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Simplified Relay Algorithm for Two-Way MIMO Relay Communications

Yue Rong
Department of Electrical and Computer Engineering, Curtin University, Bentley, WA 6102, Australia

***Abstract*—In this paper, we investigate the challenging problem of joint source and relay optimization for two-way linear non-regenerative multiple-input multiple-output (MIMO) relay communication systems. First, a novel relay amplifying matrix is proposed which significantly reduces the computational complexity of the optimal relay design with only a marginal performance degradation. Interestingly, we show that the proposed relay matrix is indeed optimal for some special cases. Second, a semi-definite programming (SDP)-based source matrices optimization algorithm is developed. Then the source and relay optimization algorithms are carried out iteratively to minimized the sum mean-squared error of the signal waveform estimation in a two-way MIMO relay system. The performance of the proposed algorithm is demonstrated by numerical simulations.**

I. Introduction

In a two-way relay communication system, two source nodes exchange their information through an assisting relay node. By resorting to the idea of analog network coding [1], the information exchange can be completed in two time slots with a half-duplex relay. This leads to a high spectral efficiency.

When nodes in the relay network are equipped with multiple antennas, we have a two-way multiple-input multiple-output (MIMO) relay system [2]-[4]. Distributed space-time coding has been designed in [5] for two-way relay communication with multiple single-antenna relay nodes. For a two-way (and in general N-way) relay system with a multi-antenna relay node and single-antenna source nodes, the relay beamforming issue has been investigated in [6] and [7]. For two-way MIMO relay systems, the optimal relay and source matrices have been developed in [2] and [8] to maximize the two-way sum mutual information (SMI). A minimal sum mean-squared error (MSMSE) based two-way MIMO relay system was proposed in [4]. An algebraic norm-maximization relaying algorithm has been developed in [9]. Two-way relay communication in a multiuser scenario was recently studied in [10].

In this paper, we first propose a new simplified relay amplifying matrix design which significantly reduces the computational complexity of the optimal relay design in [2], [4] with only a marginal performance degradation. This is important for practical relay systems. Interestingly, we show that the proposed relay matrix is indeed optimal for some special cases. Second, a semi-definite programming (SDP)-based source matrices optimization algorithm is developed in this paper. Then the source and relay optimization algorithms are carried out iteratively to minimized the SMSE of the signal waveform estimation in a two-way MIMO relay system.

Numerical simulations are carried out to study the performance of the proposed source and relay matrices design algorithm. It is shown that the proposed iterative algorithm converges in only a few iterations, which is important for practical two-way relay systems. We also show that the MSMSE-based relay algorithm has a better bit-error-rate (BER) performance compared with the algorithm using the maximal SMI (MSMI) criterion [2].

The rest of this paper is organized as follows. In Section II, we introduce the model of a two-way linear non-regenerative MIMO relay communication system. The source and relay matrices design algorithms are developed in Section III. In Section IV, we show some numerical examples. Conclusions are drawn in Section V.

II. System Model

We consider a three-node MIMO communication system where nodes 1 and 2 exchange information with the aid of one relay node. We assume that both nodes 1 and 2 are equipped with N antennas, and the relay node has M antennas. The information exchange between nodes 1 and 2 is completed in two time slots. In the first time slot, nodes 1 and 2 concurrently transmit, and the signal vector from node i is $\mathbf{x}_i = \mathbf{B}_i\mathbf{s}_i$, $i = 1, 2$, where $\mathbf{s}_i$ is the $N \times 1$ source signal vector, and $\mathbf{B}_i$ is the $N \times N$ source precoding matrix at node i. The signal vector $\mathbf{y}_r$ received at the relay node can be written as

$$\mathbf{y}_r = \mathbf{H}_{r,1}\mathbf{B}_1\mathbf{s}_1 + \mathbf{H}_{r,2}\mathbf{B}_2\mathbf{s}_2 + \mathbf{v}_r \tag{1}$$

where $\mathbf{H}_{r,i}$, $i = 1, 2$, is the $M \times N$ channel matrix between the relay node and node i, and $\mathbf{v}_r$ is the $M \times 1$ noise vector at the relay node.

In the second time slot, the relay node linearly amplifies $\mathbf{y}_r$ with an $M \times M$ matrix $\mathbf{F}$ and broadcasts the amplified signal vector $\mathbf{x}_r = \mathbf{F}\mathbf{y}_r$ to nodes 1 and 2. The effective received signal vectors at two nodes after the removal of the self-interference is given by

$$\mathbf{y}_1 = \mathbf{H}_{1,r}\mathbf{F}\mathbf{H}_{r,2}\mathbf{B}_2\mathbf{s}_2 + \mathbf{H}_{1,r}\mathbf{F}\mathbf{v}_r + \mathbf{v}_1 \triangleq \tilde{\mathbf{H}}_1\mathbf{s}_2 + \tilde{\mathbf{v}}_1 \tag{2}$$
$$\mathbf{y}_2 = \mathbf{H}_{2,r}\mathbf{F}\mathbf{H}_{r,1}\mathbf{B}_1\mathbf{s}_1 + \mathbf{H}_{2,r}\mathbf{F}\mathbf{v}_r + \mathbf{v}_2 \triangleq \tilde{\mathbf{H}}_2\mathbf{s}_1 + \tilde{\mathbf{v}}_2 \tag{3}$$

where $\mathbf{H}_{i,r}$, $i = 1, 2$, is the $N \times M$ channel matrix between node i and the relay, $\mathbf{v}_i$, $i = 1, 2$, is the $N \times 1$ noise vector at node i, $\tilde{\mathbf{H}}_i$ is the equivalent MIMO channel seen at node i, and $\tilde{\mathbf{v}}_i$ is the equivalent noise vector at node i with

$$\tilde{\mathbf{H}}_1 \triangleq \mathbf{H}_{1,r}\mathbf{F}\mathbf{H}_{r,2}\mathbf{B}_2 \qquad \tilde{\mathbf{v}}_1 \triangleq \mathbf{H}_{1,r}\mathbf{F}\mathbf{v}_r + \mathbf{v}_1$$
$$\tilde{\mathbf{H}}_2 \triangleq \mathbf{H}_{2,r}\mathbf{F}\mathbf{H}_{r,1}\mathbf{B}_1 \qquad \tilde{\mathbf{v}}_2 \triangleq \mathbf{H}_{2,r}\mathbf{F}\mathbf{v}_r + \mathbf{v}_2 .$$

