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Joint MMSE Transceiver Design in Non-Regenerative MIMO Relay Systems with Covariance Feedback

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***Abstract*— In this paper, the problem of transceiver design in a non-regenerative MIMO relay system is addressed, where linear signal processing is applied at the relay and destination to minimize the mean-squared error (MSE) of the signal waveform estimation. The optimal structure of the relay precoding matrix is derived with the assumption that the relay knows the channel covariance information of the relay-destination link and the full channel state information (CSI) of the source-relay link. Simulation results demonstrate that the proposed scheme outperforms conventional relay algorithms, and its performance is comparable to the optimal relay algorithm using the full relay-destination CSI.**

***Index Terms*— MIMO relay, MMSE, Covariance Feedback.**

I. INTRODUCTION

Wireless relaying is essential to provide reliable and cost effective, wide-area coverage for wireless networks in a variety of applications. In a cellular environment, a relay can be deployed in areas where there are strong shadowing effects, such as inside buildings and tunnels. For mobile ad-hoc networks, relaying is essential not only to overcome shadowing due to obstacles but also to reduce transmission power from source to neighbouring nodes. For tactical applications, dynamic deployment of manned or unmanned relays is useful to enhance the networks reliability, throughput, and minimize interception by unwanted users.

There are two types of relay strategies: non-regenerative scheme and regenerative scheme [1] - [3]. Compared with the regenerative scheme, the non-regenerative scheme is easy to implement, and thus is embraced by industry.

Recent studies show that performing linear precoding at the relays in a non-regenerative MIMO relay system can provide higher rate data transmission than a single-antenna system in a scattered environment. In [4] and [5] a relay precoding scheme in non-regenerative MIMO relaying has been proposed to increase the capacity between the source and destination with further signal processing. In this scheme, the relay multiplies the received signal by a linear precoding matrix and retransmits the precoded signal to the destination. In [6] - [8], the precoding matrix was designed by minimizing

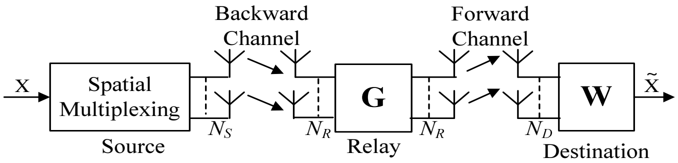

Fig. 1. Non-regenerative MIMO relay system

the MSE of the signal waveform estimation at the destination. An optimal precoding matrix based on the maximum signal-to-noise ratio (SNR) criterion is developed in [7]. A unified framework is developed in [8] to jointly optimize the source precoding matrix and the relay amplifying matrix for a broad class of objective functions. In [6] - [8], the full channel state information (CSI) for all link is assumed to be available at the relay.

In a practical system with a limited feedback rate, the assumption that the relay knows the full CSI for the relay-destination link is not feasible, especially in the situation when the mobile node is moving rapidly. The covariance matrix is more stable than the instantaneous channel matrix because the scattering environment changes more slowly compared to the mobile location. In [9] and [10], the precoding matrix is derived for maximizing the ergodic capacity when only the partial CSI for the relay-destination link is available at the relay. Recently, covariance feedback based minimum mean-squared error (MMSE) estimator is proposed in [11] and the estimator is only suitable for a MIMO relay system, where the number of antennas at the destination is greater than the relay antennas. In this paper, optimal precoder is proposed to minimize the MSE of the symbol estimation in a non-regenerative MIMO relay system, when the covariance information for the relay-destination link is available at the relay. It is assumed that the relay knows the full CSI of the source-relay link and channel covariance information (CCI) of the relay destination link. As well as by restraining power consumption at the relay node, we derive the optimal precoding matrix to minimize the MSE of the estimated symbols at the destination. The proposed algorithm is not constrained by the number of antennas at the destination as in [11]. Simulation results show the effectiveness of the proposed MSE scheme.

The rest of the paper is organized as follows. In Section II, we introduce the system model of a two-hop non-regenerative MIMO relay system. The proposed algorithm is developed in Section III. In Section IV, we show some numerical examples. The conclusions are drawn in Section V.

