Two-Way Amplify-and-Forward MIMO Relay Communications Using Linear MMSE Receiver

Yue Rong
Department of Electrical and Computer Engineering, Curtin University of Technology, Bentley, WA 6102, Australia

***Abstract*—In this paper, we derive the optimal structure of the source precoding matrices and the relay amplifying matrix for a two-way amplify-and-forward multiple-input multiple-output (MIMO) relay communication system where linear minimal mean-squared error (MMSE) receivers are used at both destinations. We show that for a broad class of frequently used objective functions for MIMO communications, the optimal relay and source matrices have a general beamforming structure. Based on this optimal structure, a new iterative algorithm is developed to optimize the relay and source matrices. The performance of the proposed algorithm is demonstrated by numerical simulations.**

I. Introduction

Relay communication is well known for being a cost-effective approach in improving the energy-efficiency of wireless communications systems [1]. When nodes in the relay network are equipped with multiple antennas, we have a multiple-input multiple-output (MIMO) relay system. For a one-way MIMO relay system (i.e., one source node sends information to one destination node), a unified framework was established in [2] to jointly optimize the source precoding matrix and the relay amplifying matrix for a broad class of objective functions.

In this paper, we study a two-way MIMO relay communication system where two source nodes exchange their information through an assisting relay node, and both source nodes and the relay node are equipped with multiple antennas. By using the idea of analog network coding [3], the information exchange is completed in two time slots. In the first time slot, both source nodes concurrently transmit signals to the relay node. In the second time slot, the relay node linearly amplifies the received signals and broadcasts the amplified signals to both source nodes. Since each node knows its own transmitted signals, the self-interference can be easily cancelled. Then the message from the other node can be decoded.

Distributed space-time coding has been designed in [4] for two-way relay communication with multiple single-antenna relays. For a two-way (and in general N-way) relay system with a multi-antenna relay node and single-antenna source nodes, the relay beamforming issue has been investigated in [5] and [6]. For two-way MIMO relay systems, the optimal relay and source matrices have been developed in [7] to maximize the two-way sum mutual information (SMI). However, the maximal SMI (MSMI)-based algorithm is optimal only when the codewords are infinitely long. However, in practical communication systems, due to the delay constraint, codewords always have a finite length. Thus, the performance of the MSMI-based algorithm will degrade in practical systems.

In this paper, we derive the optimal structure of the source precoding matrices and the relay amplifying matrix for a two-way MIMO relay communication channel where all nodes have multiple antennas and linear minimal mean-squared error (MMSE) receivers are used at both destinations. In contrast to [7], we consider a broad class of frequently used objective functions in MIMO system design (such as the MSE of the signal waveform estimation), and show that the optimal relay and source matrices have a general beamforming structure. This interesting outcome includes the results in [5] and [7] as special cases. Based on this optimal structure, an iterative algorithm is developed to optimize the relay and source matrices. To verify the performance of the proposed algorithm, numerical simulations are carried out using the minimal sum MSE (MSMSE) of the signal waveform estimation as the objective function. It is shown that the proposed MSMSE-based algorithm has a better bit-error-rate (BER) performance compared with the algorithm developed in [7] using the MSMI criterion.

II. System Model

We consider a three-node MIMO communication system where nodes 1 and 2 exchange information with the aid of one relay node. We assume that both nodes 1 and 2 are equipped with N antennas, the relay node has M antennas. The information exchange is completed in two time slots. In the first time slot, nodes 1 and 2 concurrently transmit, and the signal vector from node i is $\mathbf{x}_i = \mathbf{B}_i\mathbf{s}_i$, $i = 1, 2$, where $\mathbf{s}_i$ is the $N \times 1$ source signal vector, and $\mathbf{B}_i$ is the $N \times N$ source precoding matrix at node i. The signal vector $\mathbf{y}_r$ received at the relay node can be written as

$$\mathbf{y}_r = \mathbf{H}_{r,1}\mathbf{B}_1\mathbf{s}_1 + \mathbf{H}_{r,2}\mathbf{B}_2\mathbf{s}_2 + \mathbf{v}_r \tag{1}$$

where $\mathbf{H}_{r,i}$, $i = 1, 2$, is the $M \times N$ channel matrix between the relay node and node i, and $\mathbf{v}_r$ is the $M \times 1$ noise vector at the relay node.

In the second time slot, the relay node linearly amplifies $\mathbf{y}_r$ with an $M \times M$ matrix $\mathbf{F}$ and broadcasts the amplified signal vector $\mathbf{x}_r = \mathbf{F}\mathbf{y}_r$ to nodes 1 and 2. Using (1), the received signal vector at nodes 1 and 2 can be respectively written as

$$\begin{aligned}\tilde{\mathbf{y}}_1 &= \mathbf{H}_{1,r}\mathbf{F}\mathbf{H}_{r,2}\mathbf{B}_2\mathbf{s}_2 + \mathbf{H}_{1,r}\mathbf{F}\mathbf{H}_{r,1}\mathbf{B}_1\mathbf{s}_1 \\ &\quad + \mathbf{H}_{1,r}\mathbf{F}\mathbf{v}_r + \mathbf{v}_1 \end{aligned} \tag{2}$$