We assume that the source signal vectors satisfy $\mathrm{E}[\mathbf{s}_i\mathbf{s}_i^H] = \mathbf{I}_N$, $i = 1, 2$, and all noises are independent and identically distributed (i.i.d.) additive white Gaussian noise (AWGN) with zero mean and unit variance. Here $\mathrm{E}[\cdot]$ stands for the statistical expectation, $\mathbf{I}_N$ is an $N \times N$ identity matrix, and $(\cdot)^H$ denotes matrix (vector) Hermitian transpose. We also assume that the relay node knows the channel state information (CSI) of $\mathbf{H}_{r,i}$ and $\mathbf{H}_{i,r}$, $i = 1, 2$. The relay node performs the optimization of $\mathbf{F}$, $\mathbf{B}_1$, $\mathbf{B}_2$, and then transmits the information of matrices $\mathbf{B}_i$ and $\mathbf{H}_{i,r}\mathbf{F}\mathbf{H}_{r,i}\mathbf{B}_i$ to node i, $i = 1, 2$. In this paper, we do not make any assumption on the reciprocity of $\mathbf{H}_{i,r}$ and $\mathbf{H}_{r,i}$, $i = 1, 2$, and the statistical property of channel matrices (e.g. independency between $\mathbf{H}_{1,r}$ and $\mathbf{H}_{2,r}$).

Due to their lower computational complexity, linear receivers are used at nodes 1 and 2 to retrieve the transmitted signals sent from the other node. The estimated signal waveform vector is given by $\hat{\mathbf{s}}_1 = \mathbf{W}_2^H\mathbf{y}_2$ and $\hat{\mathbf{s}}_2 = \mathbf{W}_1^H\mathbf{y}_1$, where $\mathbf{W}_1$ and $\mathbf{W}_2$ are $N \times N$ weight matrices. From (2) and (3), the MSE matrix of the signal waveform estimation, denoted as $\mathbf{E}_2 = \mathrm{E}\big[(\hat{\mathbf{s}}_1 - \mathbf{s}_1)(\hat{\mathbf{s}}_1 - \mathbf{s}_1)^H\big]$ and $\mathbf{E}_1 = \mathrm{E}\big[(\hat{\mathbf{s}}_2 - \mathbf{s}_2)(\hat{\mathbf{s}}_2 - \mathbf{s}_2)^H\big]$, can be written as

$$\mathbf{E}_i = (\mathbf{W}_i^H\tilde{\mathbf{H}}_i - \mathbf{I}_{N_b})(\mathbf{W}_i^H\tilde{\mathbf{H}}_i - \mathbf{I}_{N_b})^H + \mathbf{W}_i^H\mathbf{C}_{\tilde{v}_i}\mathbf{W}_i, \; i = 1, 2 \tag{4}$$

where $\mathbf{C}_{\tilde{v}_i} \triangleq \mathrm{E}\big[\tilde{\mathbf{v}}_i\tilde{\mathbf{v}}_i^H\big] = \mathbf{H}_{i,r}\mathbf{F}\mathbf{F}^H\mathbf{H}_{i,r}^H + \mathbf{I}_N$, $i = 1, 2$, is the equivalent noise covariance matrix at node i.

The weight matrix of the optimal linear receiver which minimizes $\mathrm{tr}(\mathbf{E}_i)$ is the Wiener filter given by

$$\mathbf{W}_i^{\mathrm{opt}} = (\tilde{\mathbf{H}}_i\tilde{\mathbf{H}}_i^H + \mathbf{C}_{\tilde{v}_i})^{-1}\tilde{\mathbf{H}}_i, \qquad i = 1, 2 \tag{5}$$

where $\mathrm{tr}(\cdot)$ and $(\cdot)^{-1}$ stands for matrix trace and matrix inversion, respectively. The weight matrices in (5) are computed at the relay node and forwarded to the corresponding destination node after the optimal $\mathbf{F}$, $\mathbf{B}_1$, $\mathbf{B}_2$ are calculated. By substituting (5) back into (4), the MSE matrix of the signal waveform estimation at two nodes can be written as

$$\mathbf{E}_i = \big[\mathbf{I}_N + \tilde{\mathbf{H}}_i^H\mathbf{C}_{\tilde{v}_i}^{-1}\tilde{\mathbf{H}}_i\big]^{-1}, \qquad i = 1, 2. \tag{6}$$

III. Proposed Relay and Source Matrices

The joint source and relay optimization problem for two-way MIMO relay systems is written as

$$\min_{\mathbf{B}_1, \mathbf{B}_2, \mathbf{F}} \quad \mathrm{tr}(\mathbf{E}_1 + \mathbf{E}_2) \tag{7}$$

$$\text{s.t.} \quad \mathrm{tr}\left(\mathbf{F}\left(\sum_{i=1}^{2}\mathbf{H}_{r,i}\mathbf{B}_i\mathbf{B}_i^H\mathbf{H}_{r,i}^H + \mathbf{I}_M\right)\mathbf{F}^H\right) \le P_r \tag{8}$$

$$\mathrm{tr}\big(\mathbf{B}_i\mathbf{B}_i^H\big) \le P_i, \qquad i = 1, 2 \tag{9}$$

where (8) and (9) are the transmission power constraints at the relay node and two source nodes, respectively, and P_r, P_1, P_2 are the corresponding power budget available at each node. The problem (7)-(9) is nonconvex and a globally optimal solution of $\mathbf{F}$, $\mathbf{B}_1$, $\mathbf{B}_2$ is difficult to obtain with a reasonable computational complexity (non-exhaustive searching). In this paper, we develop an iterative algorithm to optimize (7).