II. System Model and Problem Formulation

Consider the non-regenerative MIMO relay system as shown in Fig.1, where the source, relay and destination have N_S, N_R and N_D antennas, respectively. It is assumed that there is no direct link exist between the source and destination due to long distance between these two points. The data transmission takes place over two time slots. The received signal at the relay during the first time slot is given by

$$\mathbf{y}_1 = \mathbf{H}_1\mathbf{x} + \mathbf{n}_1 \tag{1}$$

where $\mathbf{H}_1 \in \mathbb{C}^{N_R \times N_S}$ is the channel matrix of the source-relay link, $\mathbf{x} \in \mathbb{C}^{N_S \times 1}$ is the transmitted vector with covariance matrix $E\{\mathbf{x}\mathbf{x}^H\} = \sigma_x^2\mathbf{I}_{N_S}$, $\mathbf{n}_1 \in \mathbb{C}^{N_R \times 1}$ is the circularly symmetric complex Gaussian noise vector with zero mean and covariance matrix $E\{\mathbf{n}_1\mathbf{n}_1^H\} = \sigma_1^2\mathbf{I}_{N_R}$. Here $E[.]$ denotes the statistical expectation.

The received signal at the destination in the second time slot is given by

$$\mathbf{y}_2 = \mathbf{H}_2\mathbf{G}\mathbf{H}_1\mathbf{x} + \mathbf{H}_2\mathbf{G}\mathbf{n}_1 + \mathbf{n}_2 \tag{2}$$

where $\mathbf{H}_2 \in \mathbb{C}^{N_D \times N_R}$ is the channel matrix of the relay-destination link, $\mathbf{G} \in \mathbb{C}^{N_R \times N_R}$ is a precoding matrix of the relay, $\mathbf{n}_2 \in \mathbb{C}^{N_D \times 1}$ is the circularly symmetric complex Gaussian noise vector with zero mean and covariance matrix $E\{\mathbf{n}_2\mathbf{n}_2^H\} = \sigma_2^2\mathbf{I}_{N_D}$. Let us introduce

$$\mathbf{H} = \mathbf{H}_2\mathbf{G}\mathbf{H}_1 \tag{3}$$

and

$$\mathbf{n} = \mathbf{H}_2\mathbf{G}\mathbf{n}_1 + \mathbf{n}_2 \tag{4}$$

where $\mathbf{H} \in \mathbb{C}^{N_D \times N_S}$ is the equivalent MIMO channel matrix, and $\mathbf{n} \in \mathbb{C}^{N_D \times 1}$ represents the equivalent noise vector. Now (2) can be written as

$$\mathbf{y}_2 = \mathbf{H}\mathbf{x} + \mathbf{n}. \tag{5}$$

Similar to [10], let us assume that the channel of the relay-destination link is correlated at the transmit antennas and is uncorrelated at the receive antennas. The model is suitable for an environment where the relay is not encumbered by local scatters and the destination is fully surrounded by local scatters [12]. It is assumed that $\mathbf{H}_2$ can be expressed as

$$\mathbf{H}_2 = \mathbf{H}_\omega\mathbf{\Sigma}^{1/2} \tag{6}$$

where $\mathbf{H}_\omega$ is an $N_D \times N_R$ Gaussian matrix having i.i.d. circularly symmetric complex entries with zero mean and unit variance, and $\mathbf{\Sigma}$ is an $N_R \times N_R$ covariance matrix of $\mathbf{H}_2$ at the relay side.

To reduce implementation complexity, linear receiver $\mathbf{W}$ is applied at the destination, the estimated signal is given by

$$\tilde{\mathbf{x}} = \mathbf{W}\mathbf{H}\mathbf{x} + \mathbf{W}\mathbf{n}. \tag{7}$$

We assume that the average power used by the source is upper bounded by P_s, and the average power used by the relay is upper bounded by P_r. Since the transmitted signal from the relay is $\mathbf{G}\mathbf{y}_1 = \mathbf{G}\mathbf{H}_1\mathbf{x} + \mathbf{G}\mathbf{n}_1$, the power constraint on the relay can be expressed as

$$p(\mathbf{G}) = tr\Big\{\mathbf{G}(\sigma_x^2\mathbf{H}_1\mathbf{H}_1^H + \sigma_1^2\mathbf{I}_{N_R})\mathbf{G}^H\Big\} \le P_r \tag{8}$$

where $tr\{.\}$ is the trace of a matrix. Our goal is to design $\mathbf{G}$ and $\mathbf{W}$ so as to obtain the estimated signal which minimizes the following MSE function subject to the power constraint (8).

