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Gravity forward modelling

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Definition

Gravity forward modelling. Computation of the gravity field of some given mass distribution

Introduction

Gravity forward modelling (GFM) denotes the computation of the gravitational field generated by some source mass distribution. The foundation of GFM is Newton's law of universal gravitation (1687) which states that the attraction force *F* between two bodies is proportional to the product of their masses *m*, *M* and inversely proportional to the square of their distance *r*:

$$F = G\frac{mM}{r^2} \qquad (1)$$

where $G = 6.67384 \times 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$ is the universal gravitational constant (Mohr et al., 2012). In most practical applications of GFM, unit mass is assumed in the computation point, and the second body's mass *M* is considered the source of the gravitational field (Blakeley 1996). The visible topography, given in form of digital terrain models (DTM), is the most frequently used mass distribution in GFM, other distributions may include, but are not limited to, water masses, ice masses (Grombein et al., 2014), and crustal structures in the Earth's interior (Tenzer et al., 2012).

GFM yields the gravitational field of the mass distribution in terms of any functional of the field, e.g., gravitational potential, gravity disturbance (i.e., radial derivative of the potential), and vertical deflections (cf. Nagy et al., 2000). In the literature, a number of different expressions are in use to denote gravity effects or functionals computed from topography models, e.g., topographic gravity, topography-induced gravity, synthetic gravity, terrain effects and topographic reductions.

GFM is a central task in physical geodesy, e.g., in the context of geoid determination (Tziavos and Sideris, 2013) and gravity prediction (Forsberg and Tscherning, 1981). In potential field geophysics, GFM is relevant for the investigation of the interior structure of the Earth and the planets (Wieczorek, 2007). While GFM as forward method delivers the gravity field from mass models, inversion techniques seek to estimate the mass distribution from gravity observations, which however is inherently non-unique (Oldenburg, 1974). Gravity inversion involves iterative application of GFM based on different mass model assumptions until the forward modelled signal sufficiently fits the gravity observations (e.g., Ebbing et al., 2001).

This article briefly outlines representations of source mass models and describes GFM calculation techniques, grouped according to the domain of evaluation (spatial vs. spectral domain), then summarizes applications for GFM, selected results and directions.