$$\begin{aligned}\tilde{\mathbf{y}}_2 &= \mathbf{H}_{2,r}\mathbf{F}\mathbf{H}_{r,1}\mathbf{B}_1\mathbf{s}_1 + \mathbf{H}_{2,r}\mathbf{F}\mathbf{H}_{r,2}\mathbf{B}_2\mathbf{s}_2 \\ &\quad + \mathbf{H}_{2,r}\mathbf{F}\mathbf{v}_r + \mathbf{v}_2 \end{aligned} \tag{3}$$

where $\mathbf{H}_{i,r}$, $i=1,2$, is the $N \times M$ channel matrix between node i and the relay, and $\mathbf{v}_i$, $i=1,2$, is the $N \times 1$ noise vector at node i.

We assume that the source signal vectors satisfy $\mathrm{E}[\mathbf{s}_i\mathbf{s}_i^H] = \mathbf{I}_N$, $i=1,2$, and all noises are independent and identically distributed (i.i.d.) additive white Gaussian noise (AWGN) with zero mean and unit variance. Here $\mathrm{E}[\cdot]$ stands for the statistical expectation, $\mathbf{I}_N$ is an $N \times N$ identity matrix, and $(\cdot)^H$ denotes matrix (vector) Hermitian transpose. We also assume that the relay node knows the channel state information (CSI) of $\mathbf{H}_{r,i}$ and $\mathbf{H}_{i,r}$, $i=1,2$. The relay node performs the optimization of $\mathbf{F}$, $\mathbf{B}_1$, $\mathbf{B}_2$, and then transmits the information of matrix $\mathbf{H}_{i,r}\mathbf{F}\mathbf{H}_{r,i}\mathbf{B}_i$ to node i, $i=1,2$. In this paper, we do not make any assumption on the statistical property of channel matrices (e.g. independency between $\mathbf{H}_{1,r}$ and $\mathbf{H}_{2,r}$).

Since node i knows its own transmitted signal vector $\mathbf{s}_i$ and the CSI of $\mathbf{H}_{i,r}\mathbf{F}\mathbf{H}_{r,i}\mathbf{B}_i$, the self-interference components in (2) and (3) can be easily cancelled. The effective received signal vector is given by

$$\begin{aligned}
\mathbf{y}_1 &= \mathbf{H}_{1,r}\mathbf{F}\mathbf{H}_{r,2}\mathbf{B}_2\mathbf{s}_2 + \mathbf{H}_{1,r}\mathbf{F}\mathbf{v}_r + \mathbf{v}_1 \\
&\triangleq \tilde{\mathbf{H}}_2\mathbf{s}_2 + \tilde{\mathbf{v}}_1 \qquad (4)\\
\mathbf{y}_2 &= \mathbf{H}_{2,r}\mathbf{F}\mathbf{H}_{r,1}\mathbf{B}_1\mathbf{s}_1 + \mathbf{H}_{2,r}\mathbf{F}\mathbf{v}_r + \mathbf{v}_2 \\
&\triangleq \tilde{\mathbf{H}}_1\mathbf{s}_1 + \tilde{\mathbf{v}}_2 \qquad (5)
\end{aligned}$$

where $\tilde{\mathbf{H}}_i$ is the equivalent MIMO channel passed through by $\mathbf{s}_i$, and $\tilde{\mathbf{v}}_i$ is the equivalent noise vector at node i with

$$\begin{aligned}
\tilde{\mathbf{H}}_1 &\triangleq \mathbf{H}_{2,r}\mathbf{F}\mathbf{H}_{r,1}\mathbf{B}_1 \qquad & \tilde{\mathbf{v}}_1 &\triangleq \mathbf{H}_{1,r}\mathbf{F}\mathbf{v}_r + \mathbf{v}_1\\
\tilde{\mathbf{H}}_2 &\triangleq \mathbf{H}_{1,r}\mathbf{F}\mathbf{H}_{r,2}\mathbf{B}_2 \qquad & \tilde{\mathbf{v}}_2 &\triangleq \mathbf{H}_{2,r}\mathbf{F}\mathbf{v}_r + \mathbf{v}_2\,.
\end{aligned}$$

Due to the following two reasons, linear MMSE receivers are used at nodes 1 and 2 to retrieve the transmitted signals sent from the other node. First, compared with other receivers, linear receiver has a lower computational complexity and is easy to implement. Second, a broad class of frequently used objective functions in MIMO system design are closely linked to the MMSE matrix as explained later. The estimated signal waveform vector is given by $\hat{\mathbf{s}}_1 = \mathbf{W}_1^H\mathbf{y}_2$ and $\hat{\mathbf{s}}_2 = \mathbf{W}_2^H\mathbf{y}_1$, where $\mathbf{W}_i$ is the MMSE weight matrix used to decode $\mathbf{s}_i$ given by

$$\mathbf{W}_1 = (\tilde{\mathbf{H}}_1\tilde{\mathbf{H}}_1^H + \mathbf{C}_{\tilde{v}_2})^{-1}\tilde{\mathbf{H}}_1 \qquad (6)$$

