On Average One Bit Per Subcarrier Channel State Information Feedback in OFDM Wireless Communication Systems

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Abstract— **In the orthogonal frequency division multiplexing (OFDM) scheme, some subcarriers may be subject to a deep fading. Adaptive techniques can be applied to mitigate this effect if the channel state information (CSI) is available at the transmitter. In this paper, we study the performance of an OFDM-based communication system whose transmitter has only one bit of CSI per subcarrier that is obtained through a low rate feedback. Three adaptive approaches are considered to exploit such a CSI feedback: adaptive subcarrier selection, adaptive power allocation and adaptive modulation selection. Under the condition of constant raw data rate, the performances of these approaches are analyzed and compared in terms of raw bit error rate (BER). We have found that one-bit CSI feedback can greatly enhance the system performance. Among the three approaches, adaptive subcarrier selection approach is found to have the lowest BER when the feedback is perfect.**

I. Introduction

An important advantage of the OFDM communication scheme is that, due to the inverse fast Fourier transform (IFFT) at the transmitter and the fast Fourier transform (FFT) at the receiver, the frequency selective fading channel is converted into parallel flat fading channels [1], [2]. However, the OFDM approach can suffer from fading that may affect some subcarriers. This makes a reliable detection of the information-bearing symbols at these particular subcarriers very difficult. Therefore, the overall performance of the system may degrade in this case.

One of the recent approaches to mitigate the effect of fading in OFDM uses error correction coding across the subcarriers [1]. Furthermore, if some CSI is available at the transmitter, adaptive modulation and resource allocation techniques can be applied to allocate bits and transmitted powers to subcarriers [3], [4]. However, in cellular communications it can be difficult to obtain CSI at the transmitter. If time division duplex (TDD) is used as the duplex mode and the reciprocity property between the uplink and the downlink channels holds, the downlink transmit CSI can be obtained by estimating the uplink channel. However, in practical situations fast channel variability and user mobility may not enable to use the reciprocity property. Moreover, the reciprocity property does not hold if the frequency division duplex (FDD) mode is used. In the latter case, some feedback has to be exploited to transmit the downlink CSI from the mobile station (MS) to the base station (BS). As the bandwidth consumed by the feedback channel is proportional to the feedback rate, it is interesting to study the performance of a wireless communication system which enables only a low-rate CSI feedback. For example, the use of one-bit channel state feedback in Alamouti-type systems has been studied in [5], while the asymptotic lower bound on the minimum feedback rates for multicarrier transmission has been derived in [6].

In this paper, we study the performance of the OFDM wireless communication system with one bit per subcarrier CSI feedback. We consider the uncoded transmission and use the raw bit error rate (BER) as the criterion to evaluate the performance of the system. Three adaptive approaches including adaptive subcarrier selection, adaptive power allocation and adaptive modulation selection are used to exploit the CSI feedback and compared via computer simulations. Moreover, for the latter two approaches, we derive a closed-form expression for the BER and, based on it, optimize their parameters.

II. System Model

We consider the point-to-point downlink cellular communication mode, where both the BS and the MS have one antenna. The frequency selective wireless channel between the BS and the MS is characterized by the path gains h_l ($l = 1, \cdots, L$) and the delays τ_l ($l = 1, \cdots, L$), where all path gains are assumed to be independent (but not necessarily identically distributed) complex Gaussian random variables with zero-mean and variance σ_l^2. We also assume that N subcarriers are used. The tth block of information-bearing symbols $\boldsymbol{s}(t) = [s(tN), \cdots, s(tN + N - 1)]^T$ is IFFT-modulated and the cyclic prefix (CP) is inserted to form one OFDM symbol. It is assumed that the length of the CP is longer than the maximum path delay τ_l, ($l = 1, \cdots, L$). Finally, the symbol is pulse-shaped and transmitted through the channel. The channel is assumed to be constant during one OFDM symbol transmission time. Hereafter, for notational simplicity we omit block index in $\boldsymbol{s}$.

