathbf{H} - \mathbf{I}_{N_b})^H + \mathbf{W}^H \mathbf{C}_n \mathbf{W} \right\} \quad (10)$$

$$\text{s.t.} \quad \text{tr} \left\{ \mathbf{F} \left( \sum_{i=1}^K \mathbf{H}_i \mathbf{B}_i \mathbf{B}_i^H \mathbf{H}_i^H + \mathbf{I}_{N_r} \right) \mathbf{F}^H \right\} \leq P_r \quad (11)$$

$$\text{tr} \left\{ \mathbf{B}_i \mathbf{B}_i^H \right\} \leq P_i, \quad i = 1, \dots, K \quad (12)$$

where (11) and (12) are the constraints for the transmission power at the relay and the  $i$ th user, respectively, and  $P_r > 0$ ,  $P_i > 0$  are the power budget available at the relay and the  $i$ th source node, respectively. The optimization problem (10)–(12) is highly nonconvex and a closed-form solution to this problem is intractable. In the following, we develop two iterative algorithms namely the Tri-Step and the Bi-Step algorithms to optimize the source, relay, and receive matrices. In the Tri-Step algorithm, the source, relay, and receive matrices are optimized iteratively through solving convex sub-problems. In the Bi-Step algorithm, the source and relay matrices are optimized alternately and the MMSE receive matrix is calculated after the convergence of the source and relay matrices. In particular, the relay matrix is optimized by the Lagrange multiplier method in the Tri-Step algorithm, and by the majorization theory in the Bi-Step algorithm. The optimal source matrices are obtained by solving semi-definite programming (SDP) problem in the Bi-Step algorithm, and by solving quadratically constrained quadratic programming (QCQP) problem in the Tri-Step algorithm.

#### A. Iterative Optimization of Source, Relay, and Receive Matrices (Tri-Step Algorithm)

This algorithm starts at a random  $\mathbf{F}$  and  $\{\mathbf{B}_i\}$  satisfying (11) and (12). In each iteration, the source, relay, and receive matrices are updated alternately through solving convex sub-problems. Firstly, with given  $\mathbf{F}$  and  $\{\mathbf{B}_i\}$ , the optimal  $\mathbf{W}$  is obtained by solving the unconstrained convex problem (10), since  $\mathbf{W}$  does not appear in constraints (11) and (12). The solution is the well-known MMSE receiver given by [22]

$$\mathbf{W} = \left( \mathbf{H}\mathbf{H}^H + \mathbf{C}_n \right)^{-1} \mathbf{H} \quad (13)$$

where  $(\cdot)^{-1}$  denotes matrix inversion.

Secondly, with given  $\mathbf{W}$  and  $\{\mathbf{B}_i\}$ ,  $\mathbf{F}$  can be updated by solving the following problem

$$\min_{\mathbf{F}} \text{tr} \left\{ (\bar{\mathbf{G}}\mathbf{F}\bar{\mathbf{H}} - \mathbf{I}_{N_b})(\bar{\mathbf{G}}\mathbf{F}\bar{\mathbf{H}} - \mathbf{I}_{N_b})^H + \bar{\mathbf{G}}\mathbf{F}\mathbf{F}^H \bar{\mathbf{G}}^H \right\} \quad (14)$$

$$\text{s.t.} \quad \text{tr} \left\{ \mathbf{F} (\bar{\mathbf{H}}\bar{\mathbf{H}}^H + \mathbf{I}_{N_r}) \mathbf{F}^H \right\} \leq P_r \quad (15)$$

where  $\bar{\mathbf{G}} \triangleq \mathbf{W}^H \mathbf{G}$  is the equivalent relay-destination MIMO channel. Using the Lagrange multiplier method, we obtain  $\mathbf{F}$  from (14)–(15) as

$$\mathbf{F} = \bar{\mathbf{G}}^H (\bar{\mathbf{G}}\bar{\mathbf{G}}^H + \mu \mathbf{I}_{N_b})^{-1} \bar{\mathbf{H}}^H (\bar{\mathbf{H}}\bar{\mathbf{H}}^H + \mathbf{I}_{N_r})^{-1} \quad (16)$$

where  $\mu \geq 0$  is the Lagrange multiplier associated with the power constraint (15). Interestingly, (16) can be viewed as  $\mathbf{F} = \mathbf{F}_2 \mathbf{F}_1$ , where  $\mathbf{F}_1 = \bar{\mathbf{H}}^H (\bar{\mathbf{H}} \bar{\mathbf{H}}^H + \mathbf{I}_{N_r})^{-1}$  is the weight matrix of the MMSE receiver for the equivalent first-hop multiaccess MIMO channel at the relay node given in (3), and  $\mathbf{F}_2 = \bar{\mathbf{G}}^H (\bar{\mathbf{G}} \bar{\mathbf{G}}^H + \mu \mathbf{I}_{N_b})^{-1}$  can be viewed as the transmit precoding matrix for the effective second-hop MIMO system  $\mathbf{y} = \bar{\mathbf{G}} \mathbf{F}_2 \mathbf{x} + \mathbf{v}$ , where  $\mathbf{x}$  is the transmitted signal vector with  $E[\mathbf{x} \mathbf{x}^H] = \mathbf{I}_{N_b}$ , and  $\mathbf{v}$  is the noise vector with covariance matrix  $\mathbf{C}_v = E[\mathbf{v} \mathbf{v}^H]$ . In this MIMO system, the MSE of estimating  $\mathbf{x}$  is given by  $\text{tr}\{E[(\mathbf{y} - \mathbf{x})(\mathbf{y} - \mathbf{x})^H]\} = \text{tr}\{(\bar{\mathbf{G}} \mathbf{F}_2 - \mathbf{I}_{N_b})(\bar{\mathbf{G}} \mathbf{F}_2 - \mathbf{I}_{N_b})^H + \mathbf{C}_v\}$ . The optimal  $\mathbf{F}_2$  that minimizes the MSE can be obtained by solving the following problem

$$\begin{aligned} \min_{\mathbf{F}_2} \quad & \text{tr}\{(\bar{\mathbf{G}} \mathbf{F}_2 - \mathbf{I}_{N_b})(\bar{\mathbf{G}} \mathbf{F}_2 - \mathbf{I}_{N_b})^H\} \\ \text{s.t.} \quad & \text{tr}\{\mathbf{F}_2 \mathbf{F}_2^H\} \leq P_x \end{aligned}$$

where  $P_x$  is the transmission power constraint. Using the Lagrange multiplier method to solve the problem above, we obtain  $\mathbf{F}_2 = \bar{\mathbf{G}}^H (\bar{\mathbf{G}} \bar{\mathbf{G}}^H + \mu \mathbf{I}_{N_b})^{-1}$ .