$$\mathbf{W}_2 = (\tilde{\mathbf{H}}_2\tilde{\mathbf{H}}_2^H + \mathbf{C}_{\tilde{v}_1})^{-1}\tilde{\mathbf{H}}_2 \qquad (7)$$

where $\mathbf{C}_{\tilde{v}_i} \triangleq \mathrm{E}[\tilde{\mathbf{v}}_i\tilde{\mathbf{v}}_i^H] = \mathbf{H}_{i,r}\mathbf{F}\mathbf{F}^H\mathbf{H}_{i,r}^H + \mathbf{I}_N$, $i=1,2$, is the equivalent noise covariance matrix at node i, and $(\cdot)^{-1}$ stands for matrix inversion. The weight matrices (6) and (7) are computed at the relay node and forwarded to the corresponding destination node after the optimal $\mathbf{F}$, $\mathbf{B}_1$, $\mathbf{B}_2$ are obtained. From (4)-(7), the MSE matrix of the signal waveform estimation at two nodes, denoted as $\mathbf{E}_i = \mathrm{E}[(\hat{\mathbf{s}}_i - \mathbf{s}_i)(\hat{\mathbf{s}}_i - \mathbf{s}_i)^H]$, $i=1,2$, can be written as

$$\mathbf{E}_1 = \left[\mathbf{I}_N + \tilde{\mathbf{H}}_1^H\mathbf{C}_{\tilde{v}_2}^{-1}\tilde{\mathbf{H}}_1\right]^{-1} \qquad (8)$$

$$\mathbf{E}_2 = \left[\mathbf{I}_N + \tilde{\mathbf{H}}_2^H\mathbf{C}_{\tilde{v}_1}^{-1}\tilde{\mathbf{H}}_2\right]^{-1}. \qquad (9)$$

III. Joint Source And Relay Beamforming

It has been shown in [2] and [8] that a broad class of frequently used objective functions in MIMO system design are closely linked to the MSE matrix. Let us introduce $q(\mathbf{E}_1, \mathbf{E}_2)$ as a unified notation for the objective function, where q can be decomposed to the superposition of the cost function of each communication direction, and the cost function (performance measure) can be the negative MI, the MSE, the weighted signal-to-interference-noise ratio, etc. [2]. For example, the negative SMI is used as the objective function in [7], which can be written as $q(\mathbf{E}_1, \mathbf{E}_2) = \log_2|\mathbf{E}_1| + \log_2|\mathbf{E}_2|$. Obviously, any practical objective function should be an increasing function of $\mathbf{E}_1$ and $\mathbf{E}_2$, i.e., if $\mathbf{E}_1 \preceq \tilde{\mathbf{E}}_1$ and/or $\mathbf{E}_2 \preceq \tilde{\mathbf{E}}_2$, then $q(\mathbf{E}_1, \mathbf{E}_2) \leq q(\tilde{\mathbf{E}}_1, \tilde{\mathbf{E}}_2)$, where $\mathbf{E}_i \preceq \tilde{\mathbf{E}}_i$ means that $\tilde{\mathbf{E}}_i - \mathbf{E}_i$ is a positive semidefinite matrix. The joint source and relay optimization problem for two-way MIMO relay systems is written as

$$\min_{\mathbf{B}_1,\mathbf{B}_2,\mathbf{F}} \quad q(\mathbf{E}_1, \mathbf{E}_2) \qquad (10)$$

$$\text{s.t.} \quad \mathrm{tr}\left(\mathbf{F}\left(\sum_{i=1}^{2}\mathbf{H}_{r,i}\mathbf{B}_i\mathbf{B}_i^H\mathbf{H}_{r,i}^H + \mathbf{I}_M\right)\mathbf{F}^H\right) \leq P_r \qquad (11)$$

$$\mathrm{tr}(\mathbf{B}_i\mathbf{B}_i^H) \leq P_i, \qquad i=1,2 \qquad (12)$$

where (11) and (12) are the transmission power constraints at the relay node and nodes 1 and 2, respectively, and P_r, P_1, P_2 are the corresponding power budget available. The problem (10)-(12) is nonconvex and a globally optimal solution of $\mathbf{F}$, $\mathbf{B}_1$, $\mathbf{B}_2$ is difficult to obtain with a reasonable computational complexity. In the following, we develop an iterative algorithm to optimize (10).

For any given $\mathbf{B}_1$ and $\mathbf{B}_2$ satisfying (12), the relay matrix optimization problem is given by

$$\min_{\mathbf{F}} \quad q(\mathbf{E}_1, \mathbf{E}_2) \qquad (13)$$

$$\text{s.t.} \quad \mathrm{tr}\left(\mathbf{F}\left(\sum_{i=1}^{2}\mathbf{H}_{r,i}\mathbf{B}_i\mathbf{B}_i^H\mathbf{H}_{r,i}^H + \mathbf{I}_M\right)\mathbf{F}^H\right) \leq P_r. \qquad (14)$$