$$J(\mathbf{G},\mathbf{W}) = tr\Big\{E\Big[(\tilde{\mathbf{x}} - \mathbf{x})(\tilde{\mathbf{x}} - \mathbf{x})^H\Big]\Big\} \tag{9}$$

Mathematically, this problem can be formulated as

$$(\mathbf{G},\mathbf{W}) = \arg\min_{(\mathbf{G},\mathbf{W})} J(\mathbf{G},\mathbf{W}),\ s.t.\ p(\mathbf{G}) \le P_r. \tag{10}$$

After substituting (7) into (9), the MSE function (9) is simplified to

$$J(\mathbf{G},\mathbf{W}) = tr\Big\{\sigma_x^2\Big(\mathbf{W}\mathbf{H} - \mathbf{I}_{N_S}\Big)\Big(\mathbf{W}\mathbf{H} - \mathbf{I}_{N_S}\Big)^H + \mathbf{W}\mathbf{R}_n\mathbf{W}^H\Big\} \tag{11}$$

where $\mathbf{R}_n$ is the equivalent noise covariance matrix, given by

$$\begin{aligned}\mathbf{R}_n &= E\Big[\mathbf{n}\mathbf{n}^H\Big]\\ &= E\Big[\big(\mathbf{H}_2\mathbf{G}\mathbf{n}_1 + \mathbf{n}_2\big)\big(\mathbf{H}_2\mathbf{G}\mathbf{n}_1 + \mathbf{n}_2\big)^H\Big]\\ &= \sigma_1^2\mathbf{H}_2\mathbf{G}\mathbf{G}^H\mathbf{H}_2^H + \sigma_2^2\mathbf{I}_{N_D}.\end{aligned} \tag{12}$$

Note that directly solving the constrained optimization problem (10) is difficult due to the fact that both the cost function $J(\mathbf{G},\mathbf{W})$ and the power constraint are non-linear function of $\mathbf{G}$ and $\mathbf{W}$. In the following section a suboptimal approach will be used to tackle the constrained non-linear optimization problem. First, the problem will be solved for the optimal linear receiver $\mathbf{W}$ for any given precoding matrix $\mathbf{G}$ which satisfies the power constraint (8). Then, the optimal precoding matrix $\mathbf{G}$ will be found by solving a closely related constrained optimization problem.

III. Optimal Transceiver Design

For any given precoding matrix $\mathbf{G}$ which satisfies the power constraint (8), the optimal linear receiver $\mathbf{W}$ that minimizes the MSE function $J(\mathbf{G},\mathbf{W})$ is the same as the MMSE (Weiner) receiver [13], which is given by

$$\mathbf{W} = \sigma_x^2\mathbf{H}^H(\sigma_x^2\mathbf{H}\mathbf{H}^H + \mathbf{R}_n)^{-1}. \tag{13}$$

After substituting (13) into (11), the MSE function is obtained as

$$J(\mathbf{G}) = \sigma_x^2 tr\Big\{\mathbf{I}_{N_S} - \sigma_x^2\mathbf{H}^H(\sigma_x^2\mathbf{H}\mathbf{H}^H + \mathbf{R}_n)^{-1}\mathbf{H}\Big\}. \quad (14)$$

Note that $\mathbf{H} = \mathbf{H}_2\mathbf{G}\mathbf{H}_1$. Using the following matrix inversion lemma [13]

$$(\mathbf{A}+\mathbf{B}\mathbf{C}\mathbf{D})^{-1}=\mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{B} \times(\mathbf{D}\mathbf{A}^{-1}\mathbf{B}+\mathbf{C}^{-1})^{-1}\mathbf{D}\mathbf{A}^{-1}, \quad (15)$$

the MSE function (14) can be written as

$$J(\mathbf{G}) = \sigma_x^2 tr\Big\{\Big[\mathbf{I}_{N_S} + \sigma_x^2\mathbf{H}^H\mathbf{R}_n^{-1}\mathbf{H}\Big]^{-1}\Big\}. \quad (16)$$

