Optimal Multicarrier Multi-Hop Non-Regenerative MIMO Relays with QoS Constraints

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***Abstract*—In this paper, we derive the optimal source and relay matrices of a multicarrier multi-hop multiple-input multiple-output (MIMO) relay system that guarantee the predetermined quality-of-service (QoS) criteria be attained with the minimal total transmission power. We consider the linear minimal mean-squared error (MMSE) receiver at the destination node and show that the solution to the original optimization problem can be upper-bounded by using a successive geometric programming (GP) approach and lower-bounded by utilizing a dual decomposition technique. Simulation results show that both bounds are tight, and to obtain the same QoS, the proposed MIMO relay system requires substantially less power than that of the suboptimal relay scheme.**

I. Introduction

As an efficient solution for wireless backhaul networks, non-regenerative multiple-input multiple-output (MIMO) relay communication systems recently attracted much research interest [1]-[7]. For a two-hop MIMO relay system, the optimal relay amplifying matrix is obtained in [1]-[2] to maximize the mutual information (MI) between source and destination. In [3]-[4], optimal algorithms are developed to minimize the mean-squared error (MSE) of the signal waveform estimation at the destination node.

The works of [1]-[4] have been generalized by [5], where a unified framework is established for two-hop multicarrier linear non-regenerative MIMO relay systems with a broad class of objective functions. The framework in [5] has been further extended to multi-hop non-regenerative MIMO relay systems with arbitrary number of hops [6]. Both [5] and [6] consider a linear minimal mean-squared error (MMSE) receiver at the destination node. Recently, it has been shown in [7] that by using a nonlinear decision feedback equalizer (DFE) based on the MMSE criterion at the destination node, the system bit-error-rate (BER) performance can be significantly improved.

The aim of [1]-[7] is to optimize a given objective function, subjecting to the transmission power constraint at each node. However, the quality-of-service (QoS) constraints are not addressed by [1]-[7]. Note that in practical communication systems, QoS criteria are very important. In our recent paper [8], the optimal source and relay matrices which guarantee that the predetermined QoS criteria be attained with the minimal total transmission power have been developed for a single-carrier multi-hop MIMO relay system.

In this paper, we follow our recent result in [8] to present additional details of its multicarrier version. We will address both subcarrier-independent and subcarrier-cooperative multi-hop MIMO relay systems. Since the optimization problem with QoS constraints is nonconvex, the globally optimal solution is difficult to obtain with a reasonable computational complexity. We show that the solution to the original optimization problem can be upper-bounded by using a successive geometric programming (GP) approach and lower-bounded by utilizing a dual decomposition technique. Simulation results show that the upper and lower bounds are tight, and to obtain the same QoS, the proposed MIMO relay system requires substantially less power than that of the suboptimal relay scheme.

We would like to mention that the optimal source matrix design for a single-hop (point-to-point) MIMO system under QoS constraints is addressed in [9] for linear MMSE receiver. Our paper generalizes the results in [9] from single-hop MIMO channel to multicarrier multi-hop non-regenerative MIMO relay communication systems with any number of hops. Note that the proof of the theorems for multi-hop MIMO relay system is much more involved than that for the single-hop MIMO channel [8]. The generalization from a single-hop MIMO system to multicarrier multi-hop MIMO relay systems is significant.

II. System Model

We consider a wireless communication system with one source node, one destination node, and $L-1$ relay nodes ($L \geq 2$). We assume that due to the propagation path-loss, signals transmitted by the ith node can only be received by its direct forward node, i.e., the $(i+1)$-th node. Thus, signals transmitted by the source pass through L hops until they reach the destination node. We also assume that the number of antennas at each node is N_i, $1 \leq i \leq L+1$, and the signal sequence is modulated by N_c subcarriers. We denote $N_b^{(n)}, 1 \leq n \leq N_c$, as the number of source symbols in the nth subcarrier. Hereafter, the superscript (n) denotes the corresponding variables for the nth subcarrier. Like [1]-[7], a linear non-regenerative relay matrix is used at each relay. Based on whether the subcarriers cooperate with each other in processing the signals at the source and relay nodes, we can have either subcarrier-independent or subcarrier-cooperative systems.

A. Subcarrier-Independent System

At the source node, the $N_1 \times 1$ signal vectors transmitted by the source node is

$$\mathbf{s}_1^{(n)} = \mathbf{F}_1^{(n)}\mathbf{b}^{(n)}, \qquad 1 \leq n \leq N_c \tag{1}$$

where $\mathbf{b}^{(n)}$ is the $N_b^{(n)} \times 1$ source symbol vector, and $\mathbf{F}_1^{(n)}$ is the $N_1 \times N_b^{(n)}$ source precoding matrix. We assume that $\mathrm{E}[\mathbf{b}^{(n)}(\mathbf{b}^{(n)})^H] = \mathbf{I}_{N_b^{(n)}}$, where $\mathrm{E}[\cdot]$ stands for the statistical expectation, $(\cdot)^H$ denotes the Hermitian transpose, and $\mathbf{I}_n$ is an $n \times n$ identity matrix. The $N_i \times 1$ signal vectors received at the ith node is written as

$$\mathbf{y}_i^{(n)} = \mathbf{H}_{i-1}^{(n)}\mathbf{s}_{i-1}^{(n)} + \mathbf{v}_i^{(n)}, \quad 2 \le i \le L+1, \ \ 1 \le n \le N_c \quad (2)$$

where $\mathbf{H}_{i-1}^{(n)}$ is the $N_i \times N_{i-1}$ MIMO channel matrix between the ith and the $(i-1)$-th nodes, i.e., the $(i-1)$-th hop, $\mathbf{v}_i^{(n)}$ is the $N_i \times 1$ independent and identically distributed (i.i.d.) additive white Gaussian noise (AWGN) vector at the ith node, and $\mathbf{s}_{i-1}^{(n)}$ is the $N_{i-1} \times 1$ signal vector transmitted by the $(i-1)$-th node. We assume that the noises are complex circularly symmetric with zero mean and unit variance.