After removing the CP, the received $N \times 1$ signal block $\boldsymbol{y}$ at the MS can be written as

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{F}^H\boldsymbol{P}^{1/2}\boldsymbol{s} + \tilde{\boldsymbol{v}} \tag{1}$$

where $\boldsymbol{P}$ is the $N \times N$ diagonal matrix of the transmit power of the symbol from different subcarriers, $\boldsymbol{F}$ is the $N \times N$ normalized FFT matrix with $\boldsymbol{F}_{i,l} = (1/\sqrt{N})\exp(-j2\pi(i-1)(l-1)/N)$, $j = \sqrt{-1}$, $\boldsymbol{H}$ is the $N \times N$ circulant channel matrix between the MS and BS with its (k,l)-th entry given by $h_{(k-l+1)\mathrm{mod}N}$, and $\tilde{\boldsymbol{v}}$ is the $N \times 1$ vector of additive white Gaussian noise (AWGN) at the MS whose variance is σ_v^2. After the FFT operation, the $N \times 1$ output symbol vector $\boldsymbol{r}$ can be written as

$$\boldsymbol{r} = \boldsymbol{F}\boldsymbol{y} \tag{2}$$

Inserting (1) into (2) and using the fact that $\boldsymbol{F}\boldsymbol{H}\boldsymbol{F}^H = \boldsymbol{D}$ where $\boldsymbol{D} = \mathrm{diag}(d_1, d_2, \cdots, d_N)$ is the diagonal matrix, the received symbol block can be written as

$$\boldsymbol{r} = \boldsymbol{D}\boldsymbol{P}^{1/2}\boldsymbol{s} + \boldsymbol{v} \tag{3}$$

where $\boldsymbol{v} = \boldsymbol{F}\tilde{\boldsymbol{v}}$ with $\mathrm{E}\{\boldsymbol{v}\boldsymbol{v}^H\} = \sigma_v^2 \boldsymbol{I}_N$, and $\boldsymbol{I}_N$ is the $N \times N$ identity matrix.

We assume that the BS transmits at the constant data rate of n_r bits per second (bps) and N subcarriers are used. We also assume that the BS has perfect knowledge of the signal-to-noise ratio (SNR) and the MS has perfect downlink CSI knowledge. The downlink CSI is transmitted back to the BS through a low-rate feedback channel. Altogether N bits containing the CSI for all subcarriers are transmitted to the BS in one feedback cycle (i.e., one bit per subcarrier). In this paper we also assume that the feedback channel is perfect, i.e., there is no feedback error and/or feedback delay. Note that the impact of imperfect one bit per subcarrier CSI on adaptive OFDM wireless communication systems is addressed in [7].

III. One Bit Per Subcarrier Feedback

In this section, we study several efficient ways to make use of N bits (i.e., one bit per subcarrier) of the CSI feedback. Clearly, it is impossible to provide a sufficiently accurate CSI feedback to the BS with only N bits. To illustrate this fact, we note that in cellular communications, the order of the multipath channel can be about $L = 10$ [8], and the typical choice of the number of subcarriers is $N = 52$ [1]. Assuming that 16 bits are used to represent a real-valued number, 320 bits are required to feedback the full CSI and, therefore, more than six bits of feedback per subcarrier (or, equivalently, more than $6N$ bits in total) are required in this case. Thus, the question how to make use of only one feedback bit per subcarrier in an efficient way is of great importance and interest.

A. Adaptive Subcarrier Selection

As it has been mentioned above, in OFDM communications some subcarriers may suffer from deep fading. The idea of subcarrier selection is that subcarriers which are affected by such a deep fading should be excluded and only subcarriers with high channel gains should be used.

The feedback in the system with adaptive subcarrier selection is organized in the following way. The MS sorts the channel gains in all N subcarriers and picks R subcarriers with the highest channel gains. If some subcarrier has been selected, 1 is transmitted back to the BS, otherwise 0 is transmitted to indicate that this particular subcarrier should be dropped. The BS equally distributes all the transmission power among the selected subcarriers. Note that in order to keep constant data rate for different numbers of selected subcarriers, different type of constellations have to be used.

In order to select optimal R, the theoretical analysis of probability of error is needed. However, such an analysis appears to be a very difficult task because of correlation between channel gains of different subcarriers. Thus, we limit our study of the adaptive subcarrier selection by simulations.