Substituting (3) and (12) into (16), the MSE function can be expressed as

$$J(\mathbf{G})=\sigma_x^2 tr\Big\{\Big[\mathbf{I}_{N_S} + \sigma_x^2\mathbf{H}_1^H\mathbf{G}^H\mathbf{H}_2^H \times\Big(\sigma_1^2\mathbf{H}_2\mathbf{G}\mathbf{G}^H\mathbf{H}_2^H + \sigma_2^2\mathbf{I}_{N_D}\Big)^{-1}\mathbf{H}_2\mathbf{G}\mathbf{H}_1\Big]^{-1}\Big\}. (17)$$

Now the problem is reduced to find the optimal $\mathbf{G}$ that minimize $J(\mathbf{G})$ subject to the power constraint (8). Let us introduce the singular value decomposition (SVD) of $\mathbf{H}_1$

$$\mathbf{H}_1 = \mathbf{U}_1\mathbf{\Lambda}_1^{1/2}\mathbf{V}_1^H \quad (18)$$

where $\mathbf{\Lambda}_1 = \text{diag}\{\Lambda_{1,1}\cdots\Lambda_{1,N_R}\}$ is a diagonal matrix with $\Lambda_{1,1} \geq \cdots \geq \Lambda_{1,N_R}$. We introduce the eigenvalue decomposition of $\mathbf{\Sigma}$ as $\mathbf{\Sigma} = \mathbf{V}_\Sigma\mathbf{\Lambda}_\Sigma\mathbf{V}_\Sigma^H$ where $\mathbf{\Lambda}_\Sigma = \text{diag}\{\Lambda_{\Sigma,1}\cdots\Lambda_{\Sigma,N_R}\}$ with $\Lambda_{\Sigma,1} \geq \cdots \geq \Lambda_{\Sigma,N_R}$. The columns of $\mathbf{V}_\Sigma$ are the eigenvectors of $\mathbf{\Sigma}$ for the corresponding eigenvalues. Then $\mathbf{H}_2$ can be rewritten as

$$\mathbf{H}_2 = \mathbf{Z}\mathbf{\Lambda}_\Sigma^{1/2}\mathbf{V}_\Sigma^H \quad (19)$$

where $\mathbf{Z} \triangleq \mathbf{H}_2\mathbf{V}_\Sigma\mathbf{\Lambda}_\Sigma^{-1/2}$. Then $\mathbf{Z}$ has the same distribution as $\mathbf{H}_w$ because $\mathbf{H}_2\mathbf{V}_\Sigma\mathbf{\Lambda}_\Sigma^{-1/2} = \mathbf{H}_\omega\mathbf{V}_\Sigma$. Let's assume that the optimal precoding matrix $\mathbf{G}$ which minimizes (17) can be expressed as

$$\mathbf{G} = \mathbf{V}_\Sigma\mathbf{\Lambda}_G^{1/2}\mathbf{U}_1^H \quad (20)$$

where $\mathbf{\Lambda}_G = \text{diag}\{\Lambda_{G,1}\cdots\Lambda_{G,N_R}\}$. Using the matrix inversion lemma (15), the MSE function (17) can be written as

$$J(\mathbf{G})=\sigma_x^2 tr\Big\{\Big[\mathbf{I}_{N_S} + \frac{\sigma_x^2}{\sigma_1^2}\mathbf{H}_1^H\Big[\mathbf{I}_{N_R} -\Big(\mathbf{I}_{N_R} + \frac{\sigma_1^2}{\sigma_2^2}\mathbf{G}^H\mathbf{H}_2^H\mathbf{H}_2\mathbf{G}\Big)^{-1}\Big]\mathbf{H}_1\Big]^{-1}\Big\}. \quad (21)$$

Substituting (18) - (20) in (21), now the MSE function is given by

$$J(\mathbf{G})=\sigma_x^2 tr\Big\{\Big[\mathbf{I}_{N_S} + \frac{\sigma_x^2}{\sigma_1^2}\mathbf{V}_1\mathbf{\Lambda}_1^{1/2}\mathbf{U}_1^H \times\Big[\mathbf{I}_{N_R} - \mathbf{D}_1\Big]\mathbf{U}_1\mathbf{\Lambda}_1^{1/2}\mathbf{V}_1^H\Big]^{-1}\Big\} \quad (22)$$

where

$$\mathbf{D}_1 = \Big(\mathbf{I}_{N_R} + \frac{\sigma_1^2}{\sigma_2^2}\mathbf{U}_1\mathbf{\Lambda}_G^{1/2}\mathbf{\Lambda}_\Sigma^{1/2}\mathbf{Z}^H\mathbf{Z}\mathbf{\Lambda}_\Sigma^{1/2}\mathbf{\Lambda}_G^{1/2}\mathbf{U}_1^H\Big)^{-1}.$$