The input-output relationship at node i is given by

$$\mathbf{s}_i^{(n)} = \mathbf{F}_i^{(n)}\mathbf{y}_i^{(n)}, \qquad 2 \le i \le L, \quad 1 \le n \le N_c \qquad (3)$$

where $\mathbf{F}_i^{(n)}$ is the $N_i \times N_i$ amplifying matrix at node i. Combining (1)-(3), we obtain the received signal vectors at the destination node (the $(L+1)$-th node) as

$$\mathbf{y}_{L+1}^{(n)} = \bar{\mathbf{H}}^{(n)}\mathbf{b}^{(n)} + \bar{\mathbf{v}}^{(n)}, \qquad 1 \le n \le N_c \qquad (4)$$

where $\bar{\mathbf{H}}^{(n)}$ and $\bar{\mathbf{v}}^{(n)}$ are the equivalent MIMO channel matrix and the noise vector, and given respectively by

$$\begin{aligned}\bar{\mathbf{H}}^{(n)} &= \mathbf{H}_L^{(n)}\mathbf{F}_L^{(n)} \cdots \mathbf{H}_1^{(n)}\mathbf{F}_1^{(n)} = \bigotimes_{i=L}^{1}(\mathbf{H}_i^{(n)}\mathbf{F}_i^{(n)}) \\ \bar{\mathbf{v}}^{(n)} &= \mathbf{H}_L^{(n)}\mathbf{F}_L^{(n)} \cdots \mathbf{H}_2^{(n)}\mathbf{F}_2^{(n)}\mathbf{v}_2^{(n)} + \cdots \\ &\quad + \mathbf{H}_L^{(n)}\mathbf{F}_L^{(n)}\mathbf{v}_L^{(n)} + \mathbf{v}_{L+1}^{(n)} \\ &= \sum_{l=2}^{L}\Big(\bigotimes_{i=L}^{l}(\mathbf{H}_i^{(n)}\mathbf{F}_i^{(n)})\mathbf{v}_l^{(n)}\Big) + \mathbf{v}_{L+1}^{(n)}.\end{aligned}$$

Here for matrices $\mathbf{A}_i$, $\bigotimes_{i=l}^{k}(\mathbf{A}_i) \triangleq \mathbf{A}_l \cdots \mathbf{A}_k$.

From (1), we know that the power of the signals transmitted by the source node is $\sum_{n=1}^{N_c}\mathrm{tr}\big(\mathbf{F}_1^{(n)}(\mathbf{F}_1^{(n)})^H\big)$, where $\mathrm{tr}(\cdot)$ denotes the trace of a matrix. Based on (2) and (3), the power of the signal transmitted by the relay node i is given by

$$\begin{aligned}&\sum_{n=1}^{N_c}\mathrm{tr}\Big(\mathrm{E}\Big[\mathbf{s}_i^{(n)}(\mathbf{s}_i^{(n)})^H\Big]\Big) \\ &= \sum_{n=1}^{N_c}\mathrm{tr}\Big(\mathbf{F}_i^{(n)}\mathrm{E}\Big[\mathbf{y}_i^{(n)}(\mathbf{y}_i^{(n)})^H\Big](\mathbf{F}_i^{(n)})^H\Big) \\ &= \sum_{n=1}^{N_c}\mathrm{tr}\Bigg(\mathbf{F}_i^{(n)}\Bigg(\sum_{l=1}^{i-1}\Big(\bigotimes_{k=i-1}^{l}\big(\mathbf{H}_k^{(n)}\mathbf{F}_k^{(n)}\big)\bigotimes_{k=l}^{i-1}\big(\mathbf{H}_k^{(n)}\mathbf{F}_k^{(n)}\big)^H\Big) \\ &\quad + \mathbf{I}_{N_i}\Big)(\mathbf{F}_i^{(n)})^H\Bigg), \qquad 2 \le i \le L. \end{aligned} \qquad (5)$$

B. Subcarrier-Cooperative System

In a subcarrier-cooperative system, the received signal vector at the destination node is

$$\mathbf{y}_{L+1} = \bar{\mathbf{H}}\mathbf{b} + \bar{\mathbf{v}} \qquad (6)$$

where

$$\begin{aligned}\mathbf{y}_{L+1} &= \Big[\big(\mathbf{y}_{L+1}^{(1)}\big)^T, \big(\mathbf{y}_{L+1}^{(2)}\big)^T, \cdots, \big(\mathbf{y}_{L+1}^{(N_c)}\big)^T\Big]^T \\ \mathbf{b} &= \Big[\big(\mathbf{b}^{(1)}\big)^T, \big(\mathbf{b}^{(2)}\big)^T, \cdots, \big(\mathbf{b}^{(N_c)}\big)^T\Big]^T \\ \bar{\mathbf{v}} &= \Big[\big(\bar{\mathbf{v}}^{(1)}\big)^T, \big(\bar{\mathbf{v}}^{(2)}\big)^T, \cdots, \big(\bar{\mathbf{v}}^{(N_c)}\big)^T\Big]^T \\ \bar{\mathbf{H}} &= \mathbf{H}_L\mathbf{F}_L \cdots \mathbf{H}_1\mathbf{F}_1 = \bigotimes_{i=L}^{1}(\mathbf{H}_i\mathbf{F}_i). \end{aligned} \qquad (7)$$

Here $\mathbf{H}_i = \mathrm{bd}\Big(\mathbf{H}_i^{(1)}, \mathbf{H}_i^{(2)}, \cdots, \mathbf{H}_i^{(N_c)}\Big)$, $(\cdot)^T$ denotes the matrix (vector) transpose, and $\mathrm{bd}(\cdot)$ stands for a block-diagonal matrix. $\mathbf{H}_i$ is an $N_cN_{i+1} \times N_cN_i$ block-diagonal "super" channel matrix of the ith hop. From (6), we see that the cooperation among different subcarriers is performed by a "super" $N_cN_1 \times J$ source matrix $\mathbf{F}_1$ where $J = \sum_{n=1}^{N_c} N_b^{(n)}$, and "super" $N_cN_i \times N_cN_i$ relay matrices $\mathbf{F}_i$, $2 \le i \le L$.

