Multicasting MIMO Relay Optimization Based on Min-Max MSE Criterion

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***Abstract*—In this paper, we consider a multicasting multiple-input multiple-output (MIMO) relay system where the transmitter multicasts a common message to multiple receivers with the aid of a relay node, all equipped with multiple antennas. Given the power constraints at the source and the relay nodes, we aim at minimizing the maximal mean-squared error (MSE) of the signal waveform estimation among the destination nodes through joint source, relay, and receiver matrices optimization. We provide a low complexity solution to this highly nonconvex optimization problem. In particular, we show that under the (moderately) high signal-to-noise ratio (SNR) assumption, the joint source and relay optimization problem can be solved using standard semidefinite programming (SDP) technique. Numerical simulations provided demonstrate the effectiveness of the proposed algorithm.**

I. Introduction

Wireless multicasting technology has attracted much research interest recently, due to the increasing demand on mobile applications such as streaming media and location-based services involving group communications. The broadcasting nature of the wireless channel makes it naturally suitable for multicasting applications, since a single transmission may be simultaneously received by a number of users. However, the wireless channel is subject to signal fading. By exploiting the spatial diversity, multi-antenna techniques can provide significant improvement in spectral efficiency and link reliability in wireless systems. Next generation wireless standards such as WiMAX 802.16m and 3GPP LTE-Advanced have already included technologies which enable better multicasting solutions based on multi-antenna and beamforming techniques [1].

The information theoretic capacity of the multi-antenna multicasting channel has been studied in [2]. The effect of channel spatial correlation on the capacity performance has been investigated in [3]. The authors in [4] designed transmit beamformers for physical layer multicasting. In [5], the authors focused on transmit precoding design for multi-antenna multicasting systems where the channel state information (CSI) is obtained via limited feedback. The works in [4]-[5] solved the max-min signal-to-interference-plus-noise ratio (SINR)/rate beamforming problems with the aid of semidefinite relaxations (SDR). In [6], a stochastic beamforming strategy is proposed for multi-antenna physical-layer multicasting considering an achievable rate perspective where the randomization is guided by SDR.

While the works [2]-[7] focus on multicasting systems with single-antenna receiving nodes, recently multi-antenna receiving nodes have been considered in [8]-[10]. In particular, coordinated beamforming techniques have been investigated in [8] to facilitate physical layer multicasting with multi-antenna receivers. In [9], non-iterative near-optimal transmit beamformers are designed for wireless link layer multicasting with real-valued channels, and for complex-valued channels an upper bound on the multicasting rate is derived. The scaling of the achievable rate for the increasing number of users has been investigated in [10] for multiple-input multiple-output (MIMO) multicasting in which the transmission is coded at the application layer over a number of channel realizations.

The above works [2]-[10] consider single-hop multicasting systems. However, in the case of long source-destination distance, relay node(s) is necessary to efficiently combat the pathloss of wireless channel. In [11], the authors investigated multicast scheduling with multiple sessions and multiple channels where the base station may multicast data in two sessions using MIMO simultaneously through the same channel and the users are allowed to cooperatively help each other on orthogonal channels. The authors in [12] studied the lower bound for the outage probability of cooperative multiple antenna multicasting schemes based on amplify-and-forward (AF) strategy where the users are equipped with a single antenna.

In this paper, we consider a multicasting MIMO relay system where the transmitter multicasts a common message to multiple receivers with the aid of a relay node. The transmitter, relay, and receiving nodes are *all* equipped with multiple antennas. To the best of our knowledge, such two-hop multicasting MIMO relay system has not been investigated in existing works. For implementation simplicity, we choose the AF relaying strategy. We aim at jointly optimizing the source, relay, and receiver matrices to minimize the maximal mean-squared error (MSE) of the signal waveform estimation among all destination nodes. This optimization problem is highly nonconvex with matrix variables. We provide a low complexity solution for the problem under some mild approximation. It has been shown that the problem can be solved using standard semidefinite programming (SDP) techniques under

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(moderately) high signal-to-noise ration (SNR) assumption. In contrast to the existing works, the proposed algorithm supports multicasting multiple data streams. Numerical simulations are performed to demonstrate the effectiveness of the proposed algorithm. Note that in contrast to our system, the second-hop receivers are equipped with a single antenna in [11]-[12].

II. System Model

We consider a two-hop MIMO multicasting system with L receiving nodes as illustrated in Fig. 1. The source, relay, and destination nodes are equipped with N_s, N_r, and N_d antennas, respectively. The source node multicasts its information-carrying symbols to the destination nodes with the aid of a relay node. Moreover, the direct links between the source node and the destination nodes are not considered since we assume that these direct links undergo relatively larger path attenuations compared with the links via the relay node.

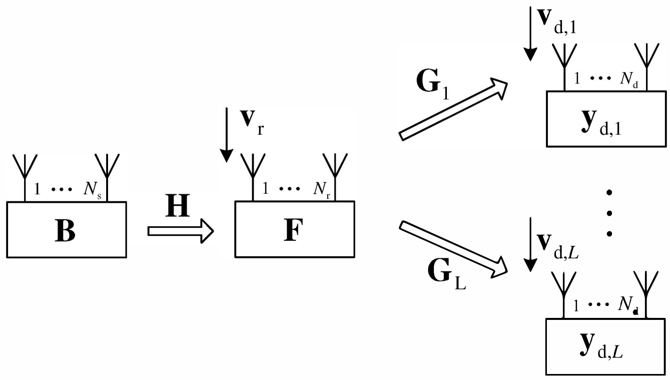

Fig. 1. Block diagram of a multicasting MIMO relay system.