Using the SVD and trace properties, the MSE function (22) can be simplified to

$$J(\mathbf{G}) = \sigma_x^2 tr\Big\{\Big[\mathbf{I}_{N_S} + \frac{\sigma_x^2}{\sigma_1^2}\Big(\mathbf{\Lambda}_1 - \mathbf{\Lambda}_1^{1/2}\mathbf{D}_2\mathbf{\Lambda}_1^{1/2}\Big)\Big]^{-1}\Big\} = \sigma_x^2\sigma_1^2 tr\Big\{\Big[\sigma_1^2\mathbf{I}_{N_S} + \sigma_x^2\Big(\mathbf{\Lambda}_1 - \mathbf{\Lambda}_1^{1/2}\mathbf{D}_2\mathbf{\Lambda}_1^{1/2}\Big)\Big]^{-1}\Big\} \quad (23)$$

where

$$\mathbf{D}_2 = \Big(\mathbf{I}_{N_R} + \frac{\sigma_1^2}{\sigma_2^2}\mathbf{\Lambda}_G^{1/2}\mathbf{\Lambda}_\Sigma^{1/2}\mathbf{Z}^H\mathbf{Z}\mathbf{\Lambda}_\Sigma^{1/2}\mathbf{\Lambda}_G^{1/2}\Big)^{-1}.$$

It can be seen from (23) that $J(\mathbf{G})$ depends on $\mathbf{Z}$, which is random and unknown. In the following, we optimize $E_{\mathbf{Z}}[J(\mathbf{G})]$, where $E_{\mathbf{Z}}[.]$ indicates that the expectation is taken with respect to the random matrix $\mathbf{Z}$. Now $E_{\mathbf{Z}}[J(\mathbf{G})]$ can be expressed as

$$E_{\mathbf{Z}}[J(\mathbf{G})]=\sigma_x^2\sigma_1^2 E_{\mathbf{Z}}\Big[tr\Big\{\Big[\sigma_1^2\mathbf{I}_{N_S} + \sigma_x^2 \times\Big(\mathbf{\Lambda}_1 - \mathbf{\Lambda}_1^{1/2}\mathbf{D}_2\mathbf{\Lambda}_1^{1/2}\Big)\Big]^{-1}\Big\}\Big] \quad (24)$$

where

$$\mathbf{D}_2 = \Big(\mathbf{I}_{N_R} + \frac{\sigma_1^2}{\sigma_2^2}\mathbf{\Lambda}_G^{1/2}\mathbf{\Lambda}_\Sigma^{1/2}\mathbf{Z}^H\mathbf{Z}\mathbf{\Lambda}_\Sigma^{1/2}\mathbf{\Lambda}_G^{1/2}\Big)^{-1}.$$

Now the work is left to determine the diagonal elements $\mathbf{\Lambda}_G$ of precoder matrix $\mathbf{G}$. The optimal precoder allocates power according to the eigenmodes of $\mathbf{H}_1\mathbf{H}_1^H$ and $\mathbf{\Sigma}$.

Direct minimization of (24) for the optimal power allocation is difficult. In the following, the lower bound of the MSE is used together with the power constraint (8) to derive the suboptimal power allocation for the precoder matrix $\mathbf{G}$. Assume that the MSE function is convex in $\mathbf{Z}^H\mathbf{Z}$ and has the following lower bound using Jensen's inequality

$$J_L(\mathbf{G}) = \sigma_x^2\sigma_1^2 tr\Big\{\Big[\sigma_1^2\mathbf{I}_{N_S} + \sigma_x^2\mathbf{\Lambda}_1 - \sigma_x^2\mathbf{\Lambda}_1^{1/2}\mathbf{D}_3\mathbf{\Lambda}_1^{1/2}\Big]^{-1}\Big\}$$

where

$$\mathbf{D}_3 = \Big(\mathbf{I}_{N_R} + \frac{\sigma_1^2}{\sigma_2^2}\mathbf{\Lambda}_G^{1/2}\mathbf{\Lambda}_\Sigma^{1/2}E_{\mathbf{Z}}[\mathbf{Z}^H\mathbf{Z}]\mathbf{\Lambda}_\Sigma^{1/2}\mathbf{\Lambda}_G^{1/2}\Big)^{-1}.$$