First we consider the scenario where $M \geq 2N$. The case of $M < 2N$ will be discussed later. Let us introduce the following singular value decompositions (SVDs)

$$\mathbf{H}_1 \triangleq [\mathbf{H}_{r,1}\mathbf{B}_1,\ \mathbf{H}_{r,2}\mathbf{B}_2] = \mathbf{U}_1\mathbf{\Sigma}_1\mathbf{V}_1^H \qquad (15)$$

$$\mathbf{H}_2 \triangleq \left[\mathbf{H}_{1,r}^T,\ \mathbf{H}_{2,r}^T\right]^T = \mathbf{U}_2\mathbf{\Sigma}_2\mathbf{V}_2^H \qquad (16)$$

where $(\cdot)^T$ denotes matrix (vector) transpose, the dimensions of $\mathbf{U}_1$, $\mathbf{\Sigma}_1$, $\mathbf{V}_1$ are $M \times 2N$, $2N \times 2N$, $2N \times 2N$, respectively, and the dimensions of $\mathbf{U}_2$, $\mathbf{\Sigma}_2$, $\mathbf{V}_2$ are $2N \times 2N$, $2N \times 2N$, $M \times 2N$, respectively. The following theorem establishes the optimal structure of $\mathbf{F}$ when $M \geq 2N$.

Theorem 1: Using the SVDs (15) and (16) and a $2N \times 2N$ matrix $\mathbf{A}$, the optimal $\mathbf{F}$ as the solution to the problem (13)-(14) is given by

$$\mathbf{F} = \mathbf{V}_2\mathbf{A}\mathbf{U}_1^H. \qquad (17)$$

PROOF: Based on (15) and (16) we have

$$\mathbf{H}_{i,r} = \mathbf{U}_{2,i}\mathbf{\Sigma}_2\mathbf{V}_2^H, \quad \mathbf{H}_{r,i}\mathbf{B}_i = \mathbf{U}_1\mathbf{\Sigma}_1\mathbf{V}_{1,i}^H, \quad i=1,2 \tag{18}$$

where $\mathbf{U}_2 = [\mathbf{U}_{2,1}^T,\ \mathbf{U}_{2,2}^T]^T$, $\mathbf{V}_1^H = [\mathbf{V}_{1,1}^H, \mathbf{V}_{1,2}^H]$, and the dimensions of $\mathbf{U}_{2,i}$ and $\mathbf{V}_{1,i}$, $i=1,2$, are all $N \times 2N$. Without loss of generality, $\mathbf{F}$ can be written as

$$\mathbf{F} = \begin{bmatrix}\mathbf{V}_2 & \mathbf{V}_2^{\perp}\end{bmatrix}\begin{bmatrix}\mathbf{A} & \mathbf{K}\\ \mathbf{G} & \mathbf{J}\end{bmatrix}\begin{bmatrix}\mathbf{U}_1^H\\ (\mathbf{U}_1^{\perp})^H\end{bmatrix} \tag{19}$$

where $\mathbf{V}_2^{\perp}(\mathbf{V}_2^{\perp})^H = \mathbf{I}_M - \mathbf{V}_2\mathbf{V}_2^H$ and $\mathbf{U}_1^{\perp}(\mathbf{U}_1^{\perp})^H = \mathbf{I}_M - \mathbf{U}_1\mathbf{U}_1^H$ such that $[\mathbf{V}_2,\ \mathbf{V}_2^{\perp}]$ and $[\mathbf{U}_1,\ \mathbf{U}_1^{\perp}]$ are $M \times M$ unitary matrices. The dimensions of $\mathbf{A}$, $\mathbf{K}$, $\mathbf{G}$ and $\mathbf{J}$ are $2N \times 2N$, $2N \times (M-2N)$, $(M-2N)\times 2N$, and $(M-2N)\times(M-2N)$, respectively. Since $[\mathbf{V}_2,\ \mathbf{V}_2^{\perp}]$ and $[\mathbf{U}_1,\ \mathbf{U}_1^{\perp}]$ are $M \times M$ unitary matrices, for any $\mathbf{F}$, we have

$$\begin{bmatrix}\mathbf{A} & \mathbf{K}\\ \mathbf{G} & \mathbf{J}\end{bmatrix} = [\mathbf{V}_2,\ \mathbf{V}_2^{\perp}]^H\mathbf{F}[\mathbf{U}_1,\ \mathbf{U}_1^{\perp}].$$

Thus, using (19) to represent $\mathbf{F}$ does not lose any generality.

Substituting (18) and (19) back into (8), we obtain that $\mathbf{H}_{2,r}\mathbf{F}\mathbf{H}_{r,1}\mathbf{B}_1 = \mathbf{U}_{2,2}\mathbf{\Sigma}_2\mathbf{A}\mathbf{\Sigma}_1\mathbf{V}_{1,1}^H$ and $\mathbf{H}_{2,r}\mathbf{F}\mathbf{F}^H\mathbf{H}_{2,r}^H = \mathbf{U}_{2,2}\mathbf{\Sigma}_2(\mathbf{A}\mathbf{A}^H + \mathbf{K}\mathbf{K}^H)\mathbf{\Sigma}_2\mathbf{U}_{2,2}^H$. Thus, $\mathbf{E}_1$ can be written as

$$\begin{aligned}\mathbf{E}_1 = \big[&\mathbf{I}_N + \mathbf{V}_{1,1}\mathbf{\Sigma}_1\mathbf{A}^H\mathbf{\Sigma}_2\mathbf{U}_{2,2}^H[\mathbf{U}_{2,2}\mathbf{\Sigma}_2(\mathbf{A}\mathbf{A}^H + \mathbf{K}\mathbf{K}^H)\\ &\times\mathbf{\Sigma}_2\mathbf{U}_{2,2}^H + \mathbf{I}_N]^{-1}\mathbf{U}_{2,2}\mathbf{\Sigma}_2\mathbf{A}\mathbf{\Sigma}_1\mathbf{V}_{1,1}^H\big]^{-1}.\end{aligned} \tag{20}$$