For any feasible $\mathbf{B}_1$ and $\mathbf{B}_2$ satisfying (9), the relay amplifying matrix optimization problem is given by

$$\min_{\mathbf{F}} \quad \mathrm{tr}(\mathbf{E}_1 + \mathbf{E}_2) \tag{10}$$

$$\text{s.t.} \quad \mathrm{tr}\left(\mathbf{F}\left(\sum_{i=1}^{2}\mathbf{H}_{r,i}\mathbf{B}_i\mathbf{B}_i^H\mathbf{H}_{r,i}^H + \mathbf{I}_M\right)\mathbf{F}^H\right) \le P_r. \tag{11}$$

First we consider the scenario where $M \ge 2N$. The case of $M < 2N$ will be discussed later. Let us introduce the following singular value decompositions (SVDs)

$$\mathbf{H}_1 \triangleq [\mathbf{H}_{r,2}\mathbf{B}_2, \; \mathbf{H}_{r,1}\mathbf{B}_1] = \mathbf{U}_1\mathbf{\Sigma}_1\mathbf{V}_1^H \tag{12}$$

$$\mathbf{H}_2 \triangleq \big[\mathbf{H}_{1,r}^T, \; \mathbf{H}_{2,r}^T\big]^T = \mathbf{U}_2\mathbf{\Sigma}_2\mathbf{V}_2^H \tag{13}$$

where $(\cdot)^T$ denotes matrix (vector) transpose, the dimensions of $\mathbf{U}_1$, $\mathbf{\Sigma}_1$, $\mathbf{V}_1$ are $M \times 2N$, $2N \times 2N$, $2N \times 2N$, respectively, and the dimensions of $\mathbf{U}_2$, $\mathbf{\Sigma}_2$, $\mathbf{V}_2$ are $2N \times 2N$, $2N \times 2N$, $M \times 2N$, respectively. It has been proven in [4] that when $M \ge 2N$, the optimal $\mathbf{F}$ has the structure of

$$\mathbf{F} = \mathbf{V}_2\mathbf{A}\mathbf{U}_1^H. \tag{14}$$

Using (14), the optimization problem (10)-(11) becomes

$$\min_{\mathbf{A}} \sum_{i=1}^{2}\mathrm{tr}\Big(\Big[\mathbf{I}_N + \mathbf{V}_{1,i}\mathbf{\Sigma}_1\mathbf{A}^H\mathbf{\Sigma}_2\mathbf{U}_{2,i}^H \times \big(\mathbf{U}_{2,i}\mathbf{\Sigma}_2\mathbf{A}\mathbf{A}^H\mathbf{\Sigma}_2\mathbf{U}_{2,i}^H + \mathbf{I}_N\big)^{-1}\mathbf{U}_{2,i}\mathbf{\Sigma}_2\mathbf{A}\mathbf{\Sigma}_1\mathbf{V}_{1,i}^H\Big]^{-1}\Big) \tag{15}$$

$$\text{s.t.} \; \mathrm{tr}\big(\mathbf{A}(\mathbf{\Sigma}_1^2 + \mathbf{I}_{2N})\mathbf{A}^H\big) \le P_r \tag{16}$$

where $\mathbf{U}_2 = \big[\mathbf{U}_{2,1}^T, \; \mathbf{U}_{2,2}^T\big]^T$, $\mathbf{V}_1^H = \big[\mathbf{V}_{1,1}^H, \mathbf{V}_{1,2}^H\big]$, and the dimensions of $\mathbf{U}_{2,i}$ and $\mathbf{V}_{1,i}$, $i = 1, 2$, are all $N \times 2N$. In general, the problem (15)-(16) is nonconvex and a globally optimal solution is difficult to obtain with a reasonable computational complexity (non-exhaustive searching). We can resort to numerical methods, such as the projected gradient algorithm [11] to find (at least) a locally optimal solution of (15)-(16).

For systems with $M \ge 2N$, since the dimension of $\mathbf{A}$ is smaller than $\mathbf{F}$, solving the problem (15)-(16) has a smaller computational complexity than solving the problem (10)-(11). For relay systems with $M < 2N$, we directly solve the problem (10)-(11) using the projected gradient algorithm to obtain (at least) a locally optimal solution of $\mathbf{F}$.

A. Simplified Relay Matrix Design

In this section, we focus on relay systems with $M \ge 2N$ and develop a relay amplifying matrix design algorithm which is suboptimal for general cases, but has a much lower computational complexity than directly solving the problem (15)-(16). Let us introduce

$$\mathbf{\Sigma}_2\mathbf{A} = \big[\mathbf{U}_{2,1}^H, \; \mathbf{U}_{2,2}^H\big]\begin{bmatrix}\mathbf{C}_1 \\ \mathbf{C}_2\end{bmatrix} = \mathbf{U}_{2,1}^H\mathbf{C}_1 + \mathbf{U}_{2,2}^H\mathbf{C}_2 \tag{17}$$

where $\mathbf{C}_1$ and $\mathbf{C}_2$ are $N \times 2N$ matrices. Note that (17) does not lose any generality since $\big[\mathbf{U}_{2,1}^H, \; \mathbf{U}_{2,2}^H\big] = \mathbf{U}_2^H$ is a unitary matrix. Substituting (17) back into (15), we obtain that $\mathbf{U}_{2,i}\mathbf{\Sigma}_2\mathbf{A} = \mathbf{C}_i$, $i = 1, 2$, and thus

$$\mathbf{E}_i = \Big[\mathbf{I}_N + \mathbf{V}_{1,i}\mathbf{\Sigma}_1\mathbf{C}_i^H\big(\mathbf{C}_i\mathbf{C}_i^H + \mathbf{I}_N\big)^{-1}\mathbf{C}_i\mathbf{\Sigma}_1\mathbf{V}_{1,i}^H\Big]^{-1}, \quad i = 1, 2. \tag{18}$$

Interestingly, it can be seen from (18) that $\mathbf{E}_i$ is only a function of $\mathbf{C}_i$, $i = 1, 2$. In other words, the optimization variables are decoupled for $\mathbf{E}_1$ and $\mathbf{E}_2$.

Let us introduce the SVDs of

$$\mathbf{\Sigma}_1\mathbf{V}_{1,i}^H = \mathbf{P}_i\mathbf{\Pi}_i\mathbf{R}_i^H, \qquad i = 1, 2 \tag{19}$$

where $\mathbf{\Pi}_i$, $\mathbf{R}_i$, $i = 1, 2$, are $N \times N$ matrices, and $\mathbf{P}_i$, $i = 1, 2$, are $2N \times N$ matrices. Based on (18) and (19), we chose $\mathbf{C}_i$ to have the SVD of

$$\mathbf{C}_i = \mathbf{U}_{c_i}\mathbf{\Delta}_i\mathbf{P}_i^H, \qquad i = 1, 2 \tag{20}$$