Now the MSE function is simplified to

$$J_L(\mathbf{G}) = \sigma_x^2\sigma_1^2 tr\Big\{\Big[\sigma_1^2\mathbf{I}_{N_S} + \sigma_x^2\mathbf{\Lambda}_1 - \sigma_x^2\mathbf{\Lambda}_1\Big(\mathbf{I}_{N_R} + \frac{\sigma_1^2}{\sigma_2^2}\mathbf{\Lambda}_G\mathbf{\Lambda}_\Sigma N_D\Big)^{-1}\Big]^{-1}\Big\} \quad (25)$$

where $E_{\mathbf{Z}}(\mathbf{Z}^H\mathbf{Z}) = N_D\mathbf{I}_{N_R}$. Inserting (18) and (20) into (8), the power constraint for the relay node can be expressed as

$$p(\mathbf{G})=tr\Big\{\mathbf{V}_\Sigma\mathbf{\Lambda}_G^{1/2}\mathbf{U}_1^H\Big(\sigma_x^2\mathbf{U}_1\mathbf{\Lambda}_1\mathbf{U}_1^H + \sigma_1^2\mathbf{I}_{N_R}\Big) \times\mathbf{U}_1\mathbf{\Lambda}_G^{1/2}\mathbf{V}_\Sigma^H\Big\} \leq P_r. \quad (26)$$

Using the SVD and trace properties, the power constraint (26) can be simplified to

$$p(\mathbf{G}) = tr\Big\{\big(\sigma_x^2\mathbf{\Lambda}_1 + \sigma_1^2\mathbf{I}_{N_R}\big)\mathbf{\Lambda}_G\Big\} \leq P_r. \quad (27)$$

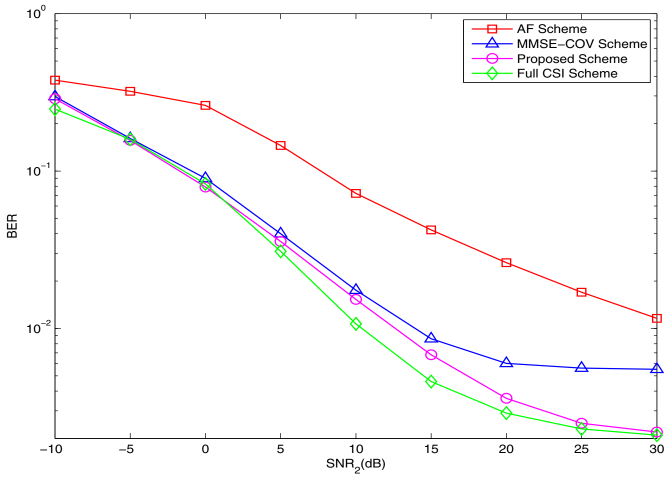

Fig. 2. BER versus SNR_2 while $SNR_1 = 20dB$.

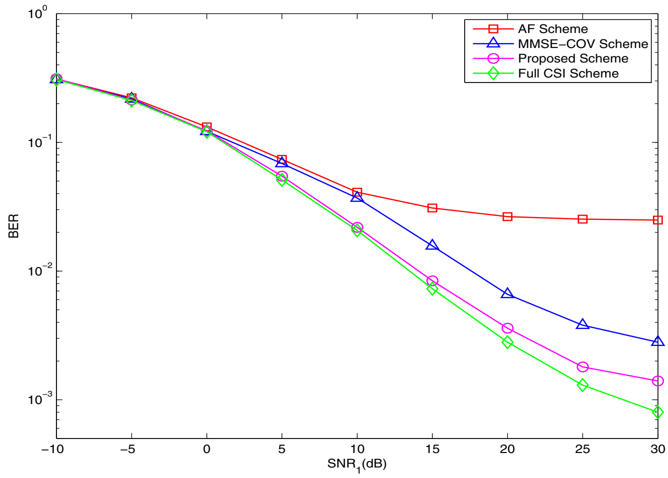

Fig. 3. BER versus SNR_1 while $SNR_2 = 20dB$.