The Lagrange multiplier  $\mu$  in (16) can be found from the following complementary slackness condition

$$\mu (\text{tr}\{\mathbf{F}(\bar{\mathbf{H}} \bar{\mathbf{H}}^H + \mathbf{I}_{N_r}) \mathbf{F}^H\} - P_r) = 0. \quad (17)$$

Assuming  $\mu = 0$ , we have the following  $\mathbf{F}$  from (16)

$$\mathbf{F} = \bar{\mathbf{G}}^H (\bar{\mathbf{G}} \bar{\mathbf{G}}^H)^{-1} \bar{\mathbf{H}}^H (\bar{\mathbf{H}} \bar{\mathbf{H}}^H + \mathbf{I}_{N_r})^{-1}. \quad (18)$$

Since in this case (17) is already satisfied, if  $\mathbf{F}$  in (18) satisfies the constraint (15), then (18) is the solution to the problem (14)–(15). Otherwise, there must be  $\mu > 0$ , and from (17) we can see that  $\text{tr}\{\mathbf{F}(\bar{\mathbf{H}} \bar{\mathbf{H}}^H + \mathbf{I}_{N_r}) \mathbf{F}^H\} = P_r$  must hold. In this case,  $\mu$  can be obtained from (15) by solving the following non-linear equation

$$\begin{aligned} \text{tr}\left\{ \bar{\mathbf{G}}^H (\bar{\mathbf{G}} \bar{\mathbf{G}}^H + \mu \mathbf{I}_{N_b})^{-1} \bar{\mathbf{H}}^H (\bar{\mathbf{H}} \bar{\mathbf{H}}^H + \mathbf{I}_{N_r})^{-1} \bar{\mathbf{H}} \right. \\ \left. \times (\bar{\mathbf{G}} \bar{\mathbf{G}}^H + \mu \mathbf{I}_{N_b})^{-1} \bar{\mathbf{G}} \right\} = P_r. \quad (19) \end{aligned}$$

Let us now define the singular value decomposition (SVD) of  $\bar{\mathbf{G}} \triangleq \mathbf{U} \mathbf{\Lambda} \mathbf{V}^H$ , where the dimensions of  $\mathbf{U}$ ,  $\mathbf{\Lambda}$ ,  $\mathbf{V}$  are  $N_b \times N_b$ ,  $N_b \times N_r$ , and  $N_r \times N_r$ , respectively. Then we have from (19) that

$$\begin{aligned} \text{tr}\left\{ \mathbf{\Lambda} (\mathbf{\Lambda}^2 + \mu \mathbf{I}_{N_b})^{-1} \mathbf{U}^H \bar{\mathbf{H}}^H (\bar{\mathbf{H}} \bar{\mathbf{H}}^H + \mathbf{I}_{N_r})^{-1} \bar{\mathbf{H}} \mathbf{U} \right. \\ \left. \times (\mathbf{\Lambda}^2 + \mu \mathbf{I}_{N_b})^{-1} \mathbf{\Lambda} \right\} = P_r. \quad (20) \end{aligned}$$

Denoting  $\Phi \triangleq \mathbf{U}^H \bar{\mathbf{H}}^H (\bar{\mathbf{H}} \bar{\mathbf{H}}^H + \mathbf{I}_{N_r})^{-1} \bar{\mathbf{H}} \mathbf{U}$ , (20) can be equivalently written as

$$\sum_{i=1}^{N_b} \frac{\lambda_i^2 \Phi_{i,i}}{(\lambda_i^2 + \mu)^2} = P_r \quad (21)$$

where  $\lambda_i$  and  $\Phi_{i,i}$  are the  $i$ th main diagonal elements of  $\mathbf{\Lambda}$  and  $\Phi$ , respectively. Since the left-hand side of (21) is a monotonically decreasing function of  $\mu > 0$ , it can be efficiently solved using the bisection method [23].

Thirdly, with given  $\mathbf{W}$  and  $\mathbf{F}$ , we reformulate the problem (10)–(12) as a QCQP problem [23] to update  $\mathbf{b}_i$ ,  $i = 1, \dots, K$ , where  $\mathbf{b}_i = \text{vec}(\mathbf{B}_i)$  stands for a vector obtained by stacking all column vectors of  $\mathbf{B}_i$  on top of each other. Let  $\mathbf{A}_i \triangleq \mathbf{W}^H \mathbf{G} \mathbf{F} \mathbf{H}_i$  and  $\mathbf{A}_{ii}$  be a matrix containing the  $(\sum_{j=1}^{i-1} N_j + 1)$ -th to  $(\sum_{j=1}^i N_j)$ -th rows of  $\mathbf{A}_i$ . Using the identity of  $\text{vec}(\mathbf{A} \mathbf{B} \mathbf{C}) = (\mathbf{C}^T \otimes \mathbf{A}) \text{vec}(\mathbf{B})$  [24], where  $\otimes$  denotes the matrix Kronecker product, we obtain that

$$\begin{aligned} \text{tr}\{ \mathbf{W}^H \mathbf{G} \mathbf{F} \mathbf{H}_i \mathbf{B}_i \mathbf{B}_i^H \mathbf{H}_i^H \mathbf{F}^H \mathbf{G}^H \mathbf{W} \} \\ = \text{tr}\{ \mathbf{A}_i \mathbf{B}_i \mathbf{B}_i^H \mathbf{A}_i^H \} \\ = \mathbf{b}_i^H (\mathbf{I}_{N_i} \otimes (\mathbf{A}_i^H \mathbf{A}_i)) \mathbf{b}_i, \quad i = 1, \dots, K \\ \text{tr}\{ \mathbf{W}^H \mathbf{G} \mathbf{F} [\mathbf{H}_1 \mathbf{B}_1, \dots, \mathbf{H}_K \mathbf{B}_K] \} \\ = \text{tr}\{ [\mathbf{A}_1 \mathbf{B}_1, \dots, \mathbf{A}_K \mathbf{B}_K] \} \\ = \sum_{i=1}^K \text{tr}\{ \mathbf{A}_{ii} \mathbf{B}_i \} \\ = \sum_{i=1}^K (\text{vec}(\mathbf{A}_{ii}^T))^T \mathbf{b}_i. \end{aligned}$$

Thus the MSE in (8) can be expressed as

$$\begin{aligned} \text{MSE} = \text{tr}\left\{ \mathbf{W}^H \mathbf{G} \mathbf{F} \left( \sum_{i=1}^K \mathbf{H}_i \mathbf{B}_i \mathbf{B}_i^H \mathbf{H}_i^H \right) \mathbf{F}^H \mathbf{G}^H \mathbf{W} \right. \\ \left. - \mathbf{W}^H \mathbf{G} \mathbf{F} [\mathbf{H}_1 \mathbf{B}_1, \dots, \mathbf{H}_K \mathbf{B}_K] \right. \\ \left. - (\mathbf{W}^H \mathbf{G} \mathbf{F} [\mathbf{H}_1 \mathbf{B}_1, \dots, \mathbf{H}_K \mathbf{B}_K])^H \right. \\ \left. + \mathbf{I}_{N_b} + \mathbf{W}^H (\mathbf{G} \mathbf{F} \mathbf{F}^H \mathbf{G}^H + \mathbf{I}_{N_d}) \mathbf{W} \right\} \\ = \sum_{i=1}^K \mathbf{b}_i^H (\mathbf{I}_{N_i} \otimes (\mathbf{A}_i^H \mathbf{A}_i)) \mathbf{b}_i - \sum_{i=1}^K (\text{vec}(\mathbf{A}_{ii}^T))^T \mathbf{b}_i \\ - \sum_{i=1}^K \mathbf{b}_i^H \text{vec}(\mathbf{A}_{ii}^H) + t \\ \triangleq \mathbf{b}^H \mathbf{A} \mathbf{b} - \mathbf{c}^H \mathbf{b} - \mathbf{b}^H \mathbf{c} + t \quad (22) \end{aligned}$$

where

$$\begin{aligned} t & \triangleq \text{tr}\{ \mathbf{I}_{N_b} + \mathbf{W}^H (\mathbf{G} \mathbf{F} \mathbf{F}^H \mathbf{G}^H + \mathbf{I}_{N_d}) \mathbf{W} \} \\ \mathbf{A} & \triangleq \text{bd}(\mathbf{I}_{N_1} \otimes (\mathbf{A}_1^H \mathbf{A}_1), \dots, \mathbf{I}_{N_K} \otimes (\mathbf{A}_K^H \mathbf{A}_K)) \\ \mathbf{b} & \triangleq [\mathbf{b}_1^T, \dots, \mathbf{b}_K^T]^T \\ \mathbf{c} & \triangleq [(\text{vec}(\mathbf{A}_{11}^H))^T, \dots, (\text{vec}(\mathbf{A}_{KK}^H))^T]^T \end{aligned}$$