Similarly, by substituting (18) and (19) back into (9) we obtain

$$\begin{aligned}\mathbf{E}_2 = \big[&\mathbf{I}_N + \mathbf{V}_{1,2}\mathbf{\Sigma}_1\mathbf{A}^H\mathbf{\Sigma}_2\mathbf{U}_{2,1}^H[\mathbf{U}_{2,1}\mathbf{\Sigma}_2(\mathbf{A}\mathbf{A}^H + \mathbf{K}\mathbf{K}^H)\\ &\times\mathbf{\Sigma}_2\mathbf{U}_{2,1}^H + \mathbf{I}_N]^{-1}\mathbf{U}_{2,1}\mathbf{\Sigma}_2\mathbf{A}\mathbf{\Sigma}_1\mathbf{V}_{1,2}^H\big]^{-1}.\end{aligned} \tag{21}$$

Substituting (18) back into the left-hand-side of the transmission power constraint (14), we have

$$\begin{aligned}\mathrm{tr}\Bigg(\mathbf{F}\Bigg(\sum_{i=1}^{2}\mathbf{H}_{r,i}\mathbf{B}_i\mathbf{B}_i^H\mathbf{H}_{r,i}^H + \mathbf{I}_M\Bigg)\mathbf{F}^H\Bigg) &= \mathrm{tr}\big(\mathbf{A}(\mathbf{\Sigma}_1^2\\ +\mathbf{I}_{2N})\mathbf{A}^H + \mathbf{G}(\mathbf{\Sigma}_1^2 + \mathbf{I}_{2N})\mathbf{G}^H &+ \mathbf{K}\mathbf{K}^H + \mathbf{J}\mathbf{J}^H\big).\end{aligned} \tag{22}$$

It can be clearly seen from (20) and (21) that $\mathbf{G}$ and $\mathbf{J}$ are irrelevant to $\mathbf{E}_1$ and $\mathbf{E}_2$, and $\mathbf{E}_1$, $\mathbf{E}_2$ are minimized if $\mathbf{K} = \mathbf{0}$. Moreover, from (22) we find that $\mathbf{K} = \mathbf{0}$, $\mathbf{G} = \mathbf{0}$, and $\mathbf{J} = \mathbf{0}$ minimize the transmit power consumption at the relay node. Thus, we have $\mathbf{F} = \mathbf{V}_2\mathbf{A}\mathbf{U}_1^H$. □

Theorem 1 shows that the optimal relay amplifying matrix can be viewed as a general form of beamforming. The relay first performs receive beamforming using the Hermitian transpose of the left singular matrix of the effective source-relay channel $\mathbf{H}_1$ (15). Then the relay conducts a linear precoding operation using $\mathbf{A}$. Finally, a transmit beamforming is performed by the relay using the right singular matrix of the relay-destination channel $\mathbf{H}_2$ (16). We would like to mention that unlike the relay scheme developed in [9], Theorem 1 is not affected by the statistics of channel matrices. Interestingly, (17) extends the result in [5] from the case of single antenna to the scenario of multiple antennas at both source nodes. Moreover, (17) generalizes the result in [7] from the MSMI objective to a broad class of frequently used objective functions for MIMO systems. Note that there are two differences between Theorem 1 and the result in [2]. First, using the notation in this paper, $\mathbf{A}$ is a diagonal matrix in [2]. While in Theorem 1, $\mathbf{A}$ is not necessarily diagonal. Second, $\mathbf{U}_1$ in [2] depends only on the channel matrix. While in Theorem 1, $\mathbf{U}_1$ is a function of both source precoding matrices $\mathbf{B}_1$ and $\mathbf{B}_2$. These differences make optimizing the source and relay matrices in a two-way MIMO relay channel much more challenging than in a one-way MIMO relay system.

Using (17), the optimization problem (13)-(14) becomes

$$\min_{\mathbf{A}} \quad q(\mathbf{E}_1, \mathbf{E}_2) \qquad \text{s.t.} \quad \mathrm{tr}\big(\mathbf{A}(\mathbf{\Sigma}_1^2 + \mathbf{I}_{2N})\mathbf{A}^H\big) \le P_r \tag{23}$$

where $\mathbf{E}_1$ and $\mathbf{E}_2$ are given by (20) and (21), respectively, with $\mathbf{K} = \mathbf{0}$. In general, the problem (23) is nonconvex and a globally optimal solution is difficult to obtain with a reasonable computational complexity. We can resort to numerical methods, such as the projected gradient method [10] to find (at least) a locally optimal solution of (23). Since the dimension of $\mathbf{A}$ is smaller than $\mathbf{F}$, solving the problem (23) has a smaller computational complexity than solving the problem (13)-(14). For relay systems with $M < 2N$, we directly solve the problem (13)-(14) using the projected gradient method to obtain (at least) a locally optimal solution of $\mathbf{F}$.