A subcarrier-cooperative MIMO relay system is a generalization of a subcarrier-independent system, since if we impose a block-diagonal structure on $\mathbf{F}_i$, $1 \le i \le L$ such that $\mathbf{F}_i = \mathrm{bd}\big(\mathbf{F}_i^{(1)}, \mathbf{F}_i^{(2)}, \cdots, \mathbf{F}_i^{(N_c)}\big)$. Then (6) becomes (4). Hence we anticipate that a subcarrier-cooperative system has a better performance than a subcarrier-independent system. Interestingly, from a mathematical point of view, the subcarrier-independent system model (4) is more general, since (6) can be obtained from (4) by simply setting $N_c = 1$. Thus, in the following, we use (4) to derive the optimal source and relay matrices. After obtaining the optimal source and relay matrices for subcarrier independent system, we revisit (6) to derive the optimal structure of $\mathbf{F}_i$, $1 \le i \le L$, for subcarrier-cooperative systems.

III. Optimal Source and Relay Matrices with QoS Constraints

We design MIMO relay systems that meet the QoS requirements with the minimal total transmission power. Based on the strong link between the diagonal elements of the MMSE matrix and most commonly used MIMO communication system objective functions [5]-[8], the QoS criteria are set up as the upper-bound of MSE of each data stream.

Using a linear MMSE receiver, the estimated signal vector is

$$\hat{\mathbf{b}}^{(n)} = (\mathbf{W}^{(n)})^H\mathbf{y}_{L+1}^{(n)}, \qquad 1 \le n \le N_c \qquad (8)$$

where $\mathbf{W}^{(n)}$ is the $N_{L+1} \times N_b^{(n)}$ weight matrix of the linear MMSE receiver given by

$$\mathbf{W}^{(n)} = \big(\bar{\mathbf{H}}^{(n)}(\bar{\mathbf{H}}^{(n)})^H + \mathbf{C}_{\bar{v}}^{(n)}\big)^{-1}\bar{\mathbf{H}}^{(n)}, \quad 1 \le n \le N_c. \ (9)$$

Here $\mathbf{C}_{\bar{v}}^{(n)} = \mathrm{E}[\bar{\mathbf{v}}^{(n)}(\bar{\mathbf{v}}^{(n)})^H]$ is the noise covariance matrix

$$\mathbf{C}_{\bar{v}}^{(n)} = \sum_{l=2}^{L}\Big(\bigotimes_{i=L}^{l}(\mathbf{H}_i^{(n)}\mathbf{F}_i^{(n)})\bigotimes_{i=l}^{L}(\mathbf{H}_i^{(n)}\mathbf{F}_i^{(n)})^H\Big) + \mathbf{I}_{N_{L+1}} \tag{10}$$

and $(\cdot)^{-1}$ denotes the matrix inversion. Using (4) and (8)-(10) the MMSE matrix of the signal waveform estimation, denoted as $\mathbf{E}(\{\mathbf{F}_i^{(n)}\}) = \mathrm{E}\Big[(\hat{\mathbf{b}}^{(n)} - \mathbf{b}^{(n)})(\hat{\mathbf{b}}^{(n)} - \mathbf{b}^{(n)})^H\Big]$, $1 \le n \le N_c$, is given by [6]

$$\mathbf{E}\big(\{\mathbf{F}_i^{(n)}\}\big) = \Big(\mathbf{I}_{N_b^{(n)}} + (\bar{\mathbf{H}}^{(n)})^H(\mathbf{C}_{\bar{v}}^{(n)})^{-1}\bar{\mathbf{H}}^{(n)}\Big)^{-1} \tag{11}$$

where $\{\mathbf{F}_i^{(n)}\} \triangleq \{\mathbf{F}_i^{(n)}, 1 \le i \le L\}$.

The QoS-constrained multicarrier multi-hop MIMO relay optimization problem can be written as

$$\min_{\{\mathbf{F}_i^{(n)}\}} \sum_{i=1}^{N_c}\mathrm{tr}\big(\mathbf{F}_1^{(n)}(\mathbf{F}_1^{(n)})^H\big) + \sum_{i=1}^{N_c}\sum_{i=2}^{L}\mathrm{tr}\Bigg(\mathbf{F}_i^{(n)}\Bigg(\sum_{l=1}^{i-1}\Big(\bigotimes_{k=i-1}^{l}(\mathbf{H}_k^{(n)}\mathbf{F}_k^{(n)}) \bigotimes_{k=l}^{i-1}(\mathbf{H}_k^{(n)}\mathbf{F}_k^{(n)})^H\Big) + \mathbf{I}_{N_i}\Bigg)(\mathbf{F}_i^{(n)})^H\Bigg) \tag{12}$$

$$\text{s.t.} \quad \mathbf{d}\big[\mathbf{E}\big(\{\mathbf{F}_i^{(n)}\}\big)\big] \le \mathbf{q}^{(n)}, \qquad 1 \le n \le N_c \tag{13}$$

where the objective function (12) is the total transmission power consumed by the source node and all relay nodes in the relay network, $\mathbf{d}[\mathbf{A}]$ is a column vector containing all main diagonal elements of $\mathbf{A}$, and $\mathbf{q}^{(n)} = \Big[q_1^{(n)}, q_2^{(n)}, \cdots, q_{N_b^{(n)}}^{(n)}\Big]^T$ is the QoS requirement vector measured in terms of the MSE of each data stream that must be satisfied. Obviously, any meaningful $\mathbf{q}^{(n)}$ has $q_i^{(n)} > 0$, $1 \le i \le N_b^{(n)}$, $1 \le n \le N_c$. Without loss of generality, we assume that the elements of $\mathbf{q}^{(n)}$ are arranged in an *increasing* order.