B. Adaptive Power Allocation

Besides subcarrier selection, the one bit per subcarrier CSI feedback can be used to adaptively allocate transmit powers according to the channel gain at each subcarrier under the constraint that the average transmit power per subcarrier is fixed. In the practical (sufficiently high) SNR range, more power should be allocated to faded subcarriers and less power should be allocated to non-faded ones to minimize the BER [3]. However, as we will see below, at low SNRs the situation may be reversed, that is, the BER is minimized when more power is allocated to non-faded subcarriers with high channel gains.

With one bit CSI feedback per subcarrier, adaptive power allocation can be done in the following way. If the channel gain of some subcarrier is below a certain threshold κ, the feedback bit 0 is transmitted to the BS and, in this case, the BS allocates transmission power γ_1 to this particular subcarrier. Otherwise, the feedback bit 1 is transmitted to the BS and it allocates the transmission power γ_2 to this subcarrier.

In what follows, we present a theoretical study of the average BER and optimize the discussed power allocation scheme.

The channel gain d_n $(n = 1, \cdots, N)$ of the nth subcarrier is given by $d_n = \sum_{l=1}^{L} h_l e^{-j2\pi n \tau_l / NT}$, $n = 1, \cdots, N$, where T is the sampling interval. It is easy to prove that d_n is a zero-mean complex Gaussian random variable with the variance of $\sum_{l=1}^{L} \sigma_l^2$. For the sake of simplicity, we normalize the variance of the channel gain at each subcarrier so that $\sum_{l=1}^{L} \sigma_l^2 = 1$. It can be seen that $d_1, \ldots, d_N$ all have identical distributions. The absolute value of each d_n is Rayleigh-distributed with the pdf

$$p(\alpha) = 2\alpha \exp(-\alpha^2) \tag{4}$$

The exact symbol error rate (SER) in the case of M-PSK modulation can be calculated as [9]

$$P_s(\mathrm{MPSK}) = \frac{1}{\pi}\int_0^{\frac{M-1}{M}\pi}\int_0^{\infty} \exp\left(-\frac{g_{\mathrm{PSK}}\alpha^2 E_s}{\sin^2\phi\ \sigma_v^2}\right) p(\alpha)\, d\alpha\, d\phi \tag{5}$$

where E_s is the transmitted signal power, and $g_{\mathrm{PSK}} = \sin^2(\pi/M)$. If the Gray mapping is used to map bits to symbols, the BER can be approximated as [10]

$$P_b \approx \frac{1}{\log_2 M} P_s \tag{6}$$

For example, inserting $M = 4$ into (5) and using (6), we obtain that in the QPSK modulation case,

$$P_b(\text{QPSK}) = \frac{1}{2\pi}\int_0^{\frac{3}{4}\pi}\int_0^{\infty}\exp\left(-\frac{\alpha^2 E_s}{2\sin^2\phi\,\sigma_v^2}\right)p(\alpha)\,d\alpha\,d\phi \quad (7)$$

In this case, the BER of the adaptive power allocation with one bit per subcarrier feedback can be calculated as

$$\begin{aligned} &P_b^{PA}(\text{QPSK}, \kappa, \gamma_1, \gamma_2) \\ &= \frac{1}{2\pi}\left[\int_0^{\frac{3}{4}\pi}\int_0^{\kappa}\exp\left(-\frac{\alpha^2\gamma_1 E_s}{2\sin^2\phi\,\sigma_v^2}\right)p(\alpha)\,d\alpha\,d\phi\right. \\ &\left.+\int_0^{\frac{3}{4}\pi}\int_{\kappa}^{\infty}\exp\left(-\frac{\alpha^2\gamma_2 E_s}{2\sin^2\phi\,\sigma_v^2}\right)p(\alpha)\,d\alpha\,d\phi\right] \quad (8)\end{aligned}$$

where γ_1 denotes the normalized transmission power when the value of the channel gain lies in the interval $[0, \kappa)$ and γ_2 denotes the normalized transmission power when the value of the channel gain lies in the interval $[\kappa, \infty)$.