It can be seen from (8) that $\mathbf{B}_2$ is irrelevant to $\mathbf{E}_1$. Thus, for fixed $\mathbf{F}$ and $\mathbf{B}_2$, the problem of optimizing $\mathbf{B}_1$ is given by

$$\min_{\mathbf{B}_1} \quad q\big([\mathbf{I}_N + \mathbf{B}_1^H\mathbf{\Psi}_1\mathbf{B}_1]^{-1}\big) \tag{24}$$

$$\text{s.t.} \quad \mathrm{tr}(\mathbf{B}_1^H\mathbf{B}_1) \le P_1 \tag{25}$$

$$\mathrm{tr}(\mathbf{B}_1^H\mathbf{H}_{r,1}^H\mathbf{F}^H\mathbf{F}\mathbf{H}_{r,1}\mathbf{B}_1) \le \tilde{P}_r \tag{26}$$

where $\mathbf{\Psi}_1 \triangleq \mathbf{H}_{r,1}^H\mathbf{F}^H\mathbf{H}_{2,r}^H\mathbf{C}_{\tilde{v}_2}^{-1}\mathbf{H}_{2,r}\mathbf{F}\mathbf{H}_{r,1}$ and $\tilde{P}_r \triangleq P_r - \mathrm{tr}(\mathbf{F}(\mathbf{H}_{r,2}\mathbf{B}_2\mathbf{B}_2^H\mathbf{H}_{r,2}^H + \mathbf{I}_M)\mathbf{F}^H)$. When the q function is the negative source-destination MI, the problem (24)-(26) has been solved in [11]. When the MSE of the signal waveform estimation is adopted as function q, the problem (24)-(26) is solved in [12]. Using the Lagrange multiplier method, both [11] and [12] reveal that the optimal $\mathbf{B}_1$ has the structure of $\mathbf{B}_1 = \mathbf{M}^{-H}\mathbf{Q}\mathbf{D}$, where $\mathbf{M}\mathbf{M}^H = \mu_1\mathbf{I}_N + \mu_2\mathbf{H}_{r,1}^H\mathbf{F}^H\mathbf{F}\mathbf{H}_{r,1}$, $\mathbf{M}^{-1}\mathbf{\Psi}_1\mathbf{M}^{-H} = \mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^H$, and $\mathbf{D}$ is an $N \times N$ diagonal matrix. Here $\mu_1 \ge 0$, $\mu_2 \ge 0$ are the Lagrange multipliers, and $\mathbf{\Lambda}$ is a diagonal eigenvalue matrix.

In fact, if $\mathbf{B}_1$ satisfies (25) and (26), it must also satisfy the constraint of $\mathrm{tr}(\mathbf{B}_1^H\mathbf{M}\mathbf{M}^H\mathbf{B}_1) \le \mu_1 P_1 + \mu_2\tilde{P}_r$. Denoting $\tilde{\mathbf{B}}_1 \triangleq \mathbf{M}^H\mathbf{B}_1$, a relaxed problem of the original problem (24)-(26) is given by

$$\min_{\tilde{\mathbf{B}}_1} \quad q\big([\mathbf{I}_N + \tilde{\mathbf{B}}_1^H\mathbf{M}^{-1}\mathbf{\Psi}_1\mathbf{M}^{-H}\tilde{\mathbf{B}}_1]^{-1}\big) \tag{27}$$

$$\text{s.t.} \quad \mathrm{tr}(\tilde{\mathbf{B}}_1^H\tilde{\mathbf{B}}_1) \le \mu_1 P_1 + \mu_2\tilde{P}_r. \tag{28}$$

It has been shown in [8] that for any Schur-concave [13] objective function q, the solution to the problem (27)-(28) is given by $\tilde{\mathbf{B}}_1 = \mathbf{Q}\mathbf{D}$. While for any Schur-convex q [13], the solution is $\tilde{\mathbf{B}}_1 = \mathbf{Q}\mathbf{D}\mathbf{U}_0$, where $\mathbf{U}_0$ is a unitary matrix such that $[\mathbf{I}_N+\tilde{\mathbf{B}}_1^H\mathbf{M}^{-1}\mathbf{\Psi}_1\mathbf{M}^{-H}\tilde{\mathbf{B}}_1]^{-1}$ has identical diagonal elements [8]. Therefore, for Schur-concave and Schur-convex q, we have

$$\mathbf{B}_1 = \mathbf{M}^{-H}\mathbf{Q}\mathbf{D} \quad \text{and} \quad \mathbf{B}_1 = \mathbf{M}^{-H}\mathbf{Q}\mathbf{D}\mathbf{U}_0 \qquad (29)$$

respectively. Interestingly, both negative MI and MSE are Schur-concave functions [8]. Thus, the problems discussed in [11] and [12] are special cases of the problem (24)-(26), and the results obtained in [11] and [12] are special instances of (29).