and  $\text{bd}(\cdot)$  forms a block-diagonal matrix. Now the MSE in (22) can be equivalently rewritten as

$$\begin{aligned} \text{MSE} = \mathbf{b}^H \mathbf{A}^{\frac{1}{2}} \mathbf{A}^{\frac{1}{2}} \mathbf{b} - \mathbf{c}^H \mathbf{A}^{-\frac{1}{2}} \mathbf{A}^{\frac{1}{2}} \mathbf{b} - \mathbf{b}^H \mathbf{A}^{\frac{1}{2}} \mathbf{A}^{-\frac{1}{2}} \mathbf{c} \\ + \mathbf{c}^H \mathbf{A}^{-\frac{1}{2}} \mathbf{A}^{-\frac{1}{2}} \mathbf{c} - \mathbf{c}^H \mathbf{A}^{-1} \mathbf{c} + t \\ = (\mathbf{b}^H \mathbf{A}^{\frac{1}{2}} - \mathbf{c}^H \mathbf{A}^{-\frac{1}{2}}) (\mathbf{A}^{\frac{1}{2}} \mathbf{b} - \mathbf{A}^{-\frac{1}{2}} \mathbf{c}) - \mathbf{c}^H \mathbf{A}^{-1} \mathbf{c} + t \end{aligned}$$

where  $\mathbf{A}^{\frac{1}{2}} \mathbf{A}^{\frac{1}{2}} = \mathbf{A}$  and  $\mathbf{A}^{\frac{1}{2}} = \mathbf{A}^{\frac{H}{2}}$ . Note that we can ignore the term  $t - \mathbf{c}^H \mathbf{A}^{-1} \mathbf{c}$  while optimizing  $\mathbf{b}$  with given  $\mathbf{W}$  and  $\mathbf{F}$ , since it is free of the optimization variable  $\mathbf{b}$ . Assuming  $\mathbf{C}_i \triangleq$

TABLE I  
PROCEDURE OF SOLVING THE PROBLEM (10)–(12) BY THE  
TRI-STEP ALGORITHM

- 1) Initialize the algorithm with  $\mathbf{B}_i^{(0)} = \sqrt{P_i/N_i} \mathbf{I}_{N_i}$ ,  $i = 1, \dots, K$ , and  $\mathbf{F}^{(0)} = \sqrt{P_r/\text{tr}\{\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H + \mathbf{I}_{N_r}\}} \mathbf{I}_{N_r}$ ; Set  $n = 0$ .
- 2) Update  $\mathbf{W}^{(n)}$  using  $\{\mathbf{B}_i^{(n)}\}$  and  $\mathbf{F}^{(n)}$  as in (13).
- 3) Update  $\mathbf{F}^{(n+1)}$  as in (16) using given  $\mathbf{W}^{(n)}$  and  $\{\mathbf{B}_i^{(n)}\}$ .
- 4) Solve the subproblem (23)–(25) using known  $\mathbf{F}^{(n+1)}$  and  $\mathbf{W}^{(n)}$  to obtain  $\mathbf{B}_i^{(n+1)}$ ,  $i = 1, \dots, K$ .
- 5) If  $\max_i \|\mathbf{B}_i^{(n+1)} - \mathbf{B}_i^{(n)}\|_1 \leq \varepsilon$ , then end. Otherwise, let  $n := n + 1$  and go to step 2.

$\mathbf{F}\mathbf{H}_i$ ,  $i = 1, \dots, K$ , the relay transmit power constraint in (11) can be rewritten as

$$\mathbf{b}^H \mathbf{C} \mathbf{b} \leq P_r - \text{tr}\{\mathbf{F}\mathbf{F}^H\}$$

where  $\mathbf{C} \triangleq \text{bd}(\mathbf{I}_{N_1} \otimes (\mathbf{C}_1^H \mathbf{C}_1), \dots, \mathbf{I}_{N_K} \otimes (\mathbf{C}_K^H \mathbf{C}_K))$ . Thus the optimization problem (10)–(12) can be equivalently rewritten as the following QCQP problem

$$\min_{\mathbf{b}} (\mathbf{A}^{\frac{1}{2}} \mathbf{b} - \mathbf{A}^{-\frac{1}{2}} \mathbf{c})^H (\mathbf{A}^{\frac{1}{2}} \mathbf{b} - \mathbf{A}^{-\frac{1}{2}} \mathbf{c}) \quad (23)$$

$$\text{s.t. } \mathbf{b}^H \mathbf{C} \mathbf{b} \leq P_r - \text{tr}\{\mathbf{F}\mathbf{F}^H\} \quad (24)$$

$$\mathbf{b}^H \mathbf{D}_i \mathbf{b} \leq P_i, \quad i = 1, \dots, K \quad (25)$$

where  $\mathbf{D}_i \triangleq \text{bd}(\tilde{\mathbf{D}}_{i1}, \tilde{\mathbf{D}}_{i2}, \dots, \tilde{\mathbf{D}}_{iK})$  with  $\tilde{\mathbf{D}}_{ii} = \mathbf{I}_{N_i}$  and  $\tilde{\mathbf{D}}_{ij} = \mathbf{0}$ ,  $j = 1, \dots, K, j \neq i$ . The QCQP problem (23)–(25) can be efficiently solved by the disciplined convex programming toolbox CVX [25] where interior-point method-based solvers such as SeDuMi and SDPT3 are called internally. Since all subproblems (10), (14)–(15), and (23)–(25) are convex, the solution to each subproblem is optimal. Thus, the value of the objective function (10) decreases (or at least maintains) after each iteration. Moreover, the objective function is lower bounded by at least zero.

Now, assuming that  $\mathbf{W}_0$ ,  $\{\mathbf{B}_{i,0}\}$ , and  $\mathbf{F}_0$  are the optimal solution for each subproblem, we have

$$\text{tr}\{\nabla_{\mathbf{W}} J(\mathbf{X}_0)^T (\mathbf{W} - \mathbf{W}_0)\} \geq 0 \quad (26)$$

$$\text{tr}\{\nabla_{\mathbf{B}_i} J(\mathbf{X}_0)^T (\mathbf{B}_i - \mathbf{B}_{i,0})\} \geq 0 \quad (27)$$

$$\text{tr}\{\nabla_{\mathbf{F}} J(\mathbf{X}_0)^T (\mathbf{F} - \mathbf{F}_0)\} \geq 0 \quad (28)$$

where  $\mathbf{X}_0 \triangleq [\mathbf{W}_0, \{\mathbf{B}_{i,0}\}, \mathbf{F}_0]$  and  $\nabla_{\mathbf{A}} J(\mathbf{X}_0)$  is the gradient of the objective function (10) along the direction of  $\mathbf{A} \in \{\mathbf{W}, \{\mathbf{B}_i\}, \mathbf{F}\}$  at  $\mathbf{X}_0$ . Summing up (26)–(28), we obtain  $\text{tr}\{\nabla J(\mathbf{X}_0)^T (\mathbf{X} - \mathbf{X}_0)\} \geq 0$ , indicating that  $\mathbf{X}_0$  is a stationary point of (10). Moreover, it can be seen that  $\mathbf{X}_0$  must be on the edge of the feasible set specified by inequalities in (11) and (12) (i.e., (11) and (12) must be satisfied with equality at  $\mathbf{X}_0$ , since otherwise, one can simply scale  $\mathbf{F}_0$  and  $\mathbf{B}_{i,0}$  such that the value of (10) is decreased without violating (11) and (12). This indicates that  $\mathbf{X}_0$  cannot be a saddle point and is indeed the local-optimal solution. Therefore, the proposed iterative algorithm monotonically converges to (at least) a locally optimal solution. The procedure of solving the problem (10)–(12) using the proposed Tri-Step iterative algorithm is listed in Table I, where  $\|\cdot\|_1$  denotes the matrix maximum absolute column sum norm,  $\varepsilon$  is a small positive number close to zero and the superscript  $(n)$  denotes the number of iterations.