Let us write the singular value decomposition (SVD) of $\mathbf{H}_i^{(n)}$ as

$$\mathbf{H}_i^{(n)} = \mathbf{U}_i^{(n)}\mathbf{\Sigma}_i^{(n)}(\mathbf{V}_i^{(n)})^H, \quad 1 \le i \le L, \ \ 1 \le n \le N_c \tag{14}$$

where the dimensions of matrices $\mathbf{U}_i^{(n)}$, $\mathbf{\Sigma}_i^{(n)}$, and $\mathbf{V}_i^{(n)}$ are $N_{i+1} \times N_{i+1}$, $N_{i+1} \times N_i$, $N_i \times N_i$, respectively. We assume that the main diagonal elements of $\mathbf{\Sigma}_i^{(n)}$, $1 \le i \le L$, $1 \le n \le N_c$, are arranged in the *decreasing* order. We also introduce $R_h^{(n)} \triangleq \min(\mathrm{rank}(\mathbf{H}_1^{(n)}), \mathrm{rank}(\mathbf{H}_2^{(n)}), \cdots, \mathrm{rank}(\mathbf{H}_L^{(n)}))$, $1 \le n \le N_c$, where $\mathrm{rank}(\cdot)$ denotes the rank of a matrix.

THEOREM 1: Assuming $N_b^{(n)} \le R_h^{(n)}$ and $\mathrm{rank}(\mathbf{F}_i^{(n)}) = N_b^{(n)}, 1 \le i \le L$, for the linear non-regenerative multi-hop MIMO relay design problem (12), (13), the optimal source and relay matrices $\mathbf{F}_i^{(n)}$, $1 \le i \le L$, $1 \le n \le N_c$ are given by

$$\mathbf{F}_1^{(n)} = \mathbf{V}_{1,1}^{(n)}\mathbf{\Lambda}_1^{(n)}\mathbf{U}_{F_1}^{(n)}$$
$$\mathbf{F}_i^{(n)} = \mathbf{V}_{i,1}^{(n)}\mathbf{\Lambda}_i^{(n)}(\mathbf{U}_{i-1,1}^{(n)})^H, \quad 2 \le i \le L \tag{15}$$

where $\mathbf{\Lambda}_i^{(n)}$, $1 \le i \le L$, are $N_b^{(n)} \times N_b^{(n)}$ diagonal matrices, $\mathbf{U}_{F_1}^{(n)}$ is an $N_b^{(n)} \times N_b^{(n)}$ unitary matrix such that $\big[\mathbf{E}\big(\{\mathbf{F}_i^{(n)}\}\big)\big]_{k,k} = q_k^{(n)}$, $1 \le k \le N_b^{(n)}$, and $\mathbf{U}_{i,1}^{(n)}$ and $\mathbf{V}_{i,1}^{(n)}$ contain the leftmost $N_b^{(n)}$ vectors of $\mathbf{U}_i^{(n)}$ and $\mathbf{V}_i^{(n)}$, respectively. Here for a matrix $\mathbf{A}$, $[\mathbf{A}]_{k,k}$ stands for the kth main diagonal element of $\mathbf{A}$.

PROOF: The proof is similar to [8]. □

The assumption of $N_b^{(n)} \le R_h^{(n)}$ is motivated by the fact that using a linear receiver at the destination, the maximal number of independent data streams that can be sent from source to destination for any given $\{\mathbf{F}_i^{(n)}\}$ is no more than $R_h^{(n)}$. Moreover, the assumption of $\mathrm{rank}(\mathbf{F}_i^{(n)}) = N_b^{(n)}, 1 \le i \le L$, is sufficient to allow $N_b^{(n)}$ independent data streams to be sent from source to destination.

From Theorem 1 we find that the optimal source precoding matrix, the optimal relay amplifying matrices, and the MMSE receiver matrix jointly diagonalize the multi-hop MIMO relay channel $\bar{\mathbf{H}}^{(n)}$ *after a rotation* $\mathbf{U}_{F_1}^{(n)}$ of the source precoding matrix. Substituting (15) back into (5), the transmission power at the source and relay nodes can be respectively written as

$$\sum_{n=1}^{N_c}\mathrm{tr}\big(\mathbf{F}_1^{(n)}(\mathbf{F}_1^{(n)})^H\big) = \sum_{n=1}^{N_c}\sum_{k=1}^{N_b^{(n)}}(\lambda_{1,k}^{(n)})^2 \tag{16}$$

$$\sum_{n=1}^{N_c}\mathrm{tr}\Bigg(\mathbf{F}_i^{(n)}\Bigg(\sum_{l=1}^{i-1}\Big(\bigotimes_{k=i-1}^{l}(\mathbf{H}_k^{(n)}\mathbf{F}_k^{(n)})\bigotimes_{k=l}^{i-1}(\mathbf{H}_k^{(n)}\mathbf{F}_k^{(n)})^H\Big) + \mathbf{I}_{N_i}\Bigg)(\mathbf{F}_i^{(n)})^H\Bigg)$$
$$= \sum_{n=1}^{N_c}\sum_{k=1}^{N_b^{(n)}}(\lambda_{i,k}^{(n)})^2\Bigg(\sum_{j=1}^{i-1}\prod_{l=j}^{i-1}(\lambda_{l,k}^{(n)}\sigma_{l,k}^{(n)})^2 + 1\Bigg), 2 \le i \le L \tag{17}$$