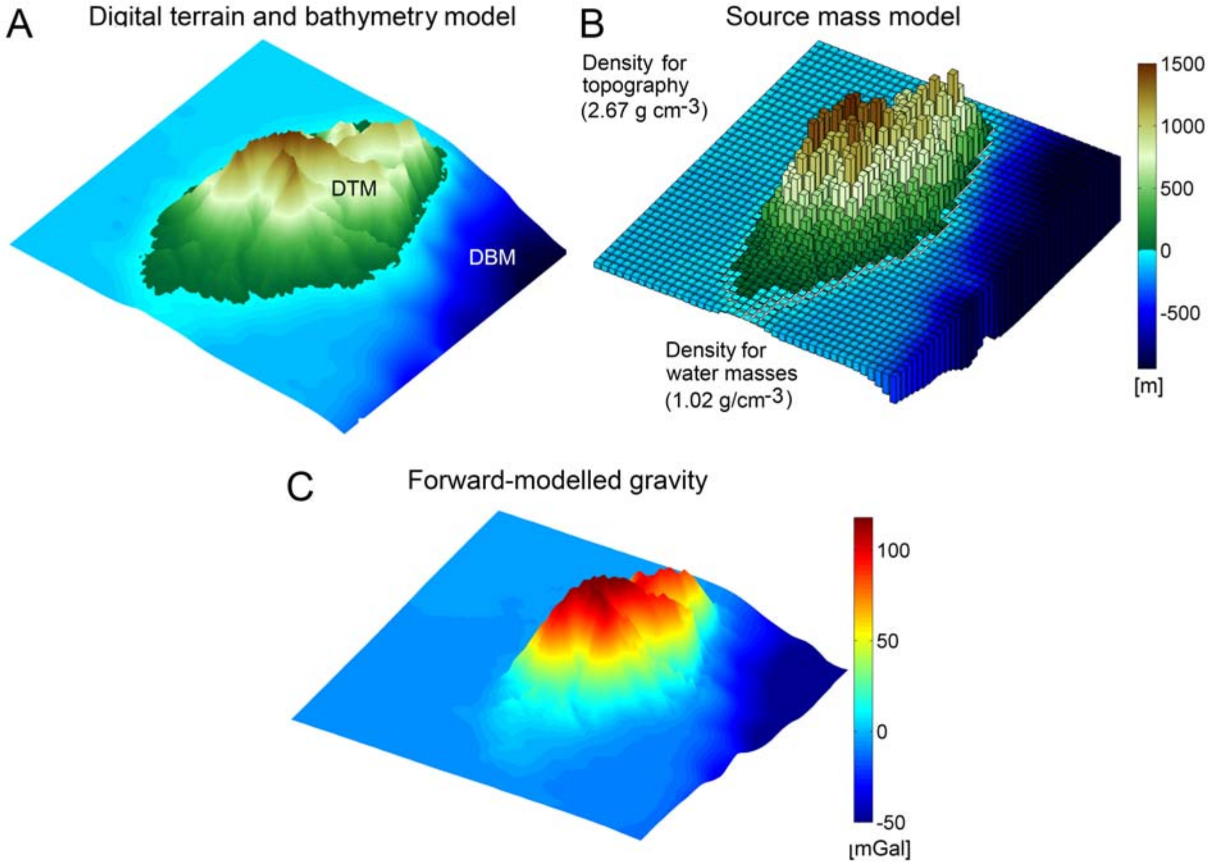

Figure 1. Principle of Gravity forward modelling. A. Digital terrain model (DTM) and digital bathymetry model (DBM) as source of mass information. B. Source mass model based on DTM and DBM geometry information, and assignment of mass-density values. The masses are approximated by rectangular prisms in this example. C. Gravity effect (gravity disturbances) as result of the forward modelling of the source masses using Newtonian integration. Examples show Samothrace Island (40.45° latitude, 25.6° longitude).

Source mass models

As prerequisite for the numerical evaluation of Newton's integral, a model of the source mass distribution must be defined, representing the attracting body. This generally includes (a) the definition of the body's geometry, and (b) assignment of a suitable mass-density value. Often the attracting body is defined as a volumetric layer with some upper and lower bound (Tenzer et al., 2012; Balmino et al., 2012), e.g., the visible topography with the geoid as lower bound and terrain elevations (from a DTM) as upper bound. When ocean water masses are modelled (Fig. 1a,b), bathymetric depths from a digital bathymetry model (DBM) define the seafloor geometry as lower bound and the geoid as upper bound (Tenzer et al., 2011; Tocho et al., 2012). Depending on the purpose of the study, the geometry of various other bodies, e.g., ice-shields (Grombein et al., 2014), sediments (Ebbing et al. 2001), crustal or mantle structures (Tenzer et al., 2012; Tsoulis, 2013), can be defined from digital data sets.

Mass-density values assigned to the body can be based on geological samples (e.g., granite 2.7 g/cm^{-3}, basalt 2.9 g/cm^{-3}, salt rock 2.4 g/cm^{-3}) or – in the absence of individual samples – on assumptions (e.g., 2.67 g cm^{-3} as an average value for rock), cf. Jacoby and Smilde (2009). While in many cases the mass-density is treated as constant value inside the attracting body, lateral (Eshagh, 2009; Göttl and Rummel, 2009) or vertical variations in mass-density (e.g., depth-dependent increase in sea water density, Tenzer et al., 2011) can be taken into account.

Following the superposition principle, gravity effects of a collection of masses (e.g., the visible topography, ocean water, and ice-shields) can be obtained through addition of gravity effects by the individual masses (e.g., Blakely, 1996). For computational reasons, water and ice masses are sometimes numerically compressed into layers of rock (rock-equivalent topography RET, Rummel et al., 1988), allowing to work with a single constant mass-density value.

The source mass model can be extended to accommodate the effect of isostatic compensation (e.g., Rummel et al., 1988; Makhloof and Ilk, 2008; Bagherbandi and Sjöberg, 2012; Grombein et al., 2014), e.g., based on the Airy-Heiskanen hypothesis through modification of the root (lower bound) of the topography, or the Pratt-Hayford hypothesis by using laterally varying mass-densities (Göttl and Rummel, 2009). If the source mass-defining DTM is high-pass filtered, e.g., through subtraction of some mean terrain surface, a residual terrain model (RTM, Forsberg and Tscherning 1981; Forsberg, 1984) is obtained. The RTM can be thought of as a model with oscillating positive and negative (in the sense of deficit) masses. In physical geodesy, the RTM is often used as for gravity field smoothing and prediction (Forsberg, 1984).