### B. Simplified Source and Relay Matrices Design (Bi-Step Algorithm)

In this subsection, we propose an iterative source and relay matrices design algorithm which has a smaller computational complexity than the Tri-Step algorithm developed in the previous subsection. In particular, using the MMSE receiver (13) at the destination node, the MSE of the signal waveform estimation (8) becomes a function of  $\{\mathbf{B}_i\}$  and  $\mathbf{F}$  as

$$\text{MMSE} = \text{tr}\left\{[\mathbf{I}_{N_b} + \mathbf{H}^H \mathbf{C}_n^{-1} \mathbf{H}]^{-1}\right\}. \quad (29)$$

Thus, the joint source and relay optimization problem is given by

$$\min_{\{\mathbf{B}_i\}, \mathbf{F}} \text{tr}\left\{[\mathbf{I}_{N_b} + \mathbf{H}^H \mathbf{C}_n^{-1} \mathbf{H}]^{-1}\right\} \quad (30)$$

$$\text{s.t. } \text{tr}\left\{\mathbf{F}\left(\sum_{i=1}^K \mathbf{H}_i \mathbf{B}_i \mathbf{B}_i^H \mathbf{H}_i^H + \mathbf{I}_{N_r}\right)\mathbf{F}^H\right\} \leq P_r \quad (31)$$

$$\text{tr}\{\mathbf{B}_i \mathbf{B}_i^H\} \leq P_i, \quad i = 1, \dots, K. \quad (32)$$

In this Bi-Step algorithm, we update the source and the relay matrices in an alternating fashion. In each iteration, for given source matrices  $\{\mathbf{B}_i\}$  satisfying (32), we optimize the relay matrix  $\mathbf{F}$  by solving the following problem

$$\min_{\mathbf{F}} \text{tr}\left\{[\mathbf{I}_{N_b} + \mathbf{H}^H \mathbf{C}_n^{-1} \mathbf{H}]^{-1}\right\} \quad (33)$$

$$\text{s.t. } \text{tr}\left\{\mathbf{F}\left(\sum_{i=1}^K \mathbf{H}_i \mathbf{B}_i \mathbf{B}_i^H \mathbf{H}_i^H + \mathbf{I}_{N_r}\right)\mathbf{F}^H\right\} \leq P_r. \quad (34)$$

Then using this  $\mathbf{F}$ , we solve the problem (30)–(32) (with only  $\{\mathbf{B}_i\}$  as the optimization variables) to obtain optimal source precoding matrices  $\{\mathbf{B}_i\}$ . Finally, the receive matrix  $\mathbf{W}$  is obtained as (13) using the value of  $\{\mathbf{B}_i\}$  and  $\mathbf{F}$  at the convergence point.

Let us now define the following SVDs

$$\tilde{\mathbf{H}} = \mathbf{U}_s \mathbf{\Lambda}_s \mathbf{V}_s^H, \quad \mathbf{G} = \mathbf{U}_r \mathbf{\Lambda}_r \mathbf{V}_r^H$$

where the dimensions of  $\mathbf{U}_s$ ,  $\mathbf{\Lambda}_s$ ,  $\mathbf{V}_s$  are  $N_r \times N_r$ ,  $N_r \times N_b$ ,  $N_b \times N_b$ , respectively, and the dimensions of  $\mathbf{U}_r$ ,  $\mathbf{\Lambda}_r$ ,  $\mathbf{V}_r$  are given as  $N_d \times N_d$ ,  $N_d \times N_r$ ,  $N_r \times N_r$ , respectively. We assume that the main diagonal elements of  $\mathbf{\Lambda}_s$  and  $\mathbf{\Lambda}_r$  are arranged in a decreasing order. Based on Theorem 1 in [10], the optimal structure of  $\mathbf{F}$  obtained from solving the problem (33)–(34) is given by

$$\mathbf{F} = \mathbf{V}_{r,1} \mathbf{\Lambda}_f \mathbf{U}_{s,1}^H \quad (35)$$

where  $\mathbf{\Lambda}_f$  is an  $N_b \times N_b$  diagonal matrix,  $\mathbf{V}_{r,1}$  and  $\mathbf{U}_{s,1}$  contain the leftmost  $N_b$  columns from  $\mathbf{V}_r$  and  $\mathbf{U}_s$ , respectively.

It can be seen from (35) that the optimal  $\mathbf{F}$  diagonalizes the equivalent source-relay-destination MIMO channel  $\mathbf{H}$ . Substituting (35) back into (33) and (34), we obtain the problem of optimizing  $\mathbf{\Lambda}_f$  as

$$\min_{\{\lambda_{f,i}\}} \sum_{i=1}^{N_b} \left(1 + \frac{\lambda_{s,i}^2 \lambda_{r,i}^2 \lambda_{f,i}^2}{1 + \lambda_{r,i}^2 \lambda_{f,i}^2}\right)^{-1} \quad (36)$$

$$\text{s.t. } \sum_{i=1}^{N_b} \lambda_{f,i}^2 (\lambda_{s,i}^2 + 1) \leq P_r, \quad \lambda_{f,i} \geq 0, \quad i = 1, \dots, N_b \quad (37)$$

where  $\lambda_{s,i}$ ,  $\lambda_{f,i}$ , and  $\lambda_{r,i}$  are the  $i$ th main diagonal elements of  $\mathbf{\Lambda}_s$ ,  $\mathbf{\Lambda}_f$ , and  $\mathbf{\Lambda}_r$ , respectively. The problem (36)–(37) has a water-filling solution which is given by

$$\lambda_{f,i} = \frac{1}{\lambda_{r,i}} \left[ \frac{1}{\lambda_{s,i}^2 + 1} \left( \frac{\lambda_{s,i} \lambda_{r,i}}{[(\lambda_{s,i}^2 + 1)\nu]^{\frac{1}{2}}} - 1 \right)^+ \right]^{\frac{1}{2}}, \quad i = 1, \dots, N_b \quad (38)$$

where for a real-valued number  $x$ ,  $(x)^+ \triangleq \max(x, 0)$ , and  $\nu > 0$  is the solution to the nonlinear problem of

$$\sum_{i=1}^{N_b} \frac{1}{\lambda_{r,i}^2} \left( \frac{\lambda_{s,i} \lambda_{r,i}}{[(\lambda_{s,i}^2 + 1)\nu]^{\frac{1}{2}}} - 1 \right)^+ = P_r. \quad (39)$$

Since (39) is a monotonically decreasing function of  $\nu$ , it can be efficiently solved using the bisection method [23].