Let us now obtain the optimal threshold κ and optimal power allocations γ_1 and γ_2 which minimize (8) subject to the average and peak transmit power constraints. Such optimal values of κ, γ_1, and γ_2 can be found as the solution to the following constrained optimization problem

$$\begin{aligned} &\min_{\kappa,\gamma_1,\gamma_2} P_b^{PA}(\text{QPSK}, \kappa, \gamma_1, \gamma_2) \\ &\text{s.t.} \quad \int_0^{\kappa}\gamma_1 p(\alpha)\,d\alpha + \int_{\kappa}^{\infty}\gamma_2 p(\alpha)\,d\alpha = 1 \quad (9) \\ &\qquad 0 < \gamma_1 < \gamma_M, \quad 0 < \gamma_2 < \gamma_M, \quad \kappa > 0 \end{aligned}$$

where γ_M denotes the normalized maximum transmission power which is determined by the transmission hardware peak power. Inserting (4) into (9), we see that the objective function is a highly nonlinear function of κ, γ_1, and γ_2. To solve the problem (9), the method of [11] can be used. The idea of this method is to quantize the parameters κ, γ_1, γ_2 and obtain the suboptimal solution using standard dynamic programming technique.

Let us now consider the effect of correlation of the channel gains between subcarriers. Using the expression for d_n, this correlation can be computed as $\mathrm{E}\{d_i d_k^*\} = \sum_{l=1}^{L}\sigma_l^2 e^{-j2\pi(i-k)\tau_l/NT}$. From the latter expression it follows that the channel gains of adjacent subcarriers are highly correlated. This fact can be exploited in the following way. We can provide CSI feedback for every other subcarrier (i.e., subcarriers with the indices $2, 4, 6, \ldots$) rather than for each subcarrier, i.e., the CSI feedback is required for $N/2$ subcarriers only. In this case, one can use 2 bits of feedback per subcarrier and still have N bits of feedback in total. Then four normalized transmission power levels γ_i $(i = 1, 2, 3, 4)$ and, correspondingly, three thresholds κ_l $(l = 1, 2, 3)$ should be used for adaptive power allocation. In this case, the BER can be computed as

$$\begin{aligned} &P_b^{PA}(\text{QPSK}, \boldsymbol{\kappa}, \boldsymbol{\gamma}) \\ &= \frac{1}{2\pi}\sum_{i=1}^{4}\int_0^{\frac{3}{4}\pi}\int_{\Omega_i}\exp\left(-\frac{\alpha^2\gamma_i E_s}{2\sin^2\phi\,\sigma_v^2}\right)p(\alpha)\,d\alpha\,d\phi \end{aligned}$$

TABLE I

Optimal Parameters of Conventional Adaptive Power Allocation

SNR (dB)	0	5	10	15	20	25
κ	0.4724	1.1774	0.7147	0.4724	0.3246	0.2265
γ_1	0.2000	1.1000	1.6000	2.6000	4.6000	10.500
γ_2	1.2000	0.7000	0.6000	0.6000	0.6000	0.5000

TABLE II

Optimal Parameters of Modified Adaptive Power Allocation

SNR (dB)	κ_1	κ_2	κ_3	γ_1	γ_2	γ_3	γ_4
0	0.3654	0.6269	1.5174	0.10	0.70	1.30	1.0
5	0.2792	0.8936	1.3774	0.30	1.30	0.90	0.60
10	0.5049	0.7732	1.1362	2.00	1.10	0.70	0.40
15	0.2265	0.4031	0.7147	5.80	2.20	1.00	0.40
20	0.1591	0.3246	0.5972	16.30	2.70	0.90	0.30
25	0.1591	0.3246	0.7147	28.30	1.50	0.40	0.10

where $\boldsymbol{\kappa} = [\kappa_1, \kappa_2, \kappa_3]^T$ is the vector of the thresholds, $\boldsymbol{\gamma} = [\gamma_1, \gamma_2, \gamma_3, \gamma_4]^T$ is the vector of the normalized transmission powers, and $\Omega_i = [\kappa_{i-1}, \kappa_i)$, $(i = 1, \cdots, 4)$ are the channel gain intervals with $\kappa_0 = 0$ and $\kappa_4 = \infty$. Then, the optimal values of the vector parameters $\boldsymbol{\kappa}$ and $\boldsymbol{\gamma}$ can be found by solving the following constrained optimization problem:

$$\begin{aligned} &\min_{\boldsymbol{\kappa},\boldsymbol{\gamma}} P_b^{PA}(\text{QPSK}, \boldsymbol{\kappa}, \boldsymbol{\gamma}) \\ &\text{s.t.} \quad \sum_{i=1}^{4}\int_{\Omega_i}\gamma_i p(\alpha)\,d\alpha = 1 \quad (10) \\ &\qquad 0 < \gamma_i < \gamma_M, \quad i = 1, 2, 3, 4 \\ &\qquad 0 < \kappa_l < \infty, \quad l = 1, 2, 3 \end{aligned}$$

Table I shows the optimal parameters of the conventional adaptive power allocation scheme which uses one-bit feedback for all N subcarriers, while Table II shows the parameters $\boldsymbol{\kappa}$ and $\boldsymbol{\gamma}$ for the modified adaptive power allocation scheme when the correlation between adjacent subcarriers is exploited and two bits of feedback for every other subcarrier is provided. In these tables, the optimal values of parameters are obtained by solving problems (9) and (10).

Another important question is whether it is beneficial to reduce the total number of subcarriers but to increase the constellation dimension. For example, if the number of subcarriers is reduced twice (to $N/2$), then the same amount of information at the same rate can be transmitted by using the constellation whose dimension is four times higher than in the case of N subcarriers. For example, if the QPSK modulation has been used in the case of N subcarriers, then 16-QAM modulation should be used in the case of $N/2$ subcarriers to maintain the same data transmission rate.

The BER for M-QAM modulation can be computed as [9]

$$\begin{aligned} &P_b(\text{MQAM}) = \frac{1}{\log_2 M} \\ &\cdot\left[\frac{4}{\pi}\left(1-\frac{1}{\sqrt{M}}\right)\int_0^{\frac{\pi}{2}}\int_0^{\infty}\exp\left(-\frac{g_{\text{QAM}}\alpha^2 E_s}{\sin^2\phi\,\sigma_v^2}\right)p(\alpha)\,d\alpha\,d\phi \quad (11)\right. \end{aligned}$$

$$-\frac{4}{\pi}\left(1-\frac{1}{\sqrt{M}}\right)^2\int_0^{\frac{\pi}{4}}\int_0^{\infty}\exp\left(-\frac{g_{\rm QAM}\alpha^2 E_s}{\sin^2\phi\ \sigma_v^2}\right)p(\alpha)\,d\alpha\,d\phi\Bigg]$$

where $g_{\rm QAM} = 3/(2(M-1))$. Inserting $M = 16$ into (11), the BER of the OFDM scheme with adaptive power allocation that uses N bits of feedback, $N/2$ subcarriers, and 16-QAM modulation can be written as

$$P_b^{PA}(16{\rm QAM}) = \frac{1}{4}\left[\frac{3}{\pi}\sum_{i=1}^{4}\int_0^{\frac{\pi}{2}}\int_{\Omega_i}\exp\left(-\frac{0.1\alpha^2\gamma_i E_s}{\sin^2\phi\ \sigma_v^2}\right)p(\alpha)\,d\alpha\,d\phi - \frac{9}{4\pi}\sum_{i=1}^{4}\int_0^{\frac{\pi}{4}}\int_{\Omega_i}\exp\left(-\frac{0.1\alpha^2\gamma_i E_s}{\sin^2\phi\ \sigma_v^2}\right)p(\alpha)\,d\alpha\,d\phi\right] \quad (12)$$

Similar to (10), the parameters $\boldsymbol{\kappa}$ and $\boldsymbol{\gamma}$ of this scheme can be optimized by solving the following constrained optimization problem

$$\begin{aligned}&\min_{\boldsymbol{\kappa},\boldsymbol{\gamma}} P_b^{PA}(16{\rm QAM},\boldsymbol{\kappa},\boldsymbol{\gamma})\\ &\text{s.t.}\quad \sum_{i=1}^{4}\int_{\Omega_i}\gamma_i p(\alpha)\,d\alpha = 1 \qquad (13)\\ &\qquad 0<\gamma_i<\gamma_M,\quad i=1,2,3,4\\ &\qquad 0<\kappa_l<\infty,\quad l=1,2,3\end{aligned}$$

C. Adaptive Modulation

The adaptive modulation scheme is based on the following idea. When a certain subcarrier is corrupted by fading, smaller dimension constellation and more transmission power can be assigned for this particular subcarrier, while constellations of higher dimension and less transmission power can be assigned to the subcarriers whose channel gain is high. Similar to the case of adaptive subcarrier selection, a low-rate one bit per subcarrier feedback can be used to divide the subcarriers into two groups which receive different constellations and transmission powers.