Gravity forward modelling in the spatial domain

Spatial domain GFM encompasses all evaluation methods for Newton's integral that yields the gravitational potential *V* generated by the source mass model *M* (e.g., Blakeley, 1996, 46)

$$V = G\iiint_M \frac{dm}{r} \tag{2}$$

where *dm* are elementary mass bodies of *M* and *r* is the separation between *dm* and a computation point. Newton's integral is practically evaluated by decomposing the source mass model *M* into elementary mass bodies *dm,* the gravitational potentials of which can be computed through analytical or numerical integration (Kuhn and Seitz, 2005). The gravitational potential of *M* is obtained through addition of all individual potential values implied by the elementary mass bodies (Newtonian integration). Frequently used elementary bodies are point masses, prisms with flat (Nagy et al., 2000) or inclined tops (Smith, 2000), tesseroids (Grombein et al., 2013), and polyhedrons (d'Urso, 2014), also see Papp (1996), Heck and Seitz (2007), Wild-Pfeiffer (2008) and Tsoulis et al. (2009). The gravitational potential of elementary bodies is usually obtained from closed-form expressions (e.g., Nagy et al., 2000), however, numerical integration methods can be an alternative (Asgharzadeh et al., 2007). A decomposition of the mass model into prisms is shown in Fig. 1b.

Gravity disturbances are routinely computed from DTM data through Newtonian integration (Fig. 1c), and subtracted from observed gravity measurements (topographic reduction), e.g., Forsberg (1984), Tziavos et al. (2010). The topographic reduction is often split into two parts, (i) the Bouguer reduction as the gravity effect induced by a solid mass slab or shell of constant thickness (e.g., Strang van Hees, 2000), and (ii) the terrain correction as the gravity effect of the actual DTM masses residual to the slab or shell (e.g., Forsberg, 1984). Terrain corrections are obtained through numerical evaluation of the terrain correction integral which can be derived from Eq. (2), cf. Nahavandchi and Sjöberg (1998).

Accurate spatial domain GFM requires Newtonian integration over the domain of all source masses, which often extend over the whole of the Earth's surface (e.g., Kuhn et al. 2009). In practice, however, integration over the source masses is sometimes done within some limited radius around the computation point (neglecting remote masses), and in planar approximation (instead of the more rigorous spherical or ellipsoidal approximation). The use of RTM as input source model in GFM keeps truncation and approximation effects small, while allowing limitation of the integration to some local radius (e.g., few 10s of km).

The computational effort for spatial domain GFM increase linearly with the number of computation points at which gravity effects are sought. (Sequential) gravity calculations in terms of densely spaced grids (e.g., 100 m resolution) and continental coverage is computationally expensive because Newton's integral has to be evaluated point by point. Parallel computation of gravity effects, e.g., on supercomputing platforms, significantly reduces computation times (e.g., Hirt and Kuhn, 2014). The limited computation radius in the RTM technique reduces the computational efforts too, however, only the high-frequency gravity signals of a mass distribution are obtained (Forsberg, 1984).

Gravity forward modelling in the spectral domain

These methods evaluate Newton's law of gravitation through integral transformation in the spectral domain (e.g., Rummel et al., 1988; Tenzer, 2005; Wieczorek, 2007). This generally require source mass models of global extent. Following Hirt and Kuhn (2014), (1) the source mass model (mostly a DTM) is raised to integer powers (1, 2, … , n) and expanded into spherical harmonic series. This requires spherical harmonic analyses of each integer power of the source model. (2) The resulting sets of coefficients are used to obtain the gravitational potential - as a series expansion of the integer powers of the source mass distribution - in spherical harmonics. (3) The gravity field (e.g., in terms of potential, gravity disturbances) in the spatial domain is computed via spherical harmonic synthesis.