Using the identity of  $\text{tr}\{\mathbf{I}_m + \mathbf{A}_{m \times n} \mathbf{B}_{n \times m}\}^{-1} = \text{tr}\{\mathbf{I}_n + \mathbf{B}_{n \times m} \mathbf{A}_{m \times n}\}^{-1} + m - n$ , for a given feasible  $\mathbf{F}$ , the objective function (29) can be rewritten as

$$\begin{aligned} \text{MMSE} &= \text{tr} \left\{ \left[ \mathbf{I}_{N_d} + \mathbf{H} \mathbf{H}^H \mathbf{C}_n^{-1} \right]^{-1} \right\} + N_b - N_d \\ &= \text{tr} \left\{ \left[ \mathbf{I}_{N_d} + \mathbf{C}_n^{-\frac{1}{2}} \mathbf{H} \mathbf{H}^H \mathbf{C}_n^{-\frac{1}{2}} \right]^{-1} \right\} + N_b - N_d \\ &= \text{tr} \left\{ \left[ \mathbf{I}_{N_d} + \mathbf{C}_n^{-\frac{1}{2}} \mathbf{G} \mathbf{F} \sum_{i=1}^K \mathbf{H}_i \mathbf{B}_i \right. \right. \\ &\quad \left. \left. \times \mathbf{B}_i^H \mathbf{H}_i^H \mathbf{F}^H \mathbf{G}^H \mathbf{C}_n^{-\frac{1}{2}} \right]^{-1} \right\} + N_b - N_d \\ &= \text{tr} \left\{ \left[ \mathbf{I}_{N_d} + \sum_{i=1}^K \tilde{\mathbf{H}}_i \mathbf{Q}_i \tilde{\mathbf{H}}_i^H \right]^{-1} \right\} + N_b - N_d \end{aligned}$$

where  $\tilde{\mathbf{H}}_i \triangleq \mathbf{C}_n^{-\frac{1}{2}} \mathbf{G} \mathbf{F} \mathbf{H}_i$  and  $\mathbf{Q}_i = \mathbf{B}_i \mathbf{B}_i^H$  is the source covariance matrix of the  $i$ th user. In the following, we focus on optimizing  $\mathbf{Q}_i$ . Once we obtain the optimal  $\mathbf{Q}_i$ , the optimal  $\mathbf{B}_i$  is calculated as  $\mathbf{B}_i = \mathbf{\Theta}_i \mathbf{\Lambda}_i^{\frac{1}{2}} \mathbf{\Phi}_i$ , where  $\mathbf{\Theta}_i \mathbf{\Lambda}_i \mathbf{\Theta}_i^H$  is the eigenvalue decomposition (EVD) of  $\mathbf{Q}_i$ , and  $\mathbf{\Phi}_i$  is an arbitrary  $N_i \times N_i$  unitary matrix. Considering the transmission power constraints in (31) and (32), the source covariance matrices  $\{\mathbf{Q}_i\} \triangleq \{\mathbf{Q}_i, i = 1, \dots, K\}$  can be optimized by solving the following problem

$$\min_{\{\mathbf{Q}_i\}} \text{tr} \left\{ \left[ \mathbf{I}_{N_d} + \sum_{i=1}^K \tilde{\mathbf{H}}_i \mathbf{Q}_i \tilde{\mathbf{H}}_i^H \right]^{-1} \right\} \quad (40)$$

$$\text{s.t.} \quad \text{tr} \left\{ \sum_{i=1}^K \mathbf{Q}_i \mathbf{\Psi}_i \right\} \leq \bar{P}_r \quad (41)$$

$$\text{tr} \{\mathbf{Q}_i\} \leq P_i, \quad \mathbf{Q}_i \succeq 0, \quad i = 1, \dots, K \quad (42)$$

where  $\mathbf{\Psi}_i \triangleq \mathbf{H}_i^H \mathbf{F}^H \mathbf{F} \mathbf{H}_i$ ,  $\bar{P}_r \triangleq P_r - \text{tr}\{\mathbf{F} \mathbf{F}^H\}$ , and for a matrix  $\mathbf{A}$ ,  $\mathbf{A} \succeq 0$  means that  $\mathbf{A}$  is a positive semi-definite (PSD) matrix.

Let us now introduce a PSD matrix  $\mathbf{X}$  that satisfies

$$\left[ \mathbf{I}_{N_d} + \sum_{i=1}^K \tilde{\mathbf{H}}_i \mathbf{Q}_i \tilde{\mathbf{H}}_i^H \right]^{-1} \preceq \mathbf{X} \quad (43)$$

TABLE II

PROCEDURE OF SOLVING THE PROBLEM (30)–(32) BY THE BI-STEP ALGORITHM

- 1) Initialize the algorithm with  $\mathbf{Q}_i^{(0)} = P_i/N_i \mathbf{I}_{N_i}$ ,  $i = 1, \dots, K$ ; Set  $n = 0$ .
- 2) Solve the subproblem (33)–(34) using given  $\mathbf{Q}_i^{(n)}$ ,  $i = 1, \dots, K$ , to obtain  $\mathbf{F}^{(n)}$  as in (35).
- 3) Solve the subproblem (44)–(47) using known  $\mathbf{F}^{(n)}$  to obtain  $\mathbf{Q}_i^{(n+1)}$ ,  $i = 1, \dots, K$ .
- 4) If  $\max_i \|\mathbf{Q}_i^{(n+1)} - \mathbf{Q}_i^{(n)}\|_1 \leq \varepsilon$ , then end. Otherwise, let  $n := n + 1$  and go to step 2.

where for two matrices  $\mathbf{A}$  and  $\mathbf{B}$ ,  $\mathbf{B} \succeq \mathbf{A}$  means that  $\mathbf{B} - \mathbf{A} \succeq 0$ . By using (43) and the Schur complement [23], the problem (40)–(42) can be equivalently converted to the following SDP problem

$$\min_{\{\mathbf{Q}_i\}, \mathbf{X}} \text{tr} \{\mathbf{X}\} \quad (44)$$

$$\text{s.t.} \quad \begin{bmatrix} \mathbf{X} & & & \\ & \mathbf{I}_{N_d} & & \\ & & \mathbf{I}_{N_d} & \\ & & & \mathbf{I}_{N_d} + \sum_{i=1}^K \tilde{\mathbf{H}}_i \mathbf{Q}_i \tilde{\mathbf{H}}_i^H \end{bmatrix} \succeq 0 \quad (45)$$

$$\text{tr} \left\{ \sum_{i=1}^K \mathbf{Q}_i \mathbf{\Psi}_i \right\} \leq \bar{P}_r \quad (46)$$

$$\text{tr} \{\mathbf{Q}_i\} \leq P_i, \quad \mathbf{Q}_i \succeq 0, \quad i = 1, \dots, K. \quad (47)$$

We use the CVX software package [25] to solve the problem (44)–(47). Now the original source and relay matrices optimization problem (30)–(32) can be solved by an iterative technique as shown in Table II.

Since the problem (36)–(37) is a convex optimization problem, the conditional update of  $\mathbf{F}^{(n)}$  will not increase (36) and hence the objective function (30). Similarly, the problem (44)–(47) is also convex, and the conditional update of  $\mathbf{Q}_i^{(n)}$  cannot increase (44) and hence the value of (30). Therefore, each conditional update of  $\mathbf{F}^{(n)}$  and  $\mathbf{Q}_i^{(n)}$  may either decrease or maintain but cannot increase the objective function (30). Note that the constraints in the problem (30)–(32) are always satisfied with every conditional update. Similar to the justification for the Tri-Step algorithm, a monotonic convergence of  $\mathbf{F}^{(n)}$  and  $\mathbf{Q}_i^{(n)}$  towards (at least) a locally optimal solution follows directly from this observation.

The numerical solution to the problem (44)–(47) does not provide sufficient insight to the structure of the optimal  $\mathbf{Q}_i$ . Interestingly, by solving the problem (40)–(42) applying the Lagrange multiplier method, we obtain the following theorem for the structure of the optimal  $\mathbf{Q}_i$ .