For example, to achieve the data rate of 2 bps per subcarrier, we can use the BPSK modulation at faded subcarriers and the 8PSK modulation at non-faded subcarriers. In this case, the threshold ξ of the channel gain that should be used to divide subcarriers into "faded" and "non-faded" groups can be found by solving the following data rate constraint equation

$$\int_0^{\xi} p(\alpha)\,d\alpha + 3\int_{\xi}^{\infty} p(\alpha)\,d\alpha = 2 \qquad (14)$$

Using (4), we obtain from (14) that $\xi = \sqrt{\ln 2}$. Then, the BER for this particular case of the adaptive modulation scheme can be written as

$$P_b^{AM}(\gamma_1,\gamma_2) = \frac{1}{\pi}\left[\int_0^{\frac{\pi}{2}}\int_0^{\sqrt{\ln 2}}\exp\left(-\frac{\alpha^2\gamma_1 E_s}{\sin^2\phi\ \sigma_v^2}\right)p(\alpha)\,d\alpha\,d\phi + \frac{1}{3}\int_0^{\frac{7\pi}{8}}\int_{\sqrt{\ln 2}}^{\infty}\exp\left(-\frac{\sin^2(\pi/8)\alpha^2\gamma_2 E_s}{\sin^2\phi\ \sigma_v^2}\right)p(\alpha)\,d\alpha\,d\phi\right] \quad (15)$$

TABLE III
OPTIMUM PARAMETERS OF ADAPTIVE MODULATION

SNR (dB)	0	5	10	15	20	25
γ_1	1.2925	1.0945	1.0554	1.2629	1.5799	1.8049
γ_2	0.7075	0.9055	0.9446	0.7371	0.4201	0.1951

The following constrained optimization problem should be solved to obtain the optimum power allocation in this case

$$\begin{aligned}&\min_{\gamma_1,\gamma_2} P_b^{AM}(\gamma_1,\gamma_2)\\ &\text{s.t.}\quad \gamma_1+\gamma_2=2,\qquad 0<\gamma_1,\gamma_2<2 \qquad (16)\end{aligned}$$

Table III lists the values of normalized optimal power allocation for BPSK and 8PSK constellations, respectively.

Since all aforementioned parameters are calculated off-line, the proposed adaptive OFDM techniques have nearly the same complexity as conventional OFDM.

IV. SIMULATIONS

The channel model used in our simulations is based on the ETSI "Vehicular A" channel environment [8]. In all examples, we assume that the BS transmits at the fixed data rate of $n_r = 128$ bps and the available number of subcarriers is $N = 64$.

Example 1. Adaptive Subcarrier Selection. Three different system configurations are used in this example: where no subcarrier selection is used, where 32 "best" subcarriers are selected, and where 16 "best" subcarriers are selected. To keep the constant data rate in each configuration, we use the QPSK constellation for no subcarrier selection, 16-QAM constellation for selection of 32 subcarriers, and 256-QAM constellation for selection of 16 subcarriers. Figure 1 shows the performance for all three system configurations in terms of BER versus SNR.

From Figure 1, the tradeoff between the number of subcarriers and the modulation used can be seen. In particular, the adaptive selection of 32 subcarriers has the best performance among the techniques tested. It is worth noting that the adaptive selection of 16 subcarriers has much worse performance than that of 32 subcarriers and at low/moderate SNRs can even perform worse than the system configuration without subcarrier selection.

Example 2. Adaptive Power Allocation. Simulation results using optimal parameters obtained in Table I and Table II are shown in Figure 2. The theoretical BER obtained by solving (10) represents a lower bound because it assumes that there is full correlation between each pair of adjacent subcarriers. From this figure, it can be seen that the proposed low-rate feedback-based schemes outperform the conventional scheme where no feedback is used. There is only a slight difference in performance between the conventional and modified adaptive power allocation schemes.