where $\lambda_{i,k}^{(n)}$ and $\sigma_{i,k}^{(n)}$, $1 \le i \le L, 1 \le k \le N_b^{(n)}$, are the kth main diagonal elements of $\mathbf{\Lambda}_i^{(n)}$ and $\mathbf{\Sigma}_i^{(n)}$, respectively. Using (16) and (17), it can be shown [8] that the optimal power loading parameters $\boldsymbol{\lambda} \triangleq \Big[\lambda_{1,1}^{(1)}, \cdots, \lambda_{L,N_b^{(N_c)}}^{(N_c)}\Big]^T$ can be obtained by solving the following problem

$$\min_{\boldsymbol{\lambda}} \sum_{n=1}^{N_c}\sum_{k=1}^{N_b^{(n)}}(\lambda_{1,k}^{(n)})^2 + \sum_{n=1}^{N_c}\sum_{i=2}^{L}\sum_{k=1}^{N_b^{(n)}}(\lambda_{i,k}^{(n)})^2\Bigg(\sum_{j=1}^{i-1}\prod_{l=j}^{i-1}\big(\lambda_{l,k}^{(n)}\sigma_{l,k}^{(n)}\big)^2 + 1\Bigg) \tag{18}$$

$$\text{s.t.} \quad \mathbf{q}^{(n)} \prec^{+(\mathrm{w})} \Bigg\{\Bigg(1 + \frac{\prod_{i=1}^{L}\big(\sigma_{i,k}^{(n)}\lambda_{i,k}^{(n)}\big)^2}{1 + \sum_{i=2}^{L}\prod_{l=i}^{L}\big(\sigma_{l,k}^{(n)}\lambda_{l,k}^{(n)}\big)^2}\Bigg)^{-1}\Bigg\}$$
$$1 \le n \le N_c \tag{19}$$

$$\lambda_{i,k}^{(n)} > 0, \quad 1 \le i \le L, 1 \le k \le N_b^{(n)}, 1 \le n \le N_c \tag{20}$$

where $\prec^{+(\mathrm{w})}$ denotes weakly additively supermajorization [10], and in (19) for a scalar x, $\big\{x_k^{(n)}\big\} \triangleq \Big[x_1^{(n)}, \cdots, x_{N_b^{(n)}}^{(n)}\Big]^T$.

To simplify notations, let us introduce the following variable substitutions for $1 \leq k \leq N_b^{(n)}$, $1 \leq n \leq N_c$

$$x_{1,k}^{(n)} \triangleq (\lambda_{1,k}^{(n)})^2, \quad a_{i,k}^{(n)} \triangleq (\sigma_{i,k}^{(n)})^2, \qquad 1 \leq i \leq L \quad (21)$$

$$x_{i,k}^{(n)} \triangleq (\lambda_{i,k}^{(n)})^2 \big(\big(a_{i-1,k}^{(n)} x_{i-1,k}^{(n)}\big)^2 + 1 \big), \quad 2 \leq i \leq L \,. \quad (22)$$

Applying (21) and (22) to (18)-(20) and expanding (19) using the definition of $\prec^{+(\mathrm{w})}$ in [10], the optimization problem (18)-(20) can be equivalently written as

$$\min_{\mathbf{x}} \sum_{n=1}^{N_c} \sum_{i=1}^{L} \sum_{k=1}^{N_b^{(n)}} x_{i,k}^{(n)} \quad (23)$$

$$\text{s.t.} \sum_{k=1}^{j} \left(1 - \prod_{i=1}^{L} \frac{a_{i,k}^{(n)} x_{i,k}^{(n)}}{1 + a_{i,k}^{(n)} x_{i,k}^{(n)}} \right) \leq \sum_{k=1}^{j} q_k^{(n)}$$

$$1 \leq j \leq N_b^{(n)}, \qquad 1 \leq n \leq N_c \quad (24)$$

$$x_{i,k}^{(n)} > 0, \quad 1 \leq i \leq L, 1 \leq k \leq N_b^{(n)}, 1 \leq n \leq N_c \quad (25)$$

where $\mathbf{x} \triangleq \left[x_{1,1}^{(1)}, \cdots, x_{L,N_b^{(N_c)}}^{(N_c)} \right]^T$. The constraints in (24) are nonconvex and difficult to handle. In the following, we provide two numerical methods to solve the problem (23)-(25). The first method is based on a successive application of geometric programming (GP) [11]. While the second method utilizes the dual decomposition technique [12].

For the first method, let us introduce an auxiliary variable vector $\mathbf{z} \triangleq \left[z_1^{(1)}, \cdots, z_{N_b^{(N_c)}}^{(N_c)} \right]^T$ with $z_k^{(n)} \leq \prod_{i=1}^{L} \frac{a_{i,k}^{(n)} x_{i,k}^{(n)}}{1+a_{i,k}^{(n)} x_{i,k}^{(n)}}$, $1 \leq k \leq N_b^{(n)}$, $1 \leq n \leq N_c$. Then the problem (23)-(25) can be equivalently written as

$$\min_{\mathbf{x},\mathbf{z}} \sum_{n=1}^{N_c} \sum_{i=1}^{L} \sum_{k=1}^{N_b^{(n)}} x_{i,k}^{(n)} \quad (26)$$

$$\text{s.t.} \sum_{k=1}^{j} z_k^{(n)} \geq \sum_{k=1}^{j} \big(1 - q_k^{(n)}\big)$$

$$1 \leq j \leq N_b^{(n)} \quad 1 \leq n \leq N_c \quad (27)$$

$$z_k^{(n)} \prod_{i=1}^{L} \frac{1 + a_{i,k}^{(n)} x_{i,k}^{(n)}}{a_{i,k}^{(n)} x_{i,k}^{(n)}} \leq 1$$

$$1 \leq k \leq N_b^{(n)}, \quad 1 \leq n \leq N_c \quad (28)$$

$$x_{i,k}^{(n)} > 0, \quad 1 \leq i \leq L, 1 \leq k \leq N_b^{(n)}, 1 \leq n \leq N_c \quad (29)$$