*Theorem 1:* The optimal source covariance matrix  $\mathbf{Q}_i$  for the  $i$ th user as the solution to the problem (40)–(42) has the following general beamforming structure

$$\mathbf{Q}_i = \mathbf{V}_{h_i} \mathbf{\Lambda}_{h_i,1}^{-1} \mathbf{U}_{h_i,1}^H (\mathbf{V}_i \mathbf{J}_i \mathbf{V}_i^H - \mathbf{D}_i)^\dagger \times \mathbf{U}_{h_i,1} \mathbf{\Lambda}_{h_i,1}^{-1} \mathbf{V}_{h_i}^H \quad i = 1, \dots, K \quad (48)$$

where  $\mathbf{D}_i \triangleq \mathbf{I}_{N_d} + \sum_{j=1, j \neq i}^K \tilde{\mathbf{H}}_j \mathbf{Q}_j \tilde{\mathbf{H}}_j^H$ ,  $(\cdot)^\dagger$  stands for the projection to the set of  $N_d \times N_d$  PSD matrices,  $\tilde{\mathbf{H}}_i = [\mathbf{U}_{h_i,1} \quad \mathbf{U}_{h_i,2}] [\mathbf{\Lambda}_{h_i,1} \quad \mathbf{0}]^T \mathbf{V}_{h_i}^H$  and  $\mathbf{K}_i^{-1} \tilde{\mathbf{H}}_i^H = \mathbf{U}_i [\mathbf{\Sigma}_i \quad \mathbf{0}] \mathbf{V}_i^H$  are the SVDs of  $\tilde{\mathbf{H}}_i$  and  $\mathbf{K}_i^{-1} \tilde{\mathbf{H}}_i^H$ , respectively, and  $\mathbf{J}_i \triangleq \text{bd}(\mathbf{\Sigma}_i, \mathbf{\Delta}_{i,2})$ . Here  $\mathbf{K}_i \mathbf{K}_i^H = \lambda_1 \mathbf{\Psi}_i + \lambda_2 \mathbf{I}_{N_i}$ ,

$\lambda_1 \geq 0, \lambda_2 \geq 0$  are the Lagrange multipliers, and  $\Delta_{i,2}$  is an  $(N_d - N_i) \times (N_d - N_i)$  diagonal matrix.

*Proof:* See Appendix A.  $\square$

The unknown Lagrange multipliers  $\lambda_1$  and  $\lambda_2$  in (48) can be found by solving the dual optimization problem associated with the problem (49)–(51) in Appendix A. Note that the optimal structure of the source covariance matrices in (48) can be viewed as  $\mathbf{Q}_i = \tilde{\mathbf{H}}_i^\# \mathbf{\Lambda}_i (\tilde{\mathbf{H}}_i^\#)^H, i = 1, \dots, K$ , where  $(\cdot)^\#$  denotes matrix pseudo-inverse and  $\mathbf{\Lambda}_i \triangleq (\mathbf{V}_i \mathbf{J}_i \mathbf{V}_i^H - \mathbf{D}_i)^\dagger$  is the power-loading matrix. Note that (48) indicates that the power distribution at each user needs to be adapted to the current power levels of all other users. The pseudo-inverse in  $\mathbf{Q}_i, i = 1, \dots, K$ , indicates that the source covariance matrix of the  $i$ th user needs to match the corresponding source-relay-destination channel.

In summary, matrices  $\mathbf{W}, \mathbf{F}$ , and  $\{\mathbf{B}_i\}$  are optimized in each iteration of the Tri-Step algorithm, where the major computation task lies in solving the QCQP problem (23)–(25). The amount of computation required for updating  $\mathbf{W}$  and  $\mathbf{F}$  is negligible compared with that of solving the QCQP problem. The complexity order of solving the problem (23)–(25) using the interior point method [26] is  $\mathcal{O}((\sum_{i=1}^K N_i^2)^3)$ .

In each iteration of the Bi-Step algorithm,  $\mathbf{F}$  and  $\{\mathbf{B}_i\}$  are optimized. Here the major computation task is solving the SDP problem (44)–(47), which has a complexity order of  $\mathcal{O}((\sum_{i=1}^K N_i^2)^{3.5})$  using the interior point method [26]. Therefore, the per iteration computational complexity of the Bi-Step algorithm is slightly higher than that of the Tri-Step algorithm. However, the overall computational complexity of both iterative algorithms also depends on the number of iterations they need till convergence, which will be studied in Section IV (see Table III).

#### IV. SIMULATIONS

In this section, we study the performance of the proposed optimal multiuser MIMO relay algorithms through numerical simulations. For simplicity, we consider a system with two users. The extension to  $K > 2$  users is straight-forward. The two users, relay and destination nodes are all equipped with multiple antennas. We simulate a flat Rayleigh fading environment where the channel matrices have entries with zero mean and variances  $\frac{\sigma_g^2}{N_r}, \frac{\sigma_{h,i}^2}{N_i}$ , for  $\mathbf{G}, \mathbf{H}_i, i = 1, 2$ , respectively. We define

$$\text{SNR}_{r-d} \triangleq \frac{\sigma_g^2 P_r N_d}{N_r}, \quad \text{SNR}_{s_i-r} \triangleq \frac{\sigma_{h,i}^2 P_i N_r}{N_i}, \quad i = 1, 2$$

as the signal-to-noise ratio (SNR) of the relay-destination and user- $i$ -relay links,  $i = 1, 2$ , respectively. For simplicity, we assume  $N_1 = N_2 = N_s$  and  $\text{SNR}_{s_1-r} = \text{SNR}_{s_2-r} = \text{SNR}_{s-r}$  throughout the simulations. All simulation results are averaged over 1000 independent channel realizations.

We compare the performance of the proposed Tri-Step and Bi-Step algorithms with the naive amplify-and-forward (NAF) algorithm, and the pseudo match-and-forward (PMF) algorithm in terms of both MSE and BER. For the Tri-Step algorithm, the procedure in Table I is carried out to obtain the optimal relay and source matrices, whereas for the Bi-Step

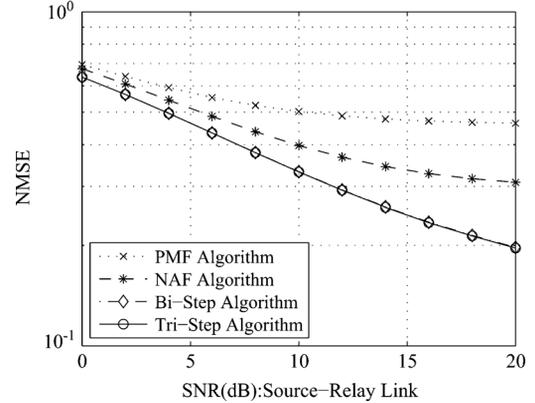


Fig. 2. Example 1: Normalized MSE versus  $\text{SNR}_{s-r}$ .  $N_s = 2, N_r = 4, N_d = 4, \text{SNR}_{r-d} = 20$  dB.

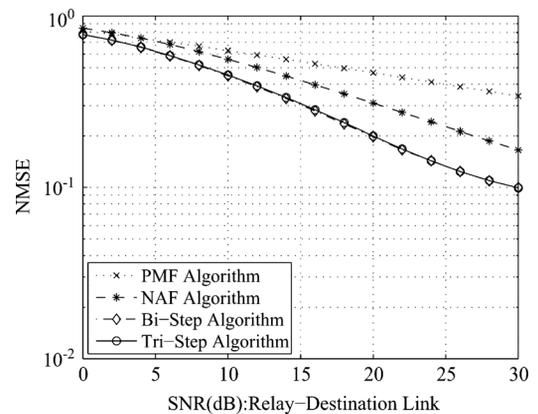


Fig. 3. Example 2: Normalized MSE versus  $\text{SNR}_{r-d}$ .  $N_s = 2, N_r = 4, N_d = 4, \text{SNR}_{s-r} = 20$  dB.

algorithm, the steps defined in Table II are followed. For both algorithms, we use the CVX Matlab toolbox for disciplined convex programming [25] to find the optimal source precoding matrices. For the NAF scheme, we use  $\mathbf{B}_i = \sqrt{\frac{P_i}{N_i}} \mathbf{I}_{N_i}, i = 1, 2$ , and  $\mathbf{F} = \sqrt{\frac{P_r}{\text{tr}\{\mathbf{H}\mathbf{H}^H + \mathbf{I}_{N_r}\}}} \mathbf{I}_{N_r}$ . For the PMF algorithm, the same  $\mathbf{B}_i$  in the NAF algorithm is taken and  $\mathbf{F} = \sqrt{\frac{P_r}{\text{tr}\{(\tilde{\mathbf{H}}\mathbf{G})^H(\mathbf{H}\mathbf{H}^H + \mathbf{I}_{N_r})\tilde{\mathbf{H}}\mathbf{G}\}}} (\tilde{\mathbf{H}}\mathbf{G})^H$ . Both the NAF and the PMF algorithms use the MMSE receiver at the destination node.