where $\mathbf{\Delta}_i$ is an $N \times N$ diagonal eigenvalue matrix and $\mathbf{U}_{c_i}$ is an $N \times N$ unitary matrix which is irrelevant to (18) and will be determined in the constraint function (16) as explained later. The reason of using (20) is that for any $\mathbf{B}_1$ and $\mathbf{B}_2$ in (12), one can always have $\bar{\mathbf{B}}_1 = \mathbf{B}_1\mathbf{R}_2$ and $\bar{\mathbf{B}}_2 = \mathbf{B}_2\mathbf{R}_1$ such that the objective function (15) with $\bar{\mathbf{B}}_1$ and $\bar{\mathbf{B}}_2$ (i.e., $\mathbf{E}_i$ becomes $\mathbf{R}_i^H\mathbf{E}_i\mathbf{R}_i$, $i = 1, 2$) is equal to

$$\sum_{i=1}^{2}\operatorname{tr}\Big(\Big[\mathbf{I}_N + \mathbf{\Pi}_i\mathbf{P}_i^H\mathbf{C}_i^H\big(\mathbf{C}_i\mathbf{C}_i^H + \mathbf{I}_N\big)^{-1}\mathbf{C}_i\mathbf{P}_i\mathbf{\Pi}_i\Big]^{-1}\Big). \tag{21}$$

It can be shown similar to [12] that the optimal $\mathbf{C}_i$ for (21) are given by (20). In this case, $\mathbf{E}_1$ and $\mathbf{E}_2$ in (21) are diagonalized by $\mathbf{C}_1$ and $\mathbf{C}_2$ respectively as $\mathbf{E}_i = \big[\mathbf{I}_N + \mathbf{\Pi}_i\mathbf{\Delta}_i\big(\mathbf{\Delta}_i^2 + \mathbf{I}_N\big)^{-1}\mathbf{\Delta}_i\mathbf{\Pi}_i\big]^{-1}$, $i = 1, 2$, and the objective function (15) can be written as

$$\sum_{i=1}^{2}\operatorname{tr}\Big(\big[\mathbf{I}_N + \mathbf{\Pi}_i\mathbf{\Delta}_i\big(\mathbf{\Delta}_i^2 + \mathbf{I}_N\big)^{-1}\mathbf{\Delta}_i\mathbf{\Pi}_i\big]^{-1}\Big). \tag{22}$$

Now we consider the power constraint (16). From (17), $\mathbf{A}$ is given by

$$\mathbf{A} = \mathbf{\Sigma}_2^{-1}\mathbf{U}_2^H\big[\mathbf{C}_1^T,\ \mathbf{C}_2^T\big]^T. \tag{23}$$

Substituting (20) into (23), which is then substituted back into (16), the transmission power consumed by the relay node can be written as

$$\begin{aligned}&\operatorname{tr}\big(\mathbf{A}(\mathbf{\Sigma}_1^2 + \mathbf{I}_{2N})\mathbf{A}^H\big) = \\ &\quad\operatorname{tr}\big(\mathbf{\Sigma}_2^{-1}\mathbf{U}_2^H\operatorname{bd}(\mathbf{U}_{c_1}\mathbf{\Delta}_1, \mathbf{U}_{c_2}\mathbf{\Delta}_2)\,\mathbf{\Phi} \\ &\quad\times\operatorname{bd}(\mathbf{\Delta}_1\mathbf{U}_{c_1}^H, \mathbf{\Delta}_2\mathbf{U}_{c_2}^H)\,\mathbf{U}_2\mathbf{\Sigma}_2^{-1}\big)\end{aligned} \tag{24}$$

where $\operatorname{bd}(\cdot)$ stands for a block diagonal matrix, and $\mathbf{\Phi} \triangleq \big[\mathbf{P}_1,\ \mathbf{P}_2\big]^H(\mathbf{\Sigma}_1^2 + \mathbf{I}_{2N})\big[\mathbf{P}_1,\ \mathbf{P}_2\big]$. From (22) and (24), the relay amplifying matrix optimization problem (15)-(16) is converted to the following problem

$$\min_{\mathbf{\Delta}_1,\mathbf{\Delta}_2,\mathbf{U}_{c_1},\mathbf{U}_{c_2}} \sum_{i=1}^{2}\operatorname{tr}\Big(\big[\mathbf{I}_N + \mathbf{\Pi}_i^2\mathbf{\Delta}_i^2\big(\mathbf{\Delta}_i^2 + \mathbf{I}_N\big)^{-1}\big]^{-1}\Big) \tag{25}$$

$$\begin{aligned}\text{s.t.}\ &\operatorname{tr}\big(\mathbf{\Sigma}_2^{-1}\mathbf{U}_2^H\operatorname{bd}(\mathbf{U}_{c_1}\mathbf{\Delta}_1, \mathbf{U}_{c_2}\mathbf{\Delta}_2)\,\mathbf{\Phi} \\ &\times\operatorname{bd}(\mathbf{\Delta}_1\mathbf{U}_{c_1}^H, \mathbf{\Delta}_2\mathbf{U}_{c_2}^H)\,\mathbf{U}_2\mathbf{\Sigma}_2^{-1}\big) \le P_r\end{aligned} \tag{26}$$

$$\mathbf{U}_{c_i}^H\mathbf{U}_{c_i} = \mathbf{I}_N, \qquad i = 1, 2 \tag{27}$$

$$\delta_{i,n} \ge 0, \quad i = 1, 2, \quad n = 1, \cdots, N \tag{28}$$

where $\delta_{i,n}$, $i = 1, \cdots, N$, denotes the nth main diagonal element of $\mathbf{\Delta}_i$, $i = 1, 2$.

Note that although the structure of $\mathbf{C}_i$ in (20) is optimal for the objective function (15), we can not prove the optimality of (20) for the constraint function (16). This is the reason that this relay amplifying matrix design is suboptimal for general cases. However, compared with the problem (15)-(16), the dimension of optimization variables in the problem (25)-(28) has reduced from $8N^2$ real numbers to $4N^2 + 2N$ real numbers, which is significant especially when N is large. It will be shown in Section IV that the suboptimal design by solving (25)-(28) has only a marginal increase of MSE compared with the optimal algorithm through solving (15)-(16). Such performance-complexity tradeoff is very important for practical two-way MIMO relay systems.

The problem (25)-(28) is nonconvex due to the unitary matrix constraints in (27). Before we develop a numerical method to solve this problem, let us have some insights into the structure of this suboptimal relay amplifying matrix. Interestingly, we will show that for two special cases, the suboptimal relay matrix is indeed optimal. By substituting (20) into (23), which is then substituted back into (14), we obtain

$$\mathbf{F} = \mathbf{V}_2\mathbf{\Sigma}_2^{-1}\mathbf{U}_2^H\begin{bmatrix}\mathbf{U}_{c_1}\mathbf{\Delta}_1 & \mathbf{0}\\ \mathbf{0} & \mathbf{U}_{c_2}\mathbf{\Delta}_2\end{bmatrix}\begin{bmatrix}\mathbf{P}_1^H\\ \mathbf{P}_2^H\end{bmatrix}\mathbf{U}_1^H. \tag{29}$$

We can also show from (19) that

$$\begin{bmatrix}\mathbf{P}_1^H\\ \mathbf{P}_2^H\end{bmatrix}\mathbf{U}_1^H = \begin{bmatrix}\mathbf{\Pi}_1^{-1}\mathbf{R}_1^H & \mathbf{0}\\ \mathbf{0} & \mathbf{\Pi}_2^{-1}\mathbf{R}_2^H\end{bmatrix}\begin{bmatrix}\mathbf{V}_{1,1}\\ \mathbf{V}_{1,2}\end{bmatrix}\mathbf{\Sigma}_1\mathbf{U}_1^H. \tag{30}$$

Finally, by substituting (30) back into (29), we can equivalently rewrite the relay amplifying matrix as

$$\begin{aligned}\mathbf{F} &= \mathbf{V}_2\mathbf{\Sigma}_2^{-1}\mathbf{U}_2^H \\ &\quad\times\begin{bmatrix}\mathbf{U}_{c_1}\mathbf{\Delta}_1\mathbf{\Pi}_1^{-1}\mathbf{R}_1^H & \mathbf{0}\\ \mathbf{0} & \mathbf{U}_{c_2}\mathbf{\Delta}_2\mathbf{\Pi}_2^{-1}\mathbf{R}_2^H\end{bmatrix}\begin{bmatrix}\mathbf{V}_{1,1}\\ \mathbf{V}_{1,2}\end{bmatrix}\mathbf{\Sigma}_1\mathbf{U}_1^H \\ &\triangleq \mathbf{F}_3\mathbf{F}_2\mathbf{F}_1\end{aligned} \tag{31}$$

where

$$\begin{aligned}\mathbf{F}_3 &= \mathbf{V}_2\mathbf{\Sigma}_2^{-1}\mathbf{U}_2^H\\ \mathbf{F}_2 &= \begin{bmatrix}\mathbf{U}_{c_1}\mathbf{\Delta}_1\mathbf{\Pi}_2^{-1}\mathbf{R}_2^H & \mathbf{0}\\ \mathbf{0} & \mathbf{U}_{c_2}\mathbf{\Delta}_2\mathbf{\Pi}_1^{-1}\mathbf{R}_1^H\end{bmatrix}\\ \mathbf{F}_1 &= \begin{bmatrix}\mathbf{V}_{1,1}\\ \mathbf{V}_{1,2}\end{bmatrix}\mathbf{\Sigma}_1\mathbf{U}_1^H.\end{aligned} \tag{32}$$

Interestingly, it can be seen from (31) and (32) that the relay amplifying matrix is composed of three linear filters. First, we know from (12) that $\mathbf{F}_1 = \mathbf{H}_1^H$, and hence $\mathbf{F}_1$ is a matched-filter for the equivalent first-hop MIMO channel $\mathbf{H}_1$. Then the signals are linearly filtered by $\mathbf{F}_2$. Finally, we can see from (13) that $\mathbf{F}_3 = \mathbf{H}_2^{\dagger}$, where $(\cdot)^{\dagger}$ denotes matrix pseudo inverse. Thus, $\mathbf{F}_3$ performs zero-forcing of the equivalent second-hop

MIMO channel $\mathbf{H}_2$. We also find from (31) and (32) that for the case of $N=1$ (i.e. both source nodes have only one antenna), the optimal relay amplifying matrix has the structure of

$$\mathbf{F}=\mathbf{H}_2^{\dagger}\begin{bmatrix} a_f & 0 \\ 0 & b_f \end{bmatrix}\mathbf{H}_1^H. \tag{33}$$

THEOREM 1: For two-way relay systems with $N=1$, the structure of $\mathbf{F}$ given by (33) is optimal for the cases of[1] (1) $\mathbf{h}_{r,1}\perp\mathbf{h}_{r,2}$, $\mathbf{h}_{1,r}\perp\mathbf{h}_{2,r}$; (2) $\mathbf{h}_{r,1}\parallel\mathbf{h}_{r,2}$, $\mathbf{h}_{1,r}\perp\mathbf{h}_{2,r}$.

PROOF: See journal version of this paper [13]. □

It has been shown in [6] that for two-way relay systems with reciprocal first and second hop channels (i.e. $\mathbf{h}_{r,1}=\mathbf{h}_{1,r}^T=\mathbf{h}_1$, $\mathbf{h}_{r,2}=\mathbf{h}_{2,r}^T=\mathbf{h}_2$), both $\mathbf{F}=\mathbf{H}_2^{\dagger}\begin{bmatrix} a_{\rm ZF} & 0 \\ 0 & b_{\rm ZF} \end{bmatrix}\mathbf{H}_1^{\dagger}$ and $\mathbf{F}=\mathbf{H}_2^H\begin{bmatrix} a_{\rm MF} & 0 \\ 0 & b_{\rm MF} \end{bmatrix}\mathbf{H}_1^H$ are optimal when $\mathbf{h}_1\perp\mathbf{h}_2$, and the latter $\mathbf{F}$ is also optimal when $\mathbf{h}_1\parallel\mathbf{h}_2$. Interestingly, Theorem 1 extends the result in [6] to two-way relay systems without any channel reciprocity and shows that (33) is optimal for the two special cases given above.

Now we show how to solve the problem (25)-(28) numerically using the projected gradient algorithm. Since $\mathbf{U}_{c_1}$ and $\mathbf{U}_{c_2}$ only appear in the constraint functions, we can optimize $\mathbf{U}_{c_i}$ and $\mathbf{\Delta}_i$ in an alternating fashion. In each iteration, we first optimize $\mathbf{\Delta}_1$ and $\mathbf{\Delta}_2$ by solving a problem consisting of (25), (26), (28) with fixed $\mathbf{U}_{c_1}$ and $\mathbf{U}_{c_2}$. This problem can be equivalently rewritten as

$$\min_{\boldsymbol{\delta}} \quad \sum_{i=1}^{2}\sum_{n=1}^{N}\frac{\delta_{i,n}^2+1}{(\pi_{i,n}^2+1)\delta_{i,n}^2+1} \tag{34}$$

$$\text{s.t.} \quad \boldsymbol{\delta}^T\left[(\mathbf{L}_1\odot\mathbf{L}_2)^H(\mathbf{L}_1\odot\mathbf{L}_2)\right]\boldsymbol{\delta}\le P_r \tag{35}$$

$$\delta_{i,n}\ge 0, \qquad i=1,2, \quad n=1,\cdots,N \tag{36}$$

where $\boldsymbol{\delta}\triangleq[\delta_{1,1},\cdots,\delta_{1,N},\delta_{2,1},\cdots,\delta_{2,N}]^T$, $\odot$ denotes matrix Khatri-Rao product, $\pi_{i,n}$, $n=1,\cdots,N$, is the nth main diagonal element of $\mathbf{\Pi}_i$, $i=1,2$, $\mathbf{L}_1\triangleq\left[\mathrm{bd}(\mathbf{U}_{c_1}^H,\mathbf{U}_{c_2}^H)\,\mathbf{U}_2\mathbf{\Sigma}_2^{-1}\right]^T$, and $\mathbf{L}_2\triangleq(\mathbf{\Sigma}_1^2+\mathbf{I}_{2N})^{\frac{1}{2}}\left[\mathbf{P}_1,\ \mathbf{P}_2\right]$. The subproblem (34)-(36) can be solved by the projected gradient algorithm [11].

With fixed $\mathbf{\Delta}_1$ and $\mathbf{\Delta}_2$, we update $\mathbf{U}_{c_1}$ and $\mathbf{U}_{c_2}$ by solving the following problem

$$\min_{\mathbf{U}_{c_1},\mathbf{U}_{c_2}} \quad \mathrm{tr}\big(\mathbf{N}_1^H\mathrm{bd}(\mathbf{U}_{c_1},\mathbf{U}_{c_2})\mathbf{N}_2\mathrm{bd}(\mathbf{U}_{c_1}^H,\mathbf{U}_{c_2}^H)\mathbf{N}_1\big) \tag{37}$$

$$\text{s.t.} \quad \mathbf{U}_{c_i}^H\mathbf{U}_{c_i}=\mathbf{I}_N, \qquad i=1,2 \tag{38}$$

where $\mathbf{N}_1\triangleq\mathbf{U}_2\mathbf{\Sigma}_2^{-1}$, $\mathbf{N}_2\triangleq\mathrm{bd}(\mathbf{\Delta}_1,\mathbf{\Delta}_2)\,\mathbf{\Phi}\,\mathrm{bd}(\mathbf{\Delta}_1,\mathbf{\Delta}_2)$, and the objective function (37) is obtained by rewriting the left-hand side of (26). The subproblem (37)-(38) can also be solved by the projected gradient algorithm. The gradient of (37) with respect to $\mathbf{U}_{c_i}$ can be calculated using the results on derivatives of matrices in [14]. The projection of an $N\times N$ matrix $\tilde{\mathbf{U}}_{c_i}$ onto the feasible set of $\bar{\mathbf{U}}_{c_i}$ given by (38) is performed by solving the following problem for $i=1,2$

$$\min_{\bar{\mathbf{U}}_{c_i}} \quad \mathrm{tr}\big((\bar{\mathbf{U}}_{c_i}-\tilde{\mathbf{U}}_{c_i})(\bar{\mathbf{U}}_{c_i}-\tilde{\mathbf{U}}_{c_i})^H\big) \tag{39}$$

$$\text{s.t.} \quad \bar{\mathbf{U}}_{c_i}^H\bar{\mathbf{U}}_{c_i}=\mathbf{I}_N. \tag{40}$$

Let $\tilde{\mathbf{U}}_{c_i}=\mathbf{\Upsilon}_i\mathbf{\Gamma}_i\mathbf{\Theta}_i^H$ be the SVD of $\tilde{\mathbf{U}}_{c_i}$. It can be easily shown using the Lagrange multiplier method that the solution to the problem (39)-(40) is given by $\bar{\mathbf{U}}_{c_i}=\mathbf{\Upsilon}_i\mathbf{\Theta}_i^H$. The procedure of solving the problem (25)-(28) using the alternating projected gradient algorithm is summarized in Table I. Here $\max\mathrm{abs}(\cdot)$ denotes the maximum among the absolute value of all elements in a matrix, and ε is a positive constant close to 0.

TABLE I
PROCEDURE OF APPLYING THE ALTERNATING PROJECTED GRADIENT ALGORITHM TO SOLVE THE PROBLEM (25)-(28).

1) Initialize the algorithm at a feasible $\mathbf{U}_{c_1}^{(0)}$, $\mathbf{U}_{c_2}^{(0)}$, and $\boldsymbol{\delta}^{(0)}$; Set $n=0$.
2) With given $\mathbf{U}_{c_1}^{(n)}$ and $\mathbf{U}_{c_2}^{(n)}$, obtain $\boldsymbol{\delta}^{(n+1)}$ by solving the subproblem (34)-(36) using the projected gradient algorithm;
   Obtain $\mathbf{U}_{c_1}^{(n+1)}$ and $\mathbf{U}_{c_2}^{(n+1)}$ through solving the subproblem (37)-(38) with known $\boldsymbol{\delta}^{(n+1)}$ using the projected gradient algorithm.
3) If $\max\mathrm{abs}\big(\boldsymbol{\delta}^{(n+1)}-\boldsymbol{\delta}^{(n)}\big)\le\varepsilon$, then end.
   Otherwise, let $n:=n+1$ and go to step 2).

B. Optimal Source Precoding Matrices

With fixed $\mathbf{F}$, the problem of updating $\mathbf{B}_1$ and $\mathbf{B}_2$ can be written as

$$\min_{\mathbf{B}_1,\mathbf{B}_2} \quad \sum_{i=1}^{2}\mathrm{tr}\big([\mathbf{I}_N+\mathbf{B}_i^H\mathbf{\Psi}_i\mathbf{B}_i]^{-1}\big) \tag{41}$$

$$\text{s.t.} \quad \mathrm{tr}\left(\sum_{i=1}^{2}\mathbf{F}\mathbf{H}_{r,i}\mathbf{B}_i\mathbf{B}_i^H\mathbf{H}_{r,i}^H\mathbf{F}^H\right)\le\breve{P}_r \tag{42}$$

$$\mathrm{tr}\big(\mathbf{B}_i\mathbf{B}_i^H\big)\le P_i, \qquad i=1,2 \tag{43}$$

where $\breve{P}_r\triangleq P_r-\mathrm{tr}(\mathbf{F}\mathbf{F}^H)$. Let us introduce $\mathbf{\Omega}_i\triangleq\mathbf{B}_i\mathbf{B}_i^H$, $i=1,2$, and positive semi-definite (PSD) matrices $\mathbf{X}_i$ with $\mathbf{X}_i\succeq\big(\mathbf{I}_N+\mathbf{\Psi}_i^{\frac{1}{2}}\mathbf{\Omega}_i\mathbf{\Psi}_i^{\frac{1}{2}}\big)^{-1}$, $i=1,2$. Here $\mathbf{A}\succeq\mathbf{B}$ means that $\mathbf{B}-\mathbf{A}$ is a PSD matrix. By using the Schur complement [15], the problem (41)-(43) can be equivalently converted to the following problem

$$\min_{\mathbf{X}_1,\mathbf{X}_2,\mathbf{\Omega}_1,\mathbf{\Omega}_2} \quad \mathrm{tr}(\mathbf{X}_1+\mathbf{X}_2) \tag{44}$$

$$\text{s.t.} \quad \begin{pmatrix} \mathbf{X}_i & \mathbf{I}_N \\ \mathbf{I}_N & \mathbf{I}_N+\mathbf{\Psi}_i^{\frac{1}{2}}\mathbf{\Omega}_i\mathbf{\Psi}_i^{\frac{1}{2}} \end{pmatrix}\succeq 0, \quad i=1,2 \tag{45}$$

$$\mathrm{tr}\left(\sum_{i=1}^{2}\mathbf{F}\mathbf{H}_{r,i}\mathbf{\Omega}_i\mathbf{H}_{r,i}^H\mathbf{F}^H\right)\le\breve{P}_r \tag{46}$$

$$\mathrm{tr}\big(\mathbf{\Omega}_i\big)\le P_i, \qquad \mathbf{\Omega}_i\succeq 0, \qquad i=1,2. \tag{47}$$

The problem (44)-(47) is a convex SDP problem which can be efficiently solved by the interior-point method [15].

Now the original joint source and relay optimization problem (7)-(9) can be solved by an iterative algorithm. This algorithm is first initialized at random feasible $\mathbf{B}_1$ and $\mathbf{B}_2$ satisfying (9). At each iteration, when $M\ge 2N$, $\mathbf{F}$ is updated

[1]For the consistency of notations, here we use vector notations for channels due to $N=1$.

according to (31) by solving the problem (25)-(28) following the steps in Table I. When $M < 2N$, $\mathbf{F}$ is updated by solving the problem (10)-(11) with fixed $\mathbf{B}_1$ and $\mathbf{B}_2$. Then $\mathbf{B}_1$ and $\mathbf{B}_2$ are updated by solving the SDP problem (44)-(47). Note that the conditional updates of each matrix may either decrease or maintain but cannot increase the objective function (7). Monotonic convergence of $\mathbf{F}$, $\mathbf{B}_1$, and $\mathbf{B}_2$ towards (at least) a locally optimal solution follows directly from this observation. The procedure of this iterative algorithm is summarized in Table II.

TABLE II
PROCEDURE OF SOLVING THE PROBLEM (7)-(9).

1) Initialize the algorithm at a feasible $\mathbf{B}_1^{(0)}$ and $\mathbf{B}_2^{(0)}$; Set $n = 0$.
2) For fixed $\mathbf{B}_1^{(n)}$ and $\mathbf{B}_2^{(n)}$, obtain $\mathbf{F}^{(n+1)}$ by solving the problem (25)-(28) using the steps in Table I;
   Update $\mathbf{B}_1^{(n+1)}$ and $\mathbf{B}_2^{(n+1)}$ by solving the problem (44)-(47) with known $\mathbf{F}^{(n+1)}$.
3) If $\max \mathrm{abs}\big(\mathbf{F}^{(n+1)} - \mathbf{F}^{(n)}\big) \le \varepsilon$, then end.
   Otherwise, let $n := n + 1$ and go to step 2).

IV. Numerical Examples

In this section, we study the performance of the proposed algorithms for two-way MIMO relay systems. All channel matrices have complex Gaussian entries with zero-mean and variances of $1/N$, $1/M$ for $\mathbf{H}_{r,i}$ and $\mathbf{H}_{i,r}$, $i = 1, 2$, respectively, and all simulation results are averaged over 1000 independent channel realizations. In the simulations, we set $P_1 = P_2 = P_s = 20$dB above the noise level and vary the value of P_r.

In the first example, we check the performance of the proposed relay amplifying matrix (31) and the algorithm in Table I by testing it for the case of $N = 1$. It is proven in Theorem 1 that (31) (or equivalently (33)) is optimal for this case, and we only need to find the optimal a_f and b_f in (33). By substituting (33) back into (10)-(11), we have the following problem to solve

$$\min_{a_f, b_f} \quad \frac{1 + c_1|a_f|^2}{1 + c_1 d_2 |a_f|^2} + \frac{1 + c_2|b_f|^2}{1 + c_2 d_1 |b_f|^2} \tag{48}$$

$$\text{s.t.} \quad d_2|a_f|^2 + d_1|b_f|^2 \le P_r \tag{49}$$

where $c_i \triangleq \|\mathbf{h}_{i,r}\|^2$ and $d_i \triangleq 1 + P_i\|\mathbf{h}_{r,i}\|^2$, $i = 1, 2$. The problem (48)-(49) has a water-filling solution given by

$$|a_f|^2 = \frac{1}{c_1 d_2}\left[\sqrt{\frac{c_1}{\kappa}\Big(1 - \frac{1}{d_2}\Big)} - 1\right]^+ \tag{50}$$

$$|b_f|^2 = \frac{1}{c_2 d_1}\left[\sqrt{\frac{c_2}{\kappa}\Big(1 - \frac{1}{d_1}\Big)} - 1\right]^+ \tag{51}$$

where $[x]^+ \triangleq \max(x, 0)$, and $\kappa > 0$ is the solution of the nonlinear equation by substituting (50) and (51) back into (49), which can be efficiently solved by the bisection method [11].