The objective function (26) is a posynomial, and the constraints in (28) are posynomial upper-bound constraints [11]. If the constraints in (27) can also be converted to posynomial upper-bound constraints, then the problem (26)-(29) becomes a GP problem. Towards this end, we apply the geometric inequality to the left-hand side of (27) such that

$$\sum_{k=1}^{j} z_k^{(n)} \geq \prod_{k=1}^{j} \left(\frac{z_k^{(n)}}{\alpha_{j,k}^{(n)}} \right)^{\alpha_{j,k}^{(n)}}, \qquad 1 \leq n \leq N_c \quad (30)$$

where $\alpha_{j,k}^{(n)} > 0$, and $\sum_{k=1}^{j} \alpha_{j,k}^{(n)} = 1, 1 \leq j \leq N_b^{(n)}$, $1 \leq n \leq N_c$. Substituting (27) with the inequalities $\prod_{k=1}^{j} \left(\frac{z_k^{(n)}}{\alpha_{j,k}^{(n)}} \right)^{\alpha_{j,k}^{(n)}} \geq \sum_{k=1}^{j} \big(1 - q_k^{(n)}\big)$, we have

$$\min_{\mathbf{x},\mathbf{z}} \sum_{n=1}^{N_c} \sum_{i=1}^{L} \sum_{k=1}^{N_b^{(n)}} x_{i,k}^{(n)} \quad (31)$$

$$\text{s.t.} \quad \beta_j^{(n)} \prod_{k=1}^{j} \big(z_k^{(n)}\big)^{-\alpha_{j,k}^{(n)}} \leq 1$$

$$1 \leq j \leq N_b^{(n)}, 1 \leq n \leq N_c \quad (32)$$

$$z_k^{(n)} \prod_{i=1}^{L} \left(1 + \big(a_{i,k}^{(n)} x_{i,k}^{(n)}\big)^{-1} \right) \leq 1$$

$$1 \leq k \leq N_b^{(n)}, 1 \leq n \leq N_c \quad (33)$$

$$x_{i,k}^{(n)} > 0, \quad 1 \leq i \leq L, 1 \leq k \leq N_b^{(n)}, 1 \leq n \leq N_c \quad (34)$$

where

$$\beta_j^{(n)} \triangleq \sum_{k=1}^{j} \big(1 - q_k^{(n)}\big) \prod_{k=1}^{j} \big(\alpha_{j,k}^{(n)}\big)^{\alpha_{j,k}^{(n)}} . \quad (35)$$

The problem (31)-(34) is a GP in standard form, which can be converted to a convex optimization problem and efficiently solved by the interior-point method [11].

The procedure of applying the successive GP approach to solve the problem (23)-(25) is summarized in Table I. Here ε is a small positive number close to zero and the variable $[m]$ denotes the number of iterations.

TABLE I
PROCEDURE OF THE SUCCESSIVE GP APPROACH.

1) Initialize the algorithm at a feasible $\mathbf{x}[0]$; Set $m = 0$.
2) Compute $z_k^{(n)}[m] = \prod_{i=1}^{L} \frac{a_{i,k}^{(n)} x_{i,k}^{(n)}[m]}{1+a_{i,k}^{(n)} x_{i,k}^{(n)}[m]}$, $1 \leq k \leq N_b^{(n)}$, $\alpha_{j,k}^{(n)}[m] = \frac{z_k^{(n)}[m]}{\sum_{k=1}^{j} z_k^{(n)}[m]}$ and $\beta_j^{(n)}[m]$ in (35), $1 \leq j \leq N_b^{(n)}$. Obtain $\mathbf{x}[m+1]$ by solving the standard GP problem (31)-(34).
3) If $\max_{i,k,n} \left| x_{i,k}^{(n)}[m+1] - x_{i,k}^{(n)}[m] \right| \leq \varepsilon$, then end. Otherwise, let $m := m + 1$ and go to step 2).

We use the MOSEK convex optimization MATLAB toolbox [13] to solve the problem (31)-(34). Note that since the constraints in (32) are stricter than those in (27), the solution to the problem (26)-(29) is upper-bounded by that of the problem (31)-(34). However, when the successive GP procedure converges, i.e., when $\mathbf{x}[m] \doteq \mathbf{x}[m+1]$, equality holds in (30). Thus, the problem (26)-(29) and the problem (31)-(34) are equivalent at the convergence of the successive GP procedure.

If we take a closer look at the problem (23)-(25), it can be seen that the variables $x_{i,k}^{(n)}$ are coupled only through the summations in (23) and (24). Such structure facilitates the application of the dual decomposition technique. First, the Lagrangian function associated with the problem (23), (24) can be written as

$$\mathcal{L}(\mathbf{x},\boldsymbol{\mu}) = \sum_{n=1}^{N_c}\sum_{i=1}^{L}\sum_{k=1}^{N_b^{(n)}} x_{i,k}^{(n)} + \sum_{n=1}^{N_c}\sum_{j=1}^{N_b^{(n)}} \mu_j^{(n)} \times \left(\sum_{k=1}^{j}\left(1-\prod_{i=1}^{L}\frac{a_{i,k}^{(n)}x_{i,k}^{(n)}}{1+a_{i,k}^{(n)}x_{i,k}^{(n)}} - q_k^{(n)}\right)\right) \triangleq \sum_{n=1}^{N_c}\sum_{k=1}^{N_b^{(n)}} \mathcal{L}_k\big(\mathbf{x}_k^{(n)},\tilde{\mu}_k^{(n)}\big) + \eta \quad (36)$$