Spectral domain GFM is the preferred technique for planetary studies, e.g., calculation of Bouguer gravity fields for the Moon (Zuber et al., 2013) and Mars (Neumann et al., 2004). This is because observed gravity data for the planets is mostly given in spherical harmonics too. On Earth, the technique is increasingly used for applications with global scope. Examples include the generation of global Bouguer gravity maps in spherical harmonics (Balmino et al., 2012), study of the topographic potential (Novák, 2010; Gruber et al., 2013), all with ultra-high resolution in the km-range. While GFM in the spectral domain usually deploys a sphere as underlying reference body, recent refinements use an ellipsoid instead

(Claessens and Hirt, 2013; Wang and Yang 2013), which take into consideration the ellipsoidal shape, e.g., of Earth and Mars.

As a benefit of spectral domain GFM, the gravitational potential induced by the mass-density distribution is directly obtained in spherical harmonics, which allows straightforward computation of various functionals through synthesis, and spectral filtering. In contrast, space domain modelling requires one computation run per functional, which can be tedious.

As a second benefit of the spectral technique over space domain GFM, there is no numerical integration involved, which can be computationally expensive for computation point grids with global coverage. However, depending on the spatial resolution of the source mass model, multiple spherical harmonic analyses are required for convergence in the spectral method (e.g., Wieczorek, 2007; Claessens and Hirt, 2013), which increase the computational costs (e.g., 7 integer powers for a 5 arc-min resolution terrestrial mass model) as the resolution increases. Spectral and spatial domain GFM were shown to be numerically equivalent techniques (agreement at the 10^{-5} level) at least when the source mass model is limited to features at ~100km scales or larger (Hirt and Kuhn, 2014). However, it is currently unclear if the techniques are equivalent for higher-resolution mass models too.

Applications, some results and directions

There are several GFM applications in geodesy and geophysics which rely on gravity functionals implied by mass models, particularly DTMs. Gravity observations reduced by the gravity effect of the topography reveal subsurface mass-anomalies (e.g., Ebbing et al., 2001; Jacoby and Smilde, 2009), and are pivotal in lithosphere studies (Wieczorek, 2007; Zuber et al., 2013). Forward-modelled gravity is effective at smoothing gravity observations before interpolation, particularly in mountainous areas (Forsberg and Tscherning, 1981). The strong correlation between gravity and topography in rugged terrain (Forsberg, 1984) can be used to approximate the short-wavelength gravity field with GFM. This is often exploited to predict a detailed gravity field where gravity measurements are limited (Pavlis et al., 2007), or to enhance spherical harmonic global gravity models at short spatial scales (Hirt, 2010). GFM is suitable to evaluate the quality of observed gravity fields (e.g., from satellite observations, Hirt et al., 2012), to generate synthetic models for geoid algorithm testing (Baran et al., 2006), and is central for topographic reductions in geoid determination (Jekeli and Serpas, 2003).

The availability of remotely-sensed DTMs (particularly from the SRTM mission) and improved supercomputing resources have stimulated the development of ultra-high resolution gravity field models based on GFM. Balmino et al. (2012) applied spectral domain GFM and global mass models to generate 2km resolution global grids of gravity effects for the construction of the World Gravity Map (WGM). Hirt et al. (2013) used the RTM technique along with observed gravity to construct a near-global gravity map with ultra-fine 220 m resolution (GGMplus).

It is foreseeable that future GFM will ultimately yield global gravity maps, and grids of terrain corrections at 100m resolution and finer, commensurate with the mass information available through remotely sensed DTMs. Accurate GFM of the topography will require the DTM data to be largely free of artefacts. In order to approximate the actual gravity field closely, detailed compilations of mass-density values are important to improve the quality of the DTM-based mass models.

Summary

Gravity forward modelling (GFM) comprises methods for the computation of the gravity field of some mass distribution, very often of the topography, but also from water, ice and crustal masses. GFM in the spatial and spectral domain are important for gravity applications in physical geodesy (e.g., gravity reduction, interpolation and prediction, e.g., for the construction of detailed gravity maps) and potential field or planetary geophysics (reduction and interpretation of gravity observations).

Cross references

Topographic Effects, Digital Terrain Models, Spherical Harmonic Models, The Remove Restore Method, Gravity Field of the Planets, Gravity Anomalies

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