In the first two examples, we compare the performance of the proposed algorithms with the other two approaches in terms of MSE normalized by the number of data streams (NMSE) for  $N_s = 2, N_r = 4$ , and  $N_d = 4$ . Fig. 2 shows the MSE performance of all tested algorithms versus  $\text{SNR}_{s-r}$  with  $\text{SNR}_{r-d} = 20$  dB, whereas Fig. 3 illustrates the MSE performance of tested algorithms versus  $\text{SNR}_{r-d}$  for an  $\text{SNR}_{s-r}$  fixed at 20 dB. Our results clearly demonstrate the better performance of the proposed iterative joint source and relay optimization algorithms. It can be seen that the proposed optimal algorithms consistently yield the lowest MSE over the entire  $\text{SNR}_{s-r}$  and  $\text{SNR}_{r-d}$  region. The NAF and PMF algorithms have much higher MSE compared with the proposed schemes even at very high SNR. Note that the MSE performance of

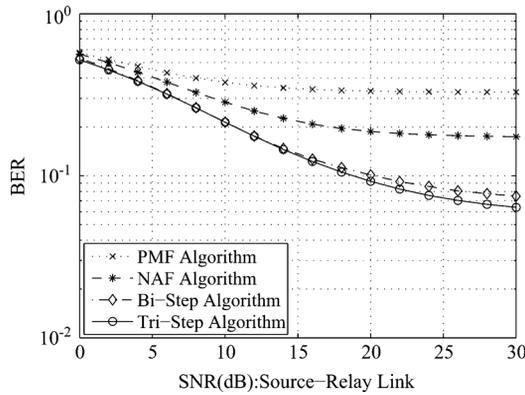


Fig. 4. Example 3: BER versus  $\text{SNR}_{s-r}$ .  $N_s = 3$ ,  $N_r = 6$ ,  $N_d = 6$ ,  $\text{SNR}_{r-d} = 20$  dB.

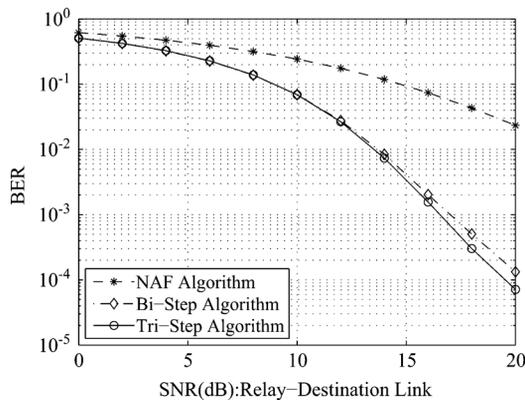


Fig. 5. Example 4: BER versus  $\text{SNR}_{r-d}$ .  $N_s = 2$ ,  $N_r = 6$ ,  $N_d = 8$ ,  $\text{SNR}_{s-r} = 20$  dB.

both the Tri-Step algorithm and the Bi-Step algorithm are almost similar to each other.

In the next example, we compare the performance of the four algorithms in terms of BER. QPSK signal constellations are used to modulate the transmitted signals. We set  $N_s = 3$ ,  $N_r = 6$ ,  $N_d = 6$ , and transmit 3000 randomly generated bits from each user in each channel realization. Fig. 4 shows the BER performance of all algorithms versus  $\text{SNR}_{s-r}$  for  $\text{SNR}_{r-d} = 20$  dB.

In the fourth example, we compare the BER performance of the algorithms varying the SNR in the relay-destination channel. This time we set  $N_s = 2$ ,  $N_r = 6$ ,  $N_d = 8$ , and transmit 3000 randomly generated bits from each user in each channel realization using QPSK signal constellations. Fig. 5 shows the BER performance of the algorithms versus  $\text{SNR}_{r-d}$  for  $\text{SNR}_{s-r} = 20$  dB. Note that in contrast to other three schemes, the PMF algorithm requires  $N_b = N_d$ , and thus, its performance cannot be included in Fig. 5.

It can be seen from Figs. 4 and 5 that the proposed joint source and relay optimization algorithms obtain the lowest BERs compared with the other approaches. Interestingly, the BER performance of the Tri-Step algorithm is slightly better than that of the Bi-Step algorithm, especially at the high SNR region. The reason is that in the Tri-Step algorithm, we update the receiver weight matrix at each iteration in addition to the source

TABLE III  
ITERATIONS REQUIRED TILL CONVERGENCE IN THE PROPOSED ALGORITHMS.

$\text{SNR}_{s-r}$ (dB)	0	5	10	15	20
Bi-Step algorithm	2	3	3	5	6
Tri-Step algorithm	6	6	10	19	22

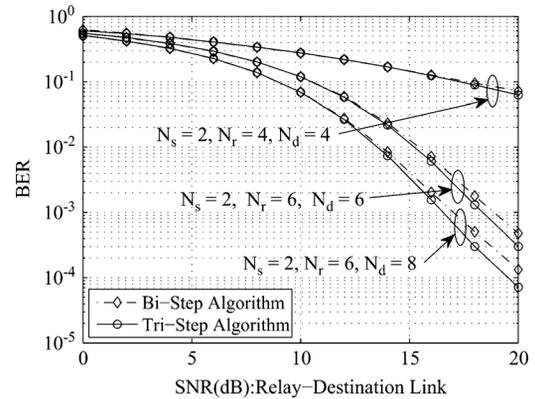


Fig. 6. Example 5: BER versus  $\text{SNR}_{r-d}$ . Varying number of antennas,  $\text{SNR}_{s-r} = 20$  dB.

and relay matrices. Although the Tri-Step algorithm performs better than the Bi-Step algorithm, the former algorithm requires a larger number of iterations than the latter one to converge to the same  $\varepsilon$ . For comparison, the number of iterations both algorithms require in a typical run to converge up to  $\varepsilon = 10^{-3}$  are listed in Table III. Here we set  $N_s = 2$ ,  $N_r = 6$ ,  $N_d = 6$  and  $\text{SNR}_{r-d} = 20$  dB. Therefore, based on the per iteration complexity of two algorithms discussed in Section III and the number of iterations they need to converge, the overall computational complexity of the Bi-Step algorithm is smaller than that of the Tri-Step algorithm when the number of antennas at each user is small (which is the case in practical uplink multiuser communication systems). Such performance-complexity tradeoff is very important for practical multiuser MIMO relay communication systems.

In the last example, we compare the BER performance of the proposed algorithms for different number of antennas at the relay and the destination nodes with a fixed number of antennas at the source nodes. Fig. 6 compares the BER performance of the proposed algorithms versus  $\text{SNR}_{r-d}$  for  $\text{SNR}_{s-r} = 20$  dB with different number of antennas. It can be clearly seen from Fig. 6 that as we increase the number of antennas at the relay and/or destination node(s), the performance of the proposed algorithms improve significantly.

## V. CONCLUSIONS

We developed the optimal structure of the source precoding matrices and the relay amplifying matrix in a multiuser MIMO relay network to jointly minimize the MSE of the signal waveform estimation. We proposed two iterative algorithms to optimize the source, relay, and receive matrices. Simulation results demonstrate that the proposed algorithms outperform the existing techniques in terms of both MSE and BER.