We assume that the relay node works in half-duplex mode. Thus the communication between the source and destination nodes is accomplished in two time slots. In the first time slot, the source node linearly precodes an $N_\mathrm{b} \times 1$ ($N_\mathrm{b} \leq \min(N_\mathrm{s}, N_\mathrm{r}, N_\mathrm{d})$) modulated signal vector $\mathbf{s}$ (common message to all destination nodes) by the $N_\mathrm{s} \times N_\mathrm{b}$ source precoding matrix $\mathbf{B}$ and transmits the precoded vector $\mathbf{x} = \mathbf{Bs}$ to the relay node. We assume that $\mathrm{E}[\mathbf{ss}^H] = \mathbf{I}_{N_\mathrm{b}}$, where $\mathrm{E}[\cdot]$ denotes statistical expectation, $(\cdot)^H$ stands for the matrix Hermitian transpose, and $\mathbf{I}_n$ is an $n \times n$ identity matrix. The received signal vector at the relay node is given by

$$\mathbf{y}_\mathrm{r} = \mathbf{HBs} + \mathbf{v}_\mathrm{r} \tag{1}$$

where $\mathbf{H}$ is the $N_\mathrm{r} \times N_\mathrm{s}$ MIMO channel matrix between the source node and the relay node, $\mathbf{y}_\mathrm{r}$ and $\mathbf{v}_\mathrm{r}$ are the $N_\mathrm{r} \times 1$ received signal and additive Gaussian noise vectors introduced at the relay node, respectively.

In the second time slot, the source node remains silent and the relay node multiplies (linearly precodes) the received signal vector $\mathbf{y}_\mathrm{r}$ by an $N_\mathrm{r} \times N_\mathrm{r}$ relay amplifying matrix $\mathbf{F}$ and transmits the precoded signal vector $\mathbf{x}_\mathrm{r} = \mathbf{F}\mathbf{y}_\mathrm{r}$ to all destination nodes. Hence the received signal vector at the ith destination node can be written as

$$\begin{aligned}\mathbf{y}_{\mathrm{d},i} &= \mathbf{G}_i\mathbf{F}(\mathbf{HBs} + \mathbf{v}_\mathrm{r}) + \mathbf{v}_{\mathrm{d},i} \\ &\triangleq \mathbf{A}_i\mathbf{s} + \mathbf{n}_i, \qquad i = 1, \cdots, L\end{aligned} \tag{2}$$

where $\mathbf{G}_i$ is the $N_\mathrm{d} \times N_\mathrm{r}$ MIMO channel matrix between the relay node and the ith destination node and $\mathbf{v}_{\mathrm{d},i}$ is the additive Gaussian noise vector at the ith destination node. Here $\mathbf{A}_i \triangleq \mathbf{G}_i\mathbf{FHB}$ is the equivalent MIMO channel between the source node and the ith destination node, and $\mathbf{n}_i \triangleq \mathbf{G}_i\mathbf{F}\mathbf{v}_\mathrm{r} + \mathbf{v}_{\mathrm{d},i}$ is the equivalent noise vector at the ith destination node.

We assume that the channel matrices $\mathbf{H}$ and $\mathbf{G}_i, i = 1, \cdots, L$, are all quasi-static, i.e., the channel matrices are constant throughout a block of transmission. In practice, the CSI of $\mathbf{G}_i$ can be obtained at the ith destination node through standard training method. The relay node can have the CSI of $\mathbf{H}$ through channel training, and obtain the CSI of $\mathbf{G}_i, i = 1, \cdots, L$, by a feedback from ith receiving node. The quasi-static channel model is valid in practice when the mobility of all communicating nodes is relatively slow. Thus, we can obtain the necessary CSI with a reasonably high precision during the channel training period. The relay node calculates the optimal source matrix $\mathbf{B}$, and the relay matrix $\mathbf{F}$, and forwards $\mathbf{B}$ to the source node and forwards $\mathbf{F}$ and $\mathbf{H}$ to the destination nodes. Thus the source node does not need any channel knowledge and each destination node needs CSI of its own channel with the relay and that of the first-hop channel. This is a very important assumption for multicasting communication since in a multicasting scenario the users are distributed and cannot cooperate. We assume that all noises are independent and identically distributed (i.i.d.) complex circularly symmetric Gaussian noise with zero mean and unit variance.

We aim at improving the system performance through optimizing the source and relay matrices. System performance is usually quantified by its quality-of-service (QoS) and the resources it uses. The most common QoS metrics include MSE of the signal waveform estimation, bit-error-rate (BER), system capacity and output SINR. Interestingly, different QoS measures can always be expressed in terms of MSE [13]. In the next section, we optimize the source and relay matrices based on the min-max MSE criterion.

III. Proposed Source and Relay Design Algorithm

Due to its simplicity, a linear receiver is considered at each destination node to retrieve the transmitted signals. Denoting $\mathbf{W}_i$ as an $N_\mathrm{d} \times N_\mathrm{b}$ weight matrix at the ith receiver, the estimated signal vector $\hat{\mathbf{s}}_i$ is given by

$$\hat{\mathbf{s}}_i = \mathbf{W}_i^H\mathbf{y}_{\mathrm{d},i}, \qquad i = 1, \cdots, L. \tag{3}$$

From (3), the MSE of the signal waveform estimation at the ith receiver is given by

$$\begin{aligned}E_i &= \mathrm{tr}\big(\mathrm{E}\big[(\hat{\mathbf{s}}_i - \mathbf{s})(\hat{\mathbf{s}}_i - