Transceiver Optimization for AF MIMO Relay Systems with Wireless Powered Relay Nodes

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***Abstract*—In this paper, we study a two-hop amplify-and-forward (AF) multiple-input multiple-output (MIMO) relay system, where the relay node has no self-power supply, and relies on harvesting the radio frequency energy transferred from the source node to forward information from source to destination. We apply the time switching (TS) protocol between wireless information and energy transfer. As a novel contribution of this paper, we propose a more general energy consumption constraint at the source node during the information and energy transfer, which includes the constant power constraints used in existing works as special cases. We study the joint optimization of the source precoding matrices, the relay amplifying matrix, and the TS factor to maximize the source-destination mutual information (MI). The optimal structure of the source and relay matrices is derived, which reduces the original transceiver optimization problem to a simpler power allocation problem. Numerical simulations show that the proposed algorithm yield higher system MI and better rate-energy tradeoff than an existing approach.**

I. Introduction

Recently, wireless power transfer techniques received increasing interests from both academia and industry due to the current proliferation of low power sensors and electronic devices [1]. An ideal receiver capable of performing information decoding (ID) and energy harvesting (EH) simultaneously has been proposed in [2]. The trade-off between the achievable information rate and the harvested energy is also characterized by a capacity-energy function in [2]. To coordinate wireless information transfer and wireless energy transfer in practical systems, a time switching (TS) protocol and a power splitting (PS) protocol have been proposed in [3].

It is well-known that both multiple-input multiple-output (MIMO) and relay communication techniques can improve the system coverage and energy efficiency [4]-[6]. The application of EH in MIMO relay systems has been studied in [7]-[11]. In [7], performance trade-offs of several receiver architectures have been discussed by applying EH in MIMO relay systems. Future research challenges in this area have also been outlined in [7]. Precoder design for decode-and-forward (DF)-based MIMO relay networks has been studied in [8] and [9].

A TS protocol and a PS protocol have been developed in [10] for an amplify-and-forward (AF) MIMO relay system, where the achievable rate is maximized for each protocol by jointly optimizing the source and relay precoding matrices. In [11], an orthogonal space-time block code (OSTBC) based AF-MIMO relay system with a multi-antenna EH receiver has been investigated.

In this paper, we consider a two-hop AF MIMO relay system with a wireless powered relay node. The TS protocol is adopted during the source phase, where the source node transfers energy and information signals to the relay node during the first and second time intervals, respectively. Then, during the relay phase, the relay node uses the harvested energy to forward the received information to the destination node. As a novel contribution of this paper, we propose an energy consumption constraint at the source node during the information and energy transfer, which is more general than the constant power constraints in [10].

We study the joint optimization of the source precoding matrices, the relay amplifying matrix, and the TS factor to maximize the source-destination mutual information (MI), subjecting to the harvested energy constraint at the relay node and the proposed source energy constraint at the source node. The optimal structure of the source and relay matrices is derived, which reduces the original problem to a simpler power allocation problem. Based on the observation that the system MI is a unimodal function of the TS factor, we develop a two-step method to efficiently solve the power allocation problem.

In particular, we show that the optimal TS factor can be efficiently found by a golden section search. Whereas for a given TS factor, we show that the remaining variables can be optimized through solving a power allocation problem in a two-hop AF MIMO relay system with sum power constraint across the source and relay nodes. We propose an upper bound based algorithm to solve the power allocation problem which has a closed-form solution. Numerical simulations show that the proposed algorithm yield much higher system MI and better rate-energy tradeoff than the approach in [10].

II. System Model

We consider a three-node two-hop MIMO communication system where the source node transmits information to the destination node with the aid of one relay node as shown in

This work is supported by the Australian Research Council's Discovery Projects funding scheme (DP140102131).

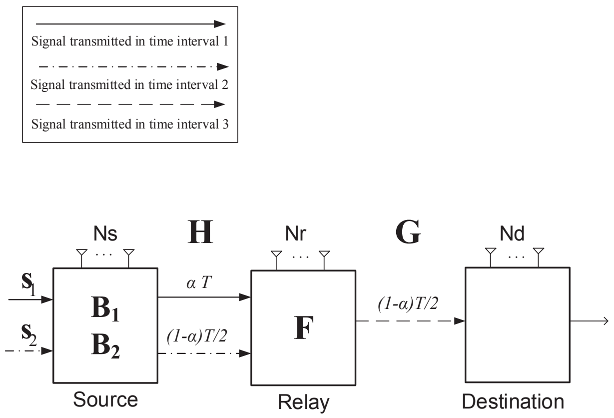

Fig. 1. A two-hop MIMO relay communication system with a wireless powered relay node.

Fig. 1. The source, relay, and destination nodes are equipped with N_s, N_r, and N_d antennas, respectively. We assume that the source node has its own power supply, while the relay node is powered by harvesting the radio energy sent from the source node. In particular, there are two phases in one communication cycle. In the source phase, energy-carrying and information-bearing signals are transmitted from the source node to the relay node. Then, in the relay phase, the information signals received at the relay node are linearly precoded and transmitted to the destination node [10]. Among various relaying protocols, the AF scheme is chosen at the relay node due to its implementation simplicity and shorter processing delay. Moreover, compared with the DF relay protocol [8], [9], the AF scheme does not need to decode and re-encode the signals at the relay node, which is energy-saving, making it suitable for wireless powered communication.

We adopt the time switching protocol [3] for the energy harvesting and information transmission at the source phase. In this protocol, the total time T of one communication cycle is divided into three intervals. In the first time interval, energy is transferred from the source node to the relay node with a duration of αT, where $0 < \alpha < 1$ denotes the time switching factor. In the second time interval, information signals are transmitted from the source node to the relay node with a duration of $(1-\alpha)T/2$. The last time interval of $(1-\alpha)T/2$ is used for relaying the information signals from the relay node to the destination node. For the simplicity of presentation, we set $T = 1$ hereafter.

During the first interval, the $N_1 \times 1$ energy-carrying signal vector $\mathbf{s}_1$ is precoded by an $N_s \times N_1$ matrix $\mathbf{B}_1$ at the source node and transmitted to the relay node. The optimal value of N_1 will be determined later. We assume that $E\{\mathbf{s}_1\mathbf{s}_1^H\} = \mathbf{I}_{N_1}$, where $E\{\cdot\}$ stands for the statistical expectation, $\mathbf{I}_n$ is an $n \times n$ identity matrix, and $(\cdot)^H$ denotes the Hermitian transpose. The received signal vector at the relay node is given by

$$\mathbf{y}_{r,1} = \mathbf{H}\mathbf{B}_1\mathbf{s}_1 + \mathbf{v}_{r,1} \tag{1}$$

where $\mathbf{H}$ is an $N_r \times N_s$ MIMO channel matrix between the source and relay nodes, $\mathbf{y}_{r,1}$ and $\mathbf{v}_{r,1}$ are the received signal and the additive Gaussian noise vectors at the relay node during the first interval, respectively. Based on [3], the energy harvested at the relay node is proportional to the baseband received signal in (1) without the noise component, and is given by

$$E_r = \eta\alpha\mathrm{tr}(\mathbf{H}\mathbf{B}_1\mathbf{B}_1^H\mathbf{H}^H) \tag{2}$$

where $\mathrm{tr}(\cdot)$ denotes the matrix trace and $0 < \eta \leq 1$ is the energy conversion efficiency.

During the second interval, an $N_2 \times 1$ information-bearing signal vector $\mathbf{s}_2$ with $E\{\mathbf{s}_2\mathbf{s}_2^H\} = \mathbf{I}_{N_2}$ is precoded by an $N_s \times N_2$ matrix $\mathbf{B}_2$ at the source node and transmitted to the relay node. The received signal vector at the relay node can be written as

$$\mathbf{y}_{r,2} = \mathbf{H}\mathbf{B}_2\mathbf{s}_2 + \mathbf{v}_{r,2} \tag{3}$$

where $\mathbf{v}_{r,2}$ is the additive white Gaussian noise (AWGN) vector at the relay node during the second interval with zero-mean and $E\{\mathbf{v}_{r,2}\mathbf{v}_{r,2}^H\} = \sigma_r^2\mathbf{I}_{N_r}$.

Finally, during the third internal, the relay node linearly precodes $\mathbf{y}_{r,2}$ with an $N_r \times N_r$ matrix $\mathbf{F}$ and transmits the precoded signal vector

$$\mathbf{x}_r = \mathbf{F}\mathbf{y}_{r,2} \tag{4}$$

to the destination node. From (3) and (4), the received signal vector at the destination node can be written as

$$\begin{aligned}\mathbf{y}_d &= \mathbf{G}\mathbf{x}_r + \mathbf{v}_d \\ &= \mathbf{G}\mathbf{F}\mathbf{H}\mathbf{B}_2\mathbf{s}_2 + \mathbf{G}\mathbf{F}\mathbf{v}_{r,2} + \mathbf{v}_d\end{aligned} \tag{5}$$

where $\mathbf{G}$ is an $N_d \times N_r$ MIMO channel matrix between the relay and destination nodes, $\mathbf{y}_d$ and $\mathbf{v}_d$ are the received signal and the AWGN vectors at the destination node, respectively, with $E\{\mathbf{v}_d\mathbf{v}_d^H\} = \sigma_d^2\mathbf{I}_{N_d}$. From (5), the mutual information between source and destination is given as [12]

$$\begin{aligned}\mathrm{MI}(\alpha, \mathbf{B}_2, \mathbf{F}) = \frac{1-\alpha}{2}\log|&\mathbf{I}_{N_2} + \mathbf{B}_2^H\mathbf{H}^H\mathbf{F}^H\mathbf{G}^H \\ &\times(\sigma_r^2\mathbf{G}\mathbf{F}\mathbf{F}^H\mathbf{G}^H + \sigma_d^2\mathbf{I}_{N_d})^{-1}\mathbf{G}\mathbf{F}\mathbf{H}\mathbf{B}_2|\end{aligned} \tag{6}$$

where $|\cdot|$ and $(\cdot)^{-1}$ denote the matrix determinant and matrix inversion, respectively.

We assume that $\mathbf{H}$ and $\mathbf{G}$ are quasi-static and known at the relay node. We also assume that without wasting the transmission power at the source and relay nodes, $N_2 \leq \min(\mathrm{rank}(\mathbf{H}), \mathrm{rank}(\mathbf{G}))$ and $\mathrm{rank}(\mathbf{F}) = \mathrm{rank}(\mathbf{B}_2) = N_2$, where $\mathrm{rank}(\cdot)$ stands for the rank of a matrix.

Note that the energy used to transmit $\mathbf{s}_1$ and $\mathbf{s}_2$ from the source node is $\alpha\mathrm{tr}(\mathbf{B}_1\mathbf{B}_1^H)$ and $\frac{1-\alpha}{2}\mathrm{tr}(\mathbf{B}_2\mathbf{B}_2^H)$, respectively. Therefore, the constraint on the energy consumed by the source node can be written as

$$\alpha\mathrm{tr}(\mathbf{B}_1\mathbf{B}_1^H) + \frac{1-\alpha}{2}\mathrm{tr}(\mathbf{B}_2\mathbf{B}_2^H) \leq \frac{1+\alpha}{2}P_s \tag{7}$$

where P_s is the nominal (average) power available at the source node and $(1+\alpha)/2$ is the total time of the first two intervals that the source node is transmitting. It is worth noting that in [10], a constant power is assumed at the source node for both energy transferring and information transmission as

$$\mathrm{tr}(\mathbf{B}_1\mathbf{B}_1^H) \le P_s, \qquad \mathrm{tr}(\mathbf{B}_2\mathbf{B}_2^H) \le P_s. \tag{8}$$

It can be seen that under the same α, both (7) and (8) lead to the same amount of energy consumption at the source node. However, (8) is a special case of (7) and the feasible region defined by (7) is larger than that of (8). In fact, in (7) the source precoding matrices $\mathbf{B}_1$ and $\mathbf{B}_2$ are linked through one energy constraint. This enables the source node to operate at different power levels adapted to the purpose of energy transferring at the first interval and information transmission at the second interval, which is more flexible than (8). Hence, transceivers designed under (7) are expected to have a better performance than that with (8) as in [10].

From (3) and (4), the energy consumed by the relay node to transmit $\mathbf{x}_r$ to the destination node is given by

$$\mathrm{tr}(E\{\mathbf{x}_r\mathbf{x}_r^H\}) = \frac{1-\alpha}{2}\mathrm{tr}(\mathbf{F}(\mathbf{H}\mathbf{B}_2\mathbf{B}_2^H\mathbf{H}^H + \sigma_r^2\mathbf{I}_{N_r})\mathbf{F}^H). \tag{9}$$

Based on (2) and (9), we obtain the following energy constraint at the relay node

$$\frac{1-\alpha}{2}\mathrm{tr}(\mathbf{F}(\mathbf{H}\mathbf{B}_2\mathbf{B}_2^H\mathbf{H}^H + \sigma_r^2\mathbf{I}_{N_r})\mathbf{F}^H) \le \alpha\eta\,\mathrm{tr}(\mathbf{H}\mathbf{B}_1\mathbf{B}_1^H\mathbf{H}^H). \tag{10}$$

From (6), (7), (10), the transceiver optimization problem for linear AF wireless information and energy transfer MIMO relay systems can be written as

$$\max_{0<\alpha<1,\mathbf{B}_1,\mathbf{B}_2,\mathbf{F}} \quad \mathrm{MI}(\alpha, \mathbf{B}_2, \mathbf{F}) \tag{11}$$

$$\text{s.t.} \;\; \alpha\mathrm{tr}(\mathbf{B}_1\mathbf{B}_1^H) + \frac{1-\alpha}{2}\mathrm{tr}(\mathbf{B}_2\mathbf{B}_2^H) \le \frac{1+\alpha}{2}P_s \tag{12}$$

$$\mathrm{tr}(\mathbf{F}(\mathbf{H}\mathbf{B}_2\mathbf{B}_2^H\mathbf{H}^H + \sigma_r^2\mathbf{I}_{N_r})\mathbf{F}^H) \le \frac{2\alpha\eta}{1-\alpha}\mathrm{tr}(\mathbf{H}\mathbf{B}_1\mathbf{B}_1^H\mathbf{H}^H) \tag{13}$$

As will be shown in the next section, the energy consumption constraint greatly increases the technical difficulty of solving the problem (11)-(13) compared with the constant power constraint in [10].

III. Proposed Algorithm

The problem (11)-(13) is non-convex with matrix variables and is challenging to solve. In the following, we derive the optimal structure of $\mathbf{B}_1$, $\mathbf{B}_2$, and $\mathbf{F}$, under which the problem (11)-(13) can be simplified to a power allocation problem. Let us introduce

$$\mathbf{H} = \mathbf{U}_h\mathbf{\Lambda}_h^{\frac{1}{2}}\mathbf{V}_h^H, \quad \mathbf{G} = \mathbf{U}_g\mathbf{\Lambda}_g^{\frac{1}{2}}\mathbf{V}_g^H \tag{14}$$

as the singular value decompositions (SVDs) of $\mathbf{H}$ and $\mathbf{G}$, respectively, with the diagonal elements of $\mathbf{\Lambda}_h$ and $\mathbf{\Lambda}_g$ sorted in decreasing order.

Theorem 1: The optimal $\mathbf{B}_1$, $\mathbf{B}_2$, and $\mathbf{F}$ as the solution to the problem (11)-(13) has the following structure

$$\mathbf{B}_1^* = \lambda_b^{\frac{1}{2}}\mathbf{v}_{h,1}, \quad \mathbf{B}_2^* = \mathbf{V}_{h,1}\mathbf{\Lambda}_2^{\frac{1}{2}}, \quad \mathbf{F}^* = \mathbf{V}_{g,1}\mathbf{\Lambda}_f^{\frac{1}{2}}\mathbf{U}_{h,1}^H \tag{15}$$

where λ_b is a positive scalar, $\mathbf{v}_{h,1}$ is the first column of $\mathbf{V}_h$, $\mathbf{\Lambda}_2$ and $\mathbf{\Lambda}_f$ are $N_2 \times N_2$ diagonal matrices, $\mathbf{V}_{g,1}$, $\mathbf{U}_{h,1}$, and $\mathbf{V}_{h,1}$ contain the leftmost N_2 columns from $\mathbf{V}_g$, $\mathbf{U}_h$, and $\mathbf{V}_h$, respectively.

It is interesting to see from (15) that the optimal $\mathbf{B}_1$ is a vector matching $\mathbf{v}_{h,1}$. This indicates that in order to maximize the energy harvested by the relay node, all transmission power at the source node should be allocated to the channel corresponding to the largest singular value of $\mathbf{H}$ during the first interval. As a result, we only need to optimize λ_b in $\mathbf{B}_1$, and the transmission power of the source during the first interval is $\mathrm{tr}(\mathbf{B}_1\mathbf{B}_1^H) = \lambda_b$. It can also be seen from (15) that the optimal structure of $\mathbf{B}_2$ and $\mathbf{F}$ is similar to that in two-hop MIMO relay systems where the relay node has self-power supply [12].

By substituting (15) back into (11)-(13), the transceiver optimization problem (11)-(13) with matrix variables is simplified to the following power allocation problem with scalar variables

$$\max_{\alpha,\lambda_b,\boldsymbol{\lambda}_2,\boldsymbol{\lambda}_f} \quad \frac{1-\alpha}{2}\sum_{i=1}^{N_2}\log\left(1 + \frac{\lambda_{2,i}\lambda_{h,i}\lambda_{f,i}\lambda_{g,i}}{1+\lambda_{f,i}\lambda_{g,i}}\right) \tag{16}$$

$$\text{s.t.} \;\; \alpha\lambda_b + \frac{1-\alpha}{2}\sum_{i=1}^{N_2}\lambda_{2,i} \le \frac{1+\alpha}{2}P_s \tag{17}$$

$$\sum_{i=1}^{N_2}\lambda_{f,i}(\lambda_{h,i}\lambda_{2,i}+1) \le \frac{2\alpha\eta}{1-\alpha}\lambda_{h,1}\lambda_b \tag{18}$$

$$0<\alpha<1, \; \lambda_{f,i} \ge 0, \; \lambda_{2,i} \ge 0, \; i = 1, \cdots, N_2 \tag{19}$$

where $\boldsymbol{\lambda}_2 = [\lambda_{2,1}, \cdots, \lambda_{2,N_2}]^T$, $\boldsymbol{\lambda}_f = [\lambda_{f,1}, \cdots, \lambda_{f,N_2}]^T$, $\lambda_{h,i} = \tilde{\lambda}_{h,i}/\sigma_r^2$, $\lambda_{g,i} = \tilde{\lambda}_{g,i}/\sigma_d^2$, $\lambda_{f,i} = \tilde{\lambda}_{f,i}\sigma_r^2$, $\lambda_{2,i}$, $\tilde{\lambda}_{f,i}$, $\tilde{\lambda}_{h,i}$, $\tilde{\lambda}_{g,i}$ denote the ith diagonal element of $\mathbf{\Lambda}_2$, $\mathbf{\Lambda}_f$, $\mathbf{\Lambda}_h$ and $\mathbf{\Lambda}_g$, respectively. By introducing $z_i = \lambda_{f,i}(\lambda_{h,i}\lambda_{2,i}+1)$, $i = 1, \cdots, N_2$, the problem (16)-(19) becomes

$$\max_{\alpha,\lambda_b,\boldsymbol{\lambda}_2,\mathbf{z}} \quad \frac{1-\alpha}{2}\sum_{i=1}^{N_2}\log\left(1 + \frac{\lambda_{2,i}\lambda_{h,i}z_i\lambda_{g,i}}{1+\lambda_{2,i}\lambda_{h,i}+z_i\lambda_{g,i}}\right) \tag{20}$$

$$\text{s.t.} \;\; \alpha\lambda_b + \frac{1-\alpha}{2}\sum_{i=1}^{N_2}\lambda_{2,i} \le \frac{1+\alpha}{2}P_s \tag{21}$$

$$\sum_{i=1}^{N_2} z_i \le \frac{2\alpha\eta}{1-\alpha}\lambda_{h,1}\lambda_b \tag{22}$$

$$0<\alpha<1, \; \lambda_{2,i} \ge 0, \; z_i \ge 0, \; i = 1, \cdots, N_2 \tag{23}$$

where $\mathbf{z} = [z_1, \cdots, z_{N_2}]^T$.

As for any λ_b, the optimal $\mathbf{z}$ maximizing (20) must satisfy equality in (22), i.e.,

$$\sum_{i=1}^{N_2} z_i = \frac{2\alpha\eta}{1-\alpha}\lambda_{h,1}\lambda_b. \tag{24}$$

Using (24), the problem (20)-(23) can be equivalently rewritten as

$$\max_{\alpha,\boldsymbol{\lambda}_2,\mathbf{z}} \frac{1-\alpha}{2}\sum_{i=1}^{N_2}\log\left(1+\frac{\lambda_{2,i}\lambda_{h,i}z_i\lambda_{g,i}}{1+\lambda_{2,i}\lambda_{h,i}+z_i\lambda_{g,i}}\right) \quad (25)$$

$$\text{s.t. } \frac{1-\alpha}{2\eta\lambda_{h,1}}\sum_{i=1}^{N_2} z_i+\frac{1-\alpha}{2}\sum_{i=1}^{N_2}\lambda_{2,i}\le\frac{1+\alpha}{2}P_s \quad (26)$$

$$0<\alpha<1, \quad \lambda_{2,i}\ge 0, \quad z_i\ge 0,\ i=1,\cdots,N_2. \quad (27)$$

By introducing $a_i=\lambda_{h,i}$, $b_i=\eta\lambda_{g,i}\lambda_{h,1}$, $x_i=\lambda_{2,i}$, $y_i=z_i/(\eta\lambda_{h,1})$, $i=1,\cdots,N_2$, the problem (25)-(27) becomes

$$\min_{\alpha,\mathbf{x},\mathbf{y}} \frac{1-\alpha}{2}\sum_{i=1}^{N_2}\log\frac{1+a_ix_i+b_iy_i}{(1+a_ix_i)(1+b_iy_i)} \quad (28)$$

$$\text{s.t. } \sum_{i=1}^{N_2}x_i+\sum_{i=1}^{N_2}y_i\le P_s\frac{1+\alpha}{1-\alpha} \quad (29)$$

$$0<\alpha<1, \quad x_i\ge 0, \quad y_i\ge 0, \quad i=1,\cdots,N_2 \quad (30)$$

where $\mathbf{x}=[x_1,\cdots,x_{N_2}]^T$ and $\mathbf{y}=[y_1,\cdots,y_{N_2}]^T$. Interestingly, although the problem (28)-(30) is still a non-convex optimization problem, it has a nice symmetric structure with respect to $\mathbf{x}$ and $\mathbf{y}$ in both the objective function (28) and the constraint (29).

Let $M(\alpha)$ be the optimal value of the problem (28)-(30) with a given α written as

$$\min_{\mathbf{x},\mathbf{y}} \sum_{i=1}^{N_2}\log\frac{1+a_ix_i+b_iy_i}{(1+a_ix_i)(1+b_iy_i)} \quad (31)$$

$$\text{s.t. } \sum_{i=1}^{N_2}x_i+\sum_{i=1}^{N_2}y_i\le P_\alpha \quad (32)$$

$$x_i\ge 0, \quad y_i\ge 0, \quad i=1,\cdots,N_2 \quad (33)$$

where

$$P_\alpha=P_s\frac{1+\alpha}{1-\alpha}. \quad (34)$$

Then the objective function (28) can be written as $F(\alpha)=\frac{1-\alpha}{2}M(\alpha)$. From (34), we know that P_α monotonically increases with α. Thus, it can be seen from (32) that the feasible region of the problem (31)-(33) expands as α increases. Therefore, $M(\alpha)$ monotonically decreases as α increases. On the other hand, $1-\alpha$ monotonically decreases as α increases. Considering the effects of α on $M(\alpha)$ and $1-\alpha$, and the fact that $M(\alpha)<0$, we can expect that $F(\alpha)$ is a unimodal function of α. However, we note that the unimodality of $F(\alpha)$ is difficult to prove rigorously for $N_2>1$ since α changes the value of $M(\alpha)$ through varying the feasible region of the optimization problem (31)-(33), and the objective function (31) is a complicated function. As a result, the closed-form expression of $M(\alpha)$ is difficult to obtain. Based on this observation, the problem (28)-(30) can be efficiently solved by a two-step algorithm, where for a given α we optimize $\mathbf{x}$ and $\mathbf{y}$ by solving the problem (31)-(33). And then a simple one dimensional search can be applied to obtain the optimal α.

Interestingly, the problem (31)-(33) can be viewed as a power allocation problem for a two-hop AF MIMO relay system with sum power constraint across the source and relay nodes. To the best of our knowledge, the globally optimal solution to such problem is not available in existing works. In the following, we propose an upper bound based algorithm to solve the power allocation problem which has a closed-form solution.

Let us introduce the upper bound of

$$\frac{1+a_ix_i+b_iy_i}{(1+a_ix_i)(1+b_iy_i)}<\frac{1+a_ix_i+b_iy_i+1}{(1+a_ix_i)(1+b_iy_i)} =\frac{1}{1+a_ix_i}+\frac{1}{1+b_iy_i}. \quad (35)$$

Using (35), the problem of optimizing $\mathbf{x}$ and $\mathbf{y}$ with a given α can be written as

$$\min_{\mathbf{x},\mathbf{y}} \sum_{i=1}^{N_2}\log\left(\frac{1}{1+a_ix_i}+\frac{1}{1+b_iy_i}\right) \quad (36)$$

$$\text{s.t. } \sum_{i=1}^{N_2}x_i+\sum_{i=1}^{N_2}y_i\le P_\alpha \quad (37)$$

$$x_i\ge 0, \quad y_i\ge 0, \quad i=1,\cdots,N_2. \quad (38)$$

The problem (36)-(38) is convex and can be efficiently solved by the Lagrange multiplier method. The Lagrangian function of (36)-(37) is

$$L(\mathbf{x},\mathbf{y},\nu)=\sum_{i=1}^{N_2}\log\left(\frac{1}{1+a_ix_i}+\frac{1}{1+b_iy_i}\right) +\nu\left(\sum_{i=1}^{N_2}x_i+\sum_{i=1}^{N_2}y_i-P_\alpha\right) \quad (39)$$

where $\nu\ge 0$ is the Lagrange multiplier.

Considering the KKT conditions, we have from (39) that for $i=1,\cdots,N_2$

$$\frac{\partial L}{\partial x_i}=-\left(\frac{1}{1+a_ix_i}+\frac{1}{1+b_iy_i}\right)^{-1}\frac{a_i}{(1+a_ix_i)^2}+\nu=0 \quad (40)$$

$$\frac{\partial L}{\partial y_i}=-\left(\frac{1}{1+a_ix_i}+\frac{1}{1+b_iy_i}\right)^{-1}\frac{b_i}{(1+b_iy_i)^2}+\nu=0. \quad (41)$$

From (40) and (41), we obtain the following equation

$$\frac{a_i}{(1+a_ix_i)^2}=\frac{b_i}{(1+b_iy_i)^2}, \quad i=1,\cdots,N_2. \quad (42)$$

By substituting (42) back into (40) and (41) and considering that $x_i\ge 0$ and $y_i\ge 0$, the optimal x_i^* and y_i^* can be obtained as

$$x_i^*=\left[\frac{\sqrt{b_i}}{\left(\sqrt{a_i}+\sqrt{b_i}\right)\nu}-\frac{1}{a_i}\right]^\dagger, \quad i=1,\cdots,N_2 \quad (43)$$

$$y_i^*=\left[\frac{\sqrt{a_i}}{\left(\sqrt{a_i}+\sqrt{b_i}\right)\nu}-\frac{1}{b_i}\right]^\dagger, \quad i=1,\cdots,N_2 \quad (44)$$

where for a real-valued number x, $[x]^\dagger=\max(x,0)$.

The Lagrange multiplier ν can be computed by solving the following nonlinear equation

$$\sum_{i=1}^{N_2}(x_i^* + y_i^*) = P_\alpha \qquad (45)$$

where x_i^* and y_i^* are given in (43) and (44), respectively. As the left hand side of (45) is monotonically decreasing with respect to ν, (45) can be efficiently solved by the bisection method [13]. Interestingly, when P_α is sufficiently large, for $i = 1, \cdots, N_2$, there is

$$x_i^* = \frac{\sqrt{b_i}}{\left(\sqrt{a_i}+\sqrt{b_i}\right)\nu} - \frac{1}{a_i}, \quad y_i^* = \frac{\sqrt{a_i}}{\left(\sqrt{a_i}+\sqrt{b_i}\right)\nu} - \frac{1}{b_i}. \qquad (46)$$

Substituting (46) back into (45), we obtain ν as

$$\nu = N_2/Q_\alpha \qquad (47)$$

where $Q_\alpha = P_\alpha + \sum_{i=1}^{N_2}(\frac{1}{a_i} + \frac{1}{b_i})$. Substituting (47) back into (46), we obtain for $i = 1, \cdots, N_2$

$$x_i = \frac{\sqrt{b_i}Q_\alpha}{(\sqrt{a_i}+\sqrt{b_i})N_2} - \frac{1}{a_i}, \quad y_i = \frac{\sqrt{a_i}Q_\alpha}{(\sqrt{a_i}+\sqrt{b_i})N_2} - \frac{1}{b_i}. \qquad (48)$$

IV. Simulations

In this section, we study the performance of the proposed algorithm through numerical simulations. We consider a scenario where the three nodes are located in a line as shown in Fig. 2. The distance between the source node and the destination node is $D_{sd} = 20$ meters, and the source-relay and relay-destination distances are $D_{sr} = 10d$ and $D_{rd} = 10(2-d)$, where the value of $0 < d < 2$ is normalized over a distance of 10 meters. Thus, the relay node is located closer to the source node if $0 < d < 1$, and closer to the destination node if $1 < d < 2$. The channel matrices $\mathbf{H}$ and $\mathbf{G}$ have independent and isotropically distributed (i.i.d.) complex Gaussian entries as $\mathcal{CN}(0, 1/(N_s D_{sr}^\zeta))$ and $\mathcal{CN}(0, 1/(N_r D_{rd}^\zeta))$, respectively, where $\zeta = 3$ is the path loss exponent. The noise power at the relay and the destination nodes is fixed as $\sigma_r^2 = \sigma_d^2 = -50$dBm. For all simulation examples, we fix $N_s = N_r = N_d = N_2 = N$. We compare the performance of the proposed algorithm with the algorithm in [10], which is denoted as the constant power based algorithm as the constraints (8) are employed in [10]. All the numerical simulation results are averaged over 1000 independent channel realizations.

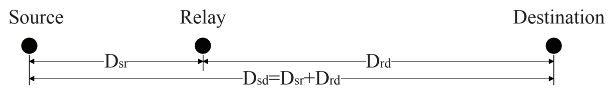

Fig. 2. Source, relay, and destination placement.

In the first example, we set $d = 1$. The MI of two algorithms versus the nominal power P_s is shown in Fig. 3 with $N = 3$ for $\eta = 0.8$ and $\eta = 0.5$, and with $N = 5$ for $\eta = 0.8$. We observe from Fig. 3 that for both $N = 3$ and $N = 5$, the proposed algorithm performs better than the constant power based algorithm in [10]. Moreover, the MI gap between the proposed algorithm and that of the constant power based algorithm increases with P_s. In fact, at $P_s = 20$dBm, the proposed algorithm achieves a 50% increase of the MI of the constant power based algorithm. As expected, the achievable rate increases with the number of antennas. It can also be seen from Fig. 3 that the system achieves a lower rate at $\eta = 0.5$ compared with that at $\eta = 0.8$. For the simplicity of presentation, we choose $\eta = 0.8$ in the following simulations.

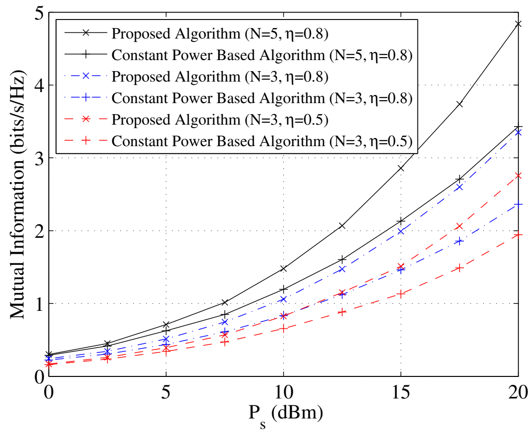

Fig. 3. Example 1: MI versus P_s, $d = 1$.

In the second example, we set $N = 3$ and $d = 1$. The time switching factors α calculated as optimal by two algorithms versus the nominal power P_s are shown in Fig. 4. It can be seen from Fig. 4 that for both algorithms, the optimal α monotonically decreases as the nominal power P_s increases. The reason is that as P_s increases, a smaller α is sufficient for the relay node to harvest the energy required for forwarding the information signals. Moreover, we observe from Fig. 4 that the variation of α in the proposed algorithm throughout the range of P_s is lager than that of the fixed power algorithm. The reason is that when P_s is large enough, λ_b (the power level at the source node at the first interval) obtained by the proposed algorithm increases. Thus, even though α is small, the energy $\alpha\lambda_{h,1}\lambda_b$ harvested by the relay node is sufficient for forwarding the signal to the destination node. Therefore, for the proposed algorithm, more time is allocated for information transmission so that a higher data rate can be achieved at large P_s.

In the third example, we investigate the energy consumption of the proposed algorithm. Fig. 5 shows the MI versus energy (measured in Joule per time T) of two algorithms for $N = 3$ and $d = 1$. It can be clearly seen from Fig. 5 that the energy efficiency of the proposed algorithm is much better than that of the constant power based algorithm. In fact, with energy of 0.05 J/T, the MI achieved by the proposed algorithm is 60% more than that of the constant power based algorithm.

In the last example, we study the impact of distance d on the achievable MI. Fig. 6 shows the MI of both algorithms versus d with $N = 3$ and $P_s = 15$dBm. It can be seen from

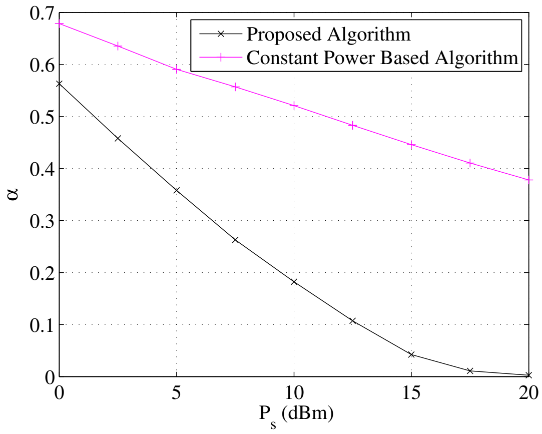

Fig. 4. Example 2: Optimal α versus P_s, $N = 3$, $d = 1$.

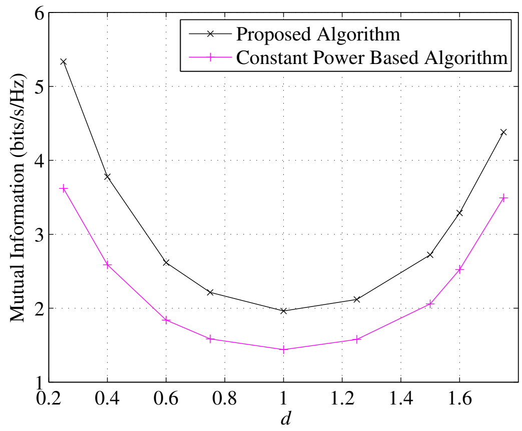

Fig. 6. Example 4: MI versus d, $P_s = 15$dBm, $N = 3$.

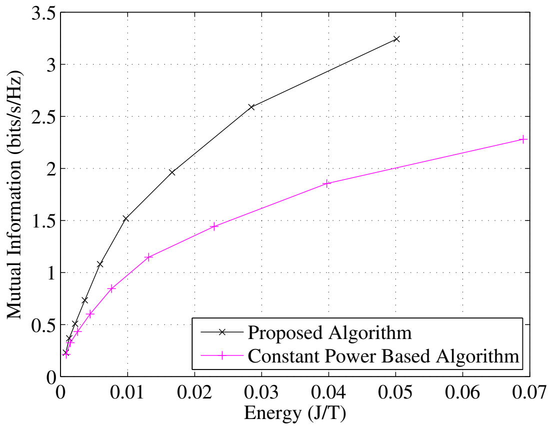

Fig. 5. Example 3: MI versus energy, $N = 3$, $d = 1$.

Fig. 6 that for both algorithms the achievable MI first decreases when d increases and then increases again with the growth of d. The reason is that when the relay node is closer to the source node, it can harvest more energy and consequently, the MI is higher. When the relay node is very close to the destination node, although the harvested energy is smaller, the shorter relay-destination distance and thus a better second-hop channel improves the system MI. Moreover, it can be observed from Fig. 6 that the proposed algorithms have an MI gain of 0.5-1 bits/s/Hz over the constant power based algorithm over the whole range of d.

V. Conclusions

We have investigated the transceiver optimization for TS-based wireless information and energy transfer in two-hop AF MIMO relay systems. Compared with the fixed power constraint at the source node used in existing works, a more general energy constraint at the source node has been proposed. The optimal structure of the source and relay precoding matrices has been derived, which reduces the original problem to a simpler power allocation problem. Based on the observation that the achievable source-destination MI is a unimodal function of the TS factor, a two-step method has been developed. Numerical simulations show that the proposed algorithm yields much higher system MI and better rate-energy tradeoff than the constant power based method.

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