where $\mu_j^{(n)} \geq 0, 1 \leq j \leq N_b^{(n)}$, $1 \leq n \leq N_c$, are the Lagrangian multipliers, $\boldsymbol{\mu} \triangleq \left[\mu_1^{(1)},\cdots,\mu_{N_b^{(N_c)}}^{(N_c)}\right]^T$, $\tilde{\mu}_k^{(n)} \triangleq \sum_{j=k}^{N_b^{(n)}} \mu_j^{(n)}$, $\eta \triangleq \sum_{n=1}^{N_c}\sum_{k=1}^{N_b^{(n)}} \tilde{\mu}_k^{(n)}\big(1-q_k^{(n)}\big)$, $\mathbf{x}_k^{(n)} \triangleq \big[x_{1,k}^{(n)},\cdots,x_{L,k}^{(n)}\big]^T$, and

$$\mathcal{L}_k\big(\mathbf{x}_k^{(n)},\tilde{\mu}_k^{(n)}\big) \triangleq \sum_{i=1}^{L} x_{i,k}^{(n)} - \tilde{\mu}_k^{(n)}\prod_{i=1}^{L}\frac{a_{i,k}^{(n)}x_{i,k}^{(n)}}{1+a_{i,k}^{(n)}x_{i,k}^{(n)}}. \quad (37)$$

The dual function [11] associated with the original problem (23)-(25) is given by

$$g(\boldsymbol{\mu}) = \min_{\mathbf{x}} \mathcal{L}(\mathbf{x},\boldsymbol{\mu}).$$

Now the optimization of $\mathbf{x}$ is carried out in two levels. At the lower level, we solve for $\mathbf{x}_k^{(n)}$, $1 \leq k \leq N_b^{(n)}$, from the following decoupled subproblem with given $\tilde{\mu}_k^{(n)}$.

$$\min_{\mathbf{x}_k^{(n)}} \quad \mathcal{L}_k\big(\mathbf{x}_k^{(n)},\tilde{\mu}_k^{(n)}\big) \qquad \text{s.t.} \quad x_{i,k}^{(n)} > 0, \qquad i = 1,\cdots,L. \quad (38)$$

At the higher level, we update the dual variable $\boldsymbol{\mu}$ by solving the master dual problem

$$\max_{\boldsymbol{\mu}} \quad g(\boldsymbol{\mu}) \qquad \text{s.t.} \quad \mu_i^{(n)} \geq 0, \qquad 1 \leq i \leq N_b^{(n)}, \quad 1 \leq n \leq N_c. \quad (39)$$

The problem (39) can be solved by the sub-gradient method [12].

The procedure of applying the dual decomposition technique to solve the problem (23)-(25) is summarized in Table II. Note that the constraints in (24) are absorbed into the Lagrangian function (36), and when this algorithm converges, (24) is always satisfied. Since the dual decomposition method essentially solves the dual optimization problem, the result we obtain is a lower-bound of the original problem (23)-(25). Interestingly, it will be shown in Section V that although the successive GP and the dual decomposition approaches provide an upper-bound and a lower-bound of the problem (23)-(25), respectively, their performance are almost identical. Thus, in practice, we can use either of the two methods.

TABLE II
PROCEDURE OF THE DUAL DECOMPOSITION APPROACH.

1) Initialize the algorithm at a feasible $\boldsymbol{\mu}[0]$; Set $m = 0$.
2) Solve the subproblems (38) $1 \leq k \leq N_b^{(n)}$, $1 \leq n \leq N_c$, using $\boldsymbol{\mu}[m]$, to obtain $\mathbf{x}[m]$.
3) Solve the master problem (39) with $\mathbf{x}[m]$ to obtain $\boldsymbol{\mu}[m+1]$
4) If $\max_{i,n}\left|\mu_i^{(n)}[m+1]-\mu_i^{(n)}[m]\right| \leq \varepsilon$, then end. Otherwise, let $m := m+1$ and go to step 2).

IV. Subcarrier-Cooperative MIMO Relay Systems

In this section, we derive the optimal structure of $\mathbf{F}_i$ for subcarrier-cooperative multi-hop MIMO relay systems. Based on the block-diagonal structure of (7) we can write the SVD of $\mathbf{H}_i$ as

$$\mathbf{H}_i = \mathbf{U}_i\boldsymbol{\Sigma}_i\mathbf{V}_i^H \quad (40)$$

where

$$\boldsymbol{\Sigma}_i = \mathrm{bd}\big(\boldsymbol{\Sigma}_i^{(1)},\boldsymbol{\Sigma}_i^{(2)},\cdots,\boldsymbol{\Sigma}_i^{(N_c)}\big)$$
$$\mathbf{U}_i = \mathrm{bd}\big(\mathbf{U}_i^{(1)},\mathbf{U}_i^{(2)},\cdots,\mathbf{U}_i^{(N_c)}\big)$$
$$\mathbf{V}_i = \mathrm{bd}\big(\mathbf{V}_i^{(1)},\mathbf{U}_i^{(2)},\cdots,\mathbf{V}_i^{(N_c)}\big).$$

Note that although the main diagonal elements of $\boldsymbol{\Sigma}_i^{(n)}$, $1 \leq n \leq N_c$, are ordered, the main diagonal elements of $\boldsymbol{\Sigma}_i$ remain unsorted. Let us introduce permutation matrices $\boldsymbol{\Pi}_{i,1}$ and $\boldsymbol{\Pi}_{i,2}$, $1 \leq i \leq L$, with commensurate dimensions such that the main diagonal elements of $\tilde{\boldsymbol{\Sigma}}_i \triangleq \boldsymbol{\Pi}_{i,1}\boldsymbol{\Sigma}_i\boldsymbol{\Pi}_{i,2}$, $1 \leq i \leq L$, are sorted in the increasing order. We can rewrite (40) as $\mathbf{H}_i = \tilde{\mathbf{U}}_i\tilde{\boldsymbol{\Sigma}}_i\tilde{\mathbf{V}}_i^H$, where $\tilde{\mathbf{U}}_i \triangleq \mathbf{U}_i\boldsymbol{\Pi}_{i,1}^T$, $\tilde{\mathbf{V}}_i \triangleq \mathbf{V}_i\boldsymbol{\Pi}_{i,2}$.