It can be seen from (29) that $\mathbf{B}_1$ has a beamforming structure, where the directions of beams are determined by $\mathbf{M}^{-H}\mathbf{Q}$ and $\mathbf{D}$ represents the power allocation at each beam. The value of μ_1, μ_2, and $\mathbf{D}$ depends on the specific expression of the objective function q and can be obtained via solving the dual optimization problem associated with the original problem (24)-(26) as proposed in [11] and [12]. Similar to (24)-(29), for fixed $\mathbf{F}$ and $\mathbf{B}_1$, we can optimize $\mathbf{B}_2$.

Now the problem (10)-(12) can be solved by an iterative algorithm. This algorithm is first initialized at random feasible $\mathbf{B}_1$ and $\mathbf{B}_2$ satisfying (12). At each iteration, when $M \geq 2N$, $\mathbf{F}$ is updated according to (17) by solving the problem (23) with fixed $\mathbf{B}_1$ and $\mathbf{B}_2$. When $M < 2N$, $\mathbf{F}$ is updated by solving the problem (13)-(14) with fixed $\mathbf{B}_1$ and $\mathbf{B}_2$. Then $\mathbf{B}_1$ is updated as (29) with fixed $\mathbf{F}$ and $\mathbf{B}_2$, and next $\mathbf{B}_2$ is updated similar to (29) with fixed $\mathbf{F}$ and $\mathbf{B}_1$. Note that the conditional updates of each matrix may either decrease or maintain but cannot increase the objective function (10). Monotonic convergence of $\mathbf{F}$, $\mathbf{B}_1$, and $\mathbf{B}_2$ towards (at least) a locally optimal solution follows directly from this observation. Multiple random initializations of $\mathbf{B}_1$ and $\mathbf{B}_2$ can be attempted to get around the local optimality.

IV. Numerical Examples

In the simulations, we study the BER performance of two-way MIMO relay systems with $N = 2$ and $M = 4, 6$, respectively. The source symbols are modulated by QPSK constellations. All channel matrices have complex Gaussian entries with zero-mean and variances of $1/N$, $1/M$ for $\mathbf{H}_{r,i}$ and $\mathbf{H}_{i,r}$, $i = 1, 2$, respectively[1], and the simulation results are averaged over 1000 independent channel realizations. In the simulations, we set $P_1 = P_2 = 20$dB above the noise level and varying the value of P_r. Note that the proposed iterative algorithm is applicable for a broad class of frequently used objective functions, and in the simulations, we consider the following two functions: (1) The negative two-way SMI adopted in [7] given by $q(\mathbf{E}_1, \mathbf{E}_2) = \log_2|\mathbf{E}_1| + \log_2|\mathbf{E}_2|$; (2) The SMSE of the signal waveform estimation written as $q(\mathbf{E}_1, \mathbf{E}_2) = \mathrm{tr}(\mathbf{E}_1 + \mathbf{E}_2)$. We refer to the algorithms using these two objectives as the MSMI algorithm and the MSMSE algorithm, respectively. In particular, for the MSMI algorithm, $\mathbf{B}_1$, $\mathbf{B}_2$, and $\mathbf{F}$ are optimized according to [7]. For the MSMSE algorithm, we study not only the BER of the system where $\mathbf{B}_1$, $\mathbf{B}_2$, and $\mathbf{F}$ are optimized using the proposed iterative algorithm (denoted as "Joint"), but also the system BER with the optimal $\mathbf{F}$ (17) based on $\mathbf{B}_i = \sqrt{P_i/N}\mathbf{I}_N$, $i = 1, 2$ (referred to as "Relay").

[1]The variances are set to normalize the effect of number of transmit antennas to the receive signal-to-noise ratio.

Figs. 1 and 2 demonstrate the performance of all algorithms in terms of BER versus P_r with $M = 4$ and $M = 6$, respectively. It can be seen from both figures that by using the MSMSE criterion, even the optimal-relay-only system outperforms the MSMI-based system [7] in terms of BER. This is because MSMI is a good criterion only for coded systems in which the number of symbols for each coding block is very large. However, in practical communication systems, due to the delay constraint, codewords always have a finite length. Thus, MSMSE is a better criterion for practical systems.

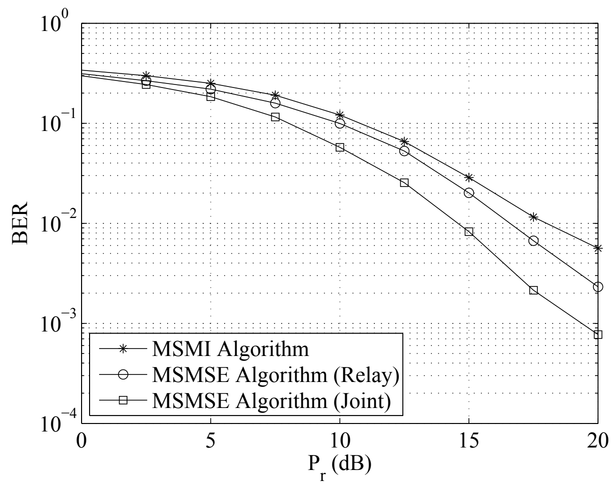

Fig. 1. BER versus P_r. $M = 4$.