From (25) and (27), we can have the following constrained optimization problem.

$$\min J_L(\mathbf{G}) = \sigma_x^2 \sum_{i=1}^{N_S} \frac{\sigma_1^2 N_D \Lambda_{\Sigma,i}\Lambda_{G,i} + \sigma_2^2}{(\sigma_x^2 \Lambda_{1,i} + \sigma_1^2) N_D \Lambda_{\Sigma,i}\Lambda_{G,i} + \sigma_2^2} \quad (28)$$

$$s.t.\ p(\mathbf{G}) = \sum_{i=1}^{N_S} (\sigma_x^2 \Lambda_{1,i} + \sigma_1^2)\Lambda_{G,i} \leq P_r. \quad (29)$$

Using the Karush-Kuhn-Tucker (KKT) conditions [14], the optimal diagonal elements of $\Lambda_{G,i}$ are obtained as

$$\Lambda_{G,i} = \frac{1}{D_4}\left(\sqrt{\frac{\sigma_x^2 \sigma_2^2 N_D \Lambda_{1,i}\Lambda_{\Sigma,i}}{\mu(\sigma_x^2 \Lambda_{1,i} + \sigma_1^2)}} - \sigma_2^2\right)^+ \quad (30)$$

where $(x)^+ = \max(x, 0)$, $D_4 = (\sigma_x^2 \Lambda_{1,i} + \sigma_1^2) N_D \Lambda_{\Sigma,i}$ and μ should be chosen to meet the power constraint (29). Inserting (30) and (18)-(20) into (13) leads to obtain the optimal receiver matrix $\mathbf{W}$.

IV. Simulation Results

In this section, we illustrate the performance of the proposed scheme by numerical examples. We simulate the MIMO relay system with $N_S = N_R = N_D$=4. The channel matrices $\mathbf{H}_1$ and $\mathbf{H}_\omega$ are generated as complex Gaussian variables with zero mean and unit variance and the symbols are generated from QPSK constellation.

The elements of covariance matrix $\mathbf{\Sigma}$ of $\mathbf{H}_2$ is generated by $\Sigma_{i,j} = j_0(\triangle\pi|i-j|)$ [12], where $j_0(.)$ is the zeroth order Bessel function of the first kind, $\triangle$ the angle of fading spread. We consider the angle spread as $\triangle = 5^o$. The SNRs for the source-relay and relay-destination links are defined as follows $SNR_1 = \frac{\sigma_x^2}{\sigma_1^2}$, $SNR_2 = \frac{P_r}{N_R \sigma_2^2}$.

We compare the performance of the proposed scheme with that of the full CSI scheme [6], the MMSE-COV scheme [11], and the traditional AF scheme. The full CSI scheme, also known as JMMSE [6] provides the lower-bound of the proposed scheme. Similar to [11] for MMSE-COV scheme, the MIMO relay system is simulated with $N_S = N_R$ =4 and N_D=5. In the conventional AF scheme, the relay precoder is obtained by $\mathbf{G} = \alpha\mathbf{I}$, where α is determined to meet the power constraint (29).

Fig.2 shows the performance of MMSE schemes in terms of BER versus SNR_2 while fixing SNR_1 = 20dB. The proposed scheme shows better BER performance over all range of SNR_2 than the MMSE-COV scheme and the AF scheme. For high SNR_2, the BER performance of the proposed scheme is closer to that of the full CSI scheme.

Fig.3 shows the BER performance for various SNR_1 while fixing SNR_2= 20dB. In Fig.3 the proposed scheme performance is similar to the MMSE-COV and AF schemes in low SNR_1 (e.g. $SNR_1 <$ 5dB) because the received signal at the relay is impaired by the noise. For high SNR_1, the proposed scheme shows better BER performance than the MMSE-COV scheme and the conventional AF scheme. In other words the proposed scheme outperforms the MMSE-COV scheme and the conventional AF scheme.

V. Conclusion

We derived the optimal structure of the non-regenerative MIMO relay matrix to minimize the MSE of the symbol estimation at the destination with the assumption that the covariance feedback of the relay-destination link is available at the relay. We assumed that the relay knows the full CSI of the source-relay link. Simulation results show that the derived optimal solution which minimize the upper-bound of the MSE is achieved and the simulation results demonstrate that the proposed scheme has better performance in terms of BER as compared to the conventional MSE schemes.

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