Based on Theorem 1, the optimal $\{\mathbf{F}_i\}$ jointly diagonalize the "super" multi-hop relay channel $\bar{\mathbf{H}}$ after a rotation of $\mathbf{F}_1$. Therefore, their optimal structure is given by

$$\mathbf{F}_1 = \tilde{\mathbf{V}}_{1,1}\boldsymbol{\Lambda}_1\mathbf{U}_0, \quad \mathbf{F}_i = \tilde{\mathbf{V}}_{i,1}\boldsymbol{\Lambda}_i\tilde{\mathbf{U}}_{i-1,1}^H, \quad 2 \leq i \leq L \quad (41)$$

where $\boldsymbol{\Lambda}_i$ are $J \times J$ diagonal matrices, $\tilde{\mathbf{V}}_{i,1}$ and $\tilde{\mathbf{U}}_{i,1}$ contain the rightmost J columns from $\tilde{\mathbf{V}}_i$ and $\tilde{\mathbf{U}}_i$, respectively, and $\mathbf{U}_0$ is a $J \times J$ unitary rotation matrix.

From (41) we find that the cooperation among subcarriers is essentially carried out by the permutation matrices $\boldsymbol{\Pi}_{i,1}$, $\boldsymbol{\Pi}_{i,2}$, $1 \leq i \leq L$. In fact, the subcarriers are reshuffled at each node such that the strong space-frequency subchannels at each link are paired together, while the weak subchannels are coupled with weak ones. The optimality of such pairing has been shown in [5] for a two-hop MIMO relay system without any QoS constraints. Here we have generalized this result to multicarrier multi-hop relay systems with QoS constraints.

After the optimal structure of $\mathbf{F}_i$ is determined, we are left with the optimization of $\{\boldsymbol{\Lambda}_i\}$, which can be efficiently solved by using either the successive GP algorithm or the dual decomposition approach in Section III.

From the computational complexity point of view, performing SVD and calculating the power loading parameters are the two most computationally intensive parts of the proposed algorithm. By exploiting the block-diagonal feature of $\mathbf{F}_i$, the complexity of SVD for the subcarrier-cooperative system is equivalent to that of the subcarrier-independent system. However, since for a subcarrier-independent system, optimization of power loading parameters are decomposed into N_c subproblems, thus it has a lower computational complexity than the subcarrier-cooperative system. On the other hand, as mentioned in Section II-B, the subcarrier-cooperative relay system has a better performance than the subcarrier independent one. Such a performance-complexity tradeoff is very useful for practical systems.

V. Numerical Examples

In the simulations, the channel between each transmit-receive antenna pair at each hop is modelled as the ETSI "Vehicular A" multipath channel environment. An OFDM communication system with $N_c = 64$ subcarriers is assumed. We assume that $N_i = N_b^{(n)} = N, 1 \leq i \leq L$, $1 \leq n \leq N_c$, and $q_k^{(n)} = q, 1 \leq k \leq N$, $1 \leq n \leq N_c$. The MIMO channel matrices $\mathbf{H}_i^{(n)}$ have i.i.d. complex Gaussian entries with zero mean and normalized variance $1/N$, $1 \leq i \leq L$. All simulation results are averaged over 1000 channel realizations. For simplicity, we only simulate subcarrier-independent systems.

In the first example, we compare the performance of the successive GP approach and the dual decomposition (DD) technique. We simulate a 2-hop ($L = 2$) relay system. Table III shows the performance of both algorithms in terms of total transmission power (dB) versus MSE. It can be seen that the dual decomposition technique is only slightly better than the successive GP approach. Since the former approach provides a lower-bound and the latter approach establishes an upper-bound for the system performance, the results in Table III indicate that both bounds are tight. Thus either of the approaches can be applied to solve the original optimization problem. However, since $\mathcal{L}_k\left(\mathbf{x}_k^{(n)}, \tilde{\mu}_k^{(n)}\right)$ in (37) is nonconvex with respect to $\mathbf{x}_k^{(n)}$, solving the subproblems (38) has a higher computational complexity than solving the GP problem (31)-(34), which can be converted to a convex optimization problem and efficiently solved by the interior-point method. Thus, in practice, the successive GP approach is preferred. In the following example, for clarity, we only show the performance of the successive GP approach.

TABLE III
Example 1: Comparison of the successive GP and the dual decomposition approaches; $N = 3$, $L = 2$.

MSE (q)	0.9	0.65	0.5	0.25	0.1
GP (dB)	19.0668	28.1708	32.2859	42.2742	48.5619
DD (dB)	19.0667	28.1274	32.2724	42.2699	48.5614

In the second example, we compare the performance of the MIMO relay system using the optimal source and relay matrices and the system using a suboptimal scheme in [8]. We choose $N = 3$, $L = 3$, and identical MSE requirement at all streams. From Fig. 1 we find that the system using the optimal scheme tremendously outperform the suboptimal scheme, especially at high MSEs.

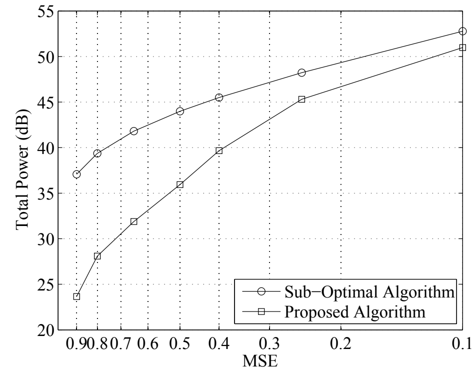

Fig. 1. Example 2: Total power versus MSE (q); $N = 3$, $L = 3$.

VI. Conclusions

We derived the optimal source and relay matrices for multicarrier multi-hop MIMO relay systems with QoS constraints using the linear MMSE receiver. The successive GP approach and the dual decomposition technique were used to solve the optimization problem. We found that at the same MSE level, the proposed system requires much less power than the suboptimal system.

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