Fig. 1 shows the normalized SMSE of the relay amplifying matrix designed by the alternating projected gradient algorithm in Table I and that of the optimal solution given by (33), (50), (51). It can be seen that for both $M = 4$ and $M = 6$, two algorithms have identical SMSE performance. This demonstrates that the algorithm in Table I achieves the global optimum for $N = 1$, and verifies the effectiveness of the projected gradient algorithm. We also observe from Fig. 1 that as expected, the SMSE decreases with increasing M.

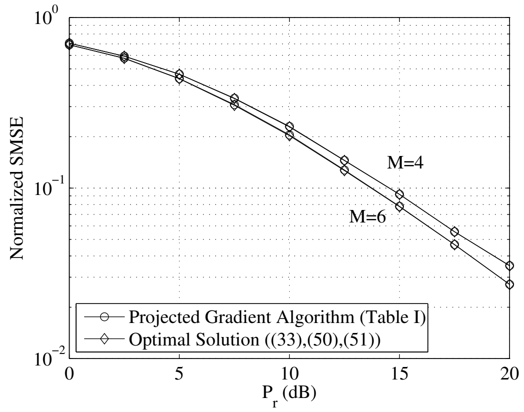

Fig. 1. Example 1: Normalized SMSE versus P_r. $N = 1$.

In the second example, we simulate a two-way MIMO relay system with $N = 2$, $M = 8$, and compare the normalized SMSE of the optimal relay matrix in [4] and the suboptimal relay design in (31) from the procedure listed in Table I. In order to study the "pure" effect of relay matrix design, we set $\mathbf{B}_1 = \mathbf{B}_2 = \sqrt{P_s/2}\,\mathbf{I}_2$ for both algorithms. It can be seen from Fig. 2 that the proposed relay amplifying matrix yields only a slightly higher MSE than the optimal relay matrix. Since the proposed relay matrix design has a substantially reduced computational complexity (20 real-valued optimization variables) than the optimal design (32 real-valued optimization variables), it is very useful in practical relay systems.

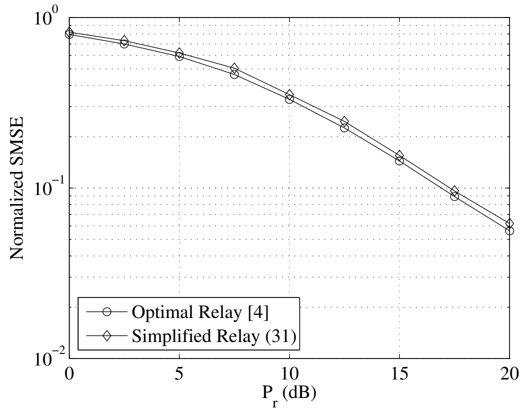

Fig. 2. Example 2: Normalized SMSE versus P_r. $N = 2$, $M = 8$.

In our third example, We set $N = 2$, $M = 4$ and investigate the performance of the joint source and relay optimization algorithm in Table II at different iterations. We observed in simulations that for most channel realizations, the algorithm converges within 10 iterations. The normalized SMSE of this algorithm after the first, second, and fifth iteration versus P_r is listed in Table III. It can be seen that the difference between iterations is very small. Thus, in practice, only a small number of iterations are required to achieve a good performance.

TABLE III
NORMALIZED SMSE OF JOINT SOURCE AND RELAY OPTIMIZATION ALGORITHM (TABLE II) AT DIFFERENT ITERATIONS.

P_r (dB)	0	5	10	15	20
NSMSE (It. 1)	0.8233	0.6248	0.3641	0.1723	0.0695
NSMSE (It. 2)	0.8232	0.6247	0.3640	0.1712	0.0689
NSMSE (It. 5)	0.8228	0.6241	0.3638	0.1698	0.0685

In our fourth example, we study the performance of two-way MIMO relay systems based on the MSMI objective [2] and the MSMSE objective, respectively. We chose $N = 2$, $M = 6$, and for both objectives, we use the procedure listed in Table II. Fig. 3 shows the SMI of both systems versus P_r. It can be seen from Fig. 3 that as expected, the MSMI-based relay design leads to a larger MI than the relay design using the MSMSE criterion. The uncoded BER of both systems versus P_r is demonstrated in Fig. 4, where the QPSK constellations are used. It can be seen from Fig. 4 that the relay system designed under the MSMSE criterion outperforms the MSMI-based system in terms of BER. In fact, the former algorithm achieves a higher diversity order than the latter one. This is because MSMI is a good criterion only for coded systems in which the number of symbols for each coding block is very large. However, in practical communication systems, due to the delay constraint, codewords always have a finite length. Thus, MSMSE is a better criterion for practical two-way MIMO relay systems.

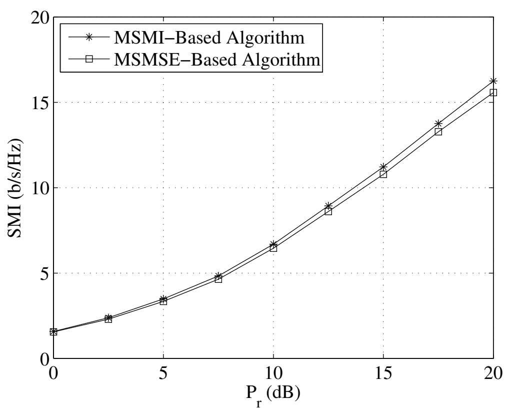

Fig. 3. Example 4: SMI versus P_r. $N = 2$, $M = 6$.

V. CONCLUSIONS

We have proposed a simplified relay amplifying matrix design for a two-way linear non-regenerative MIMO relay system. The proposed algorithm significantly reduces the computational complexity of the optimal design with only a marginal performance degradation. An iterative algorithm is developed to jointly optimize the relay and source matrices.

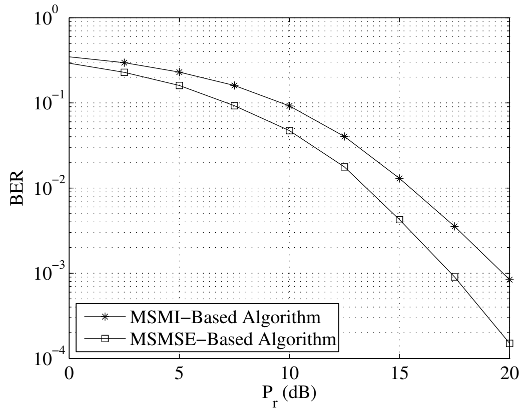

Fig. 4. Example 4: BER versus P_r. $N = 2$, $M = 6$.

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