APPENDIX  
PROOF OF THEOREM 1

To determine the structure of the optimal source covariance matrix  $\mathbf{Q}_i$  for the  $i$ th user, we rewrite the problem (40)–(42) with given  $\mathbf{Q}_j$ ,  $j = 1, \dots, K$ ,  $j \neq i$  as

$$\min_{\mathbf{Q}_i} \operatorname{tr} \left\{ \left[ \mathbf{D}_i + \tilde{\mathbf{H}}_i \mathbf{Q}_i \tilde{\mathbf{H}}_i^H \right]^{-1} \right\} \quad (49)$$

$$\text{s.t.} \quad \operatorname{tr} \{ \mathbf{Q}_i \boldsymbol{\Psi}_i \} \leq \tilde{P}_r \quad (50)$$

$$\operatorname{tr} \{ \mathbf{Q}_i \} \leq P_i, \quad \mathbf{Q}_i \succeq 0 \quad (51)$$

where  $\tilde{P}_r \triangleq \bar{P}_r - \operatorname{tr} \left\{ \sum_{j=1, j \neq i}^K \mathbf{Q}_j \boldsymbol{\Psi}_j \right\}$ . The Lagrangian function associated with the problem (49)–(51) is given by

$$\begin{aligned} \mathcal{L} = \operatorname{tr} \left\{ \left[ \mathbf{D}_i + \tilde{\mathbf{H}}_i \mathbf{Q}_i \tilde{\mathbf{H}}_i^H \right]^{-1} \right\} \\ + \lambda_1 \left( \operatorname{tr} \{ \mathbf{Q}_i \boldsymbol{\Psi}_i \} - \tilde{P}_r \right) + \lambda_2 \left( \operatorname{tr} \{ \mathbf{Q}_i \} - P_i \right) \end{aligned}$$

where  $\lambda_1 \geq 0$  and  $\lambda_2 \geq 0$  are the Lagrange multipliers. Making the derivative of  $\mathcal{L}$  with respect to  $\mathbf{Q}_i$  be zero, we obtain

$$\frac{\partial \mathcal{L}}{\partial \mathbf{Q}_i} = -\tilde{\mathbf{H}}_i^H \left( \mathbf{D}_i + \tilde{\mathbf{H}}_i \mathbf{Q}_i \tilde{\mathbf{H}}_i^H \right)^{-2} \tilde{\mathbf{H}}_i + \lambda_1 \boldsymbol{\Psi}_i + \lambda_2 \mathbf{I}_{N_i} = 0. \quad (52)$$

By introducing an invertible matrix  $\mathbf{K}_i$  with  $\mathbf{K}_i \mathbf{K}_i^H = \lambda_1 \boldsymbol{\Psi}_i + \lambda_2 \mathbf{I}_{N_i}$ , (52) becomes

$$\mathbf{K}_i^{-1} \tilde{\mathbf{H}}_i^H \left( \mathbf{D}_i + \tilde{\mathbf{H}}_i \mathbf{Q}_i \tilde{\mathbf{H}}_i^H \right)^{-2} \tilde{\mathbf{H}}_i \mathbf{K}_i^{-H} = \mathbf{I}_{N_i}. \quad (53)$$

Obviously, (53) is valid if and only if

$$\mathbf{K}_i^{-1} \tilde{\mathbf{H}}_i^H = \mathbf{P}_i \left( \mathbf{D}_i + \tilde{\mathbf{H}}_i \mathbf{Q}_i \tilde{\mathbf{H}}_i^H \right) \quad (54)$$

where  $\mathbf{P}_i$  is an  $N_i \times N_d$  semi-unitary matrix with  $\mathbf{P}_i \mathbf{P}_i^H = \mathbf{I}_{N_i}$ .

Let us introduce the following SVD and EVD

$$\begin{aligned} \mathbf{K}_i^{-1} \tilde{\mathbf{H}}_i^H &= \mathbf{U}_i [\boldsymbol{\Sigma}_i \quad \mathbf{0}] \mathbf{V}_i^H \\ \mathbf{D}_i + \tilde{\mathbf{H}}_i \mathbf{Q}_i \tilde{\mathbf{H}}_i^H &= [\mathbf{L}_{i,1} \quad \mathbf{L}_{i,2}] \operatorname{bd}(\boldsymbol{\Delta}_{i,1}, \boldsymbol{\Delta}_{i,2}) \mathbf{L}_i^H \end{aligned} \quad (55)$$

where the dimensions of  $\mathbf{U}_i$ ,  $\mathbf{V}_i$ ,  $\mathbf{L}_i$  are  $N_i \times N_i$ ,  $N_d \times N_d$ , and  $N_d \times N_d$ , respectively,  $\mathbf{L}_{i,1}$  and  $\mathbf{L}_{i,2}$  contain the leftmost  $N_i$  columns and the rightmost  $N_d - N_i$  columns of  $\mathbf{L}_i$ , respectively, and  $\boldsymbol{\Sigma}_i$ ,  $\boldsymbol{\Delta}_{i,1}$ ,  $\boldsymbol{\Delta}_{i,2}$  are  $N_i \times N_i$ ,  $N_i \times N_i$ , and  $(N_d - N_i) \times (N_d - N_i)$  diagonal matrices, respectively. Substituting (55) back into (54), we have

$$\mathbf{U}_i [\boldsymbol{\Sigma}_i \quad \mathbf{0}] \mathbf{V}_i^H = [\mathbf{P}_i \mathbf{L}_{i,1} \boldsymbol{\Delta}_{i,1} \quad \mathbf{P}_i \mathbf{L}_{i,2} \boldsymbol{\Delta}_{i,2}] \mathbf{L}_i^H. \quad (56)$$

Equation (56) holds if and only if  $\mathbf{P}_i = \mathbf{U}_i \mathbf{L}_{i,1}^H$ ,  $\boldsymbol{\Delta}_{i,1} = \boldsymbol{\Sigma}_i$ , and  $\mathbf{L}_i = \mathbf{V}_i$ . Thus, from (55) we have that

$$\mathbf{D}_i + \tilde{\mathbf{H}}_i \mathbf{Q}_i \tilde{\mathbf{H}}_i^H = \mathbf{V}_i \mathbf{J}_i \mathbf{V}_i^H \quad (57)$$

where  $\mathbf{J}_i \triangleq \operatorname{bd}(\boldsymbol{\Sigma}_i, \boldsymbol{\Delta}_{i,2})$ . Let us introduce the SVD of  $\tilde{\mathbf{H}}_i$  as

$$\tilde{\mathbf{H}}_i = [\mathbf{U}_{h_i,1} \quad \mathbf{U}_{h_i,2}] [\boldsymbol{\Lambda}_{h_i,1} \quad \mathbf{0}]^T \mathbf{V}_{h_i}^H \quad (58)$$

where the dimensions of  $\mathbf{U}_{h_i,1}$ ,  $\mathbf{U}_{h_i,2}$ ,  $\mathbf{V}_{h_i}$  are  $N_d \times N_i$ ,  $N_d \times (N_d - N_i)$ , and  $N_i \times N_i$ , respectively,  $\boldsymbol{\Lambda}_{h_i,1}$  is an  $N_i \times N_i$  diagonal matrix. By substituting (58) back into (57) and solving (57) for  $\mathbf{Q}_i$ , we have  $\mathbf{Q}_i = \mathbf{V}_{h_i} \boldsymbol{\Lambda}_{h_i,1}^{-1} \mathbf{U}_{h_i,1}^H (\mathbf{V}_i \mathbf{J}_i \mathbf{V}_i^H - \mathbf{D}_i) \mathbf{U}_{h_i,1} \boldsymbol{\Lambda}_{h_i,1}^{-1} \mathbf{V}_{h_i}^H$ . Finally, taking into account the constraint (51), we obtain (48).

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