Comparing the results of Figure 2 to Figure 1, we see that the adaptive power allocation is less efficient than the adaptive subcarrier selection. This is especially true at high SNRs.

Example 3. Adaptive Power Allocation with Reduced Number of Subcarriers. Figure 3 shows the BER versus SNR for

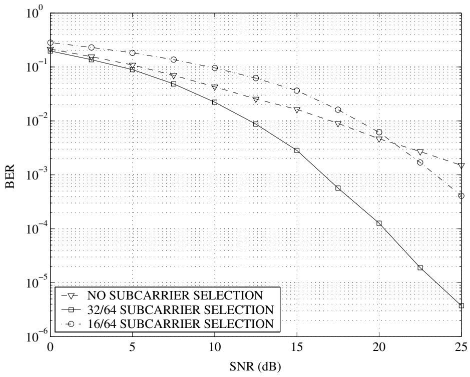

Fig. 1. BER versus SNR. First example.

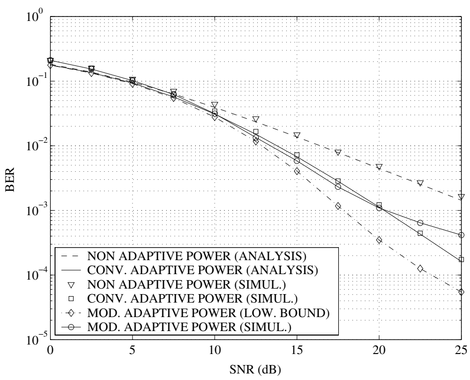

Fig. 2. BER versus SNR. Second example.

the adaptive power allocation scheme with reduced number of subcarriers. It can be seen that this scheme performs better than that without CSI feedback at moderate and high SNRs. However, it has higher BER than the conventional adaptive power allocation and the adaptive subcarrier selection schemes in the SNR interval of $[0, 20]$ dB. Note that due to the approximation (6), the theoretical and simulation curves do not coincide at low SNRs in the case when large dimensions of constellations are used.

Example 4. Adaptive Modulation. Figure 4 shows the BER versus SNR for adaptive modulation scheme. It can be seen that adaptive modulation scheme outperforms the scheme which does not use any feedback. However, it has higher BER than the adaptive power allocation and subcarrier selection schemes.

V. Conclusions

In this paper, we have studied the performance of OFDM wireless communications systems with average one bit per subcarrier CSI feedback. Three advanced approaches including adaptive subcarrier selection, adaptive power allocation, and adaptive modulation have been proposed to exploit such CSI feedback. We have found that even one bit CSI feedback can greatly improve the overall system performance. Among the three proposed approaches, adaptive subcarrier selection has the best performance.

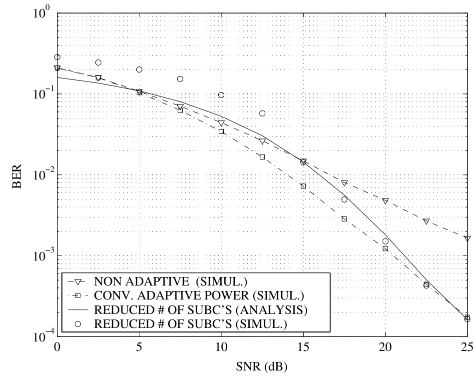

Fig. 3. BER versus SNR. Third example.

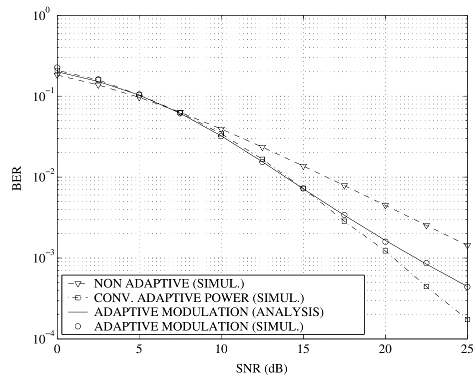

Fig. 4. BER versus SNR. Fourth example.

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