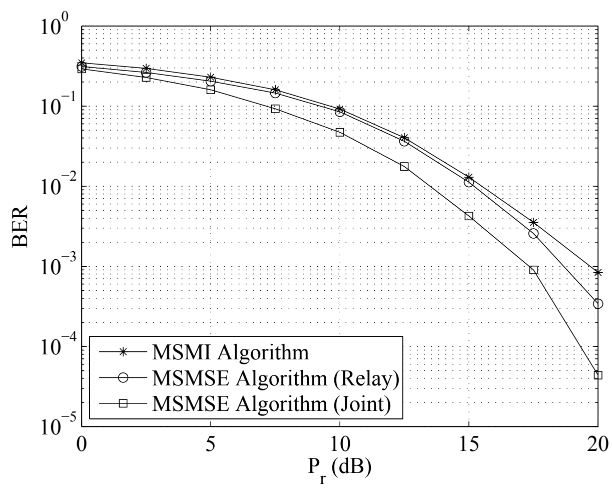

Fig. 2. BER versus P_r. $M = 6$.

From Figs. 1 and 2, we also observe that the proposed iterative algorithm further improves the system BER performance in the whole P_r range, since it jointly optimizes the source and relay matrices. In fact, it achieves a higher diversity order than the other algorithms. Comparing Fig. 1 with Fig. 2, it can be seen that as expected, the BER performance of all three systems improves with increasing antenna numbers at the relay node.

V. Conclusion

We have derived the optimal structure of the source precoding matrices and the relay amplifying matrix for a two-way amplify-and-forward MIMO relay system with a broad class of frequently used objective functions. An iterative algorithm is developed to optimize the relay and source matrices.

Acknowledgment

This work was supported under Australian Research Council's *Discovery Projects* funding scheme (project numbers DP110100736, DP110102076).

References

[1] J. Gómez-Vilardebó, A. I. Pérez-Neira, and M. Nájar, "Energy efficient communications over the AWGN relay channel," *IEEE Trans. Wireless Commun.*, vol. 9, pp. 32-37, Jan. 2010.

[2] Y. Rong, X. Tang, and Y. Hua, "A unified framework for optimizing linear non-regenerative multicarrier MIMO relay communication systems," *IEEE Trans. Signal Process.*, vol. 57, pp. 4837-4851, Dec. 2009.

[3] S. Katti, S. Gollakota, and D. Katabi, "Embracing wireless interference: Analog network coding," in *Proc. ACM SIGCOMM*, Kyoto, Japan, Aug. 27-31, 2007.

[4] T. Cui, F. Gao, T. Ho, and A. Nallanathan, "Distributed space-time coding for two-way wireless relay networks," in *Proc. IEEE ICC*, Beijing, China, May 19-23, 2008, pp. 3888-3892.

[5] R. Zhang, Y.-C. Liang, C. C. Chai, and S. Cui, "Optimal beamforming for two-way multi-antenna relay channel with analogue network coding," *IEEE J. Sel. Areas Commun.*, vol. 27, pp. 699-712, Jun. 2009.

[6] F. Gao, T. Cui, B. Jiang, and X. Gao, "On communication protocol and beamforming design for amplify-and-forward N-way relay networks," in *Proc. IEEE 3rd Intern. Workshop Computat. Advances Multi-Sensor Adaptive Process.*, Aruba, Dutch Antilles, Dec. 13-16, 2009. pp. 109-112.

[7] S. Xu and Y. Hua, "Source-relay optimization for a two-way MIMO relay system," in *Proc. IEEE ICASSP*, Dallas, TX, USA, Mar. 14-19, 2010.

[8] D. P. Palomar, J. M. Cioffi, and M. A. Lagunas, "Joint Tx-Rx beamforming design for multicarrier MIMO channels: A unified framework for convex optimization," *IEEE Trans. Signal Process.*, vol. 51, pp. 2381-2401, Sep. 2003.

[9] V. Havary-Nassab, S. Shahbazpanahi, and A. Grami, "Joint receive-transmit beamforming for multi-antenna relaying schemes," *IEEE Trans. Signal Process.*, vol. 58, pp. 4966-4972, Sep. 2010.

[10] D. P. Bertsekas, *Nonlinear Programming*. 2nd. Edition, Athena Scientific, Belmont, 1995.

[11] Y. Yu and Y. Hua, "Power allocation for a MIMO relay system with multiple-antenna users," *IEEE Trans. Signal Process.*, vol. 58, pp. 2823-2835, May 2010.

[12] Y. Rong, "Optimal joint source and relay beamforming for MIMO relays with direct link," *IEEE Commun. Lett.*, vol. 14, pp. 390-392, May 2010.

[13] A. W. Marshall and I. Olkin, *Inequalities: Theory of Majorization and Its Applications*. Academic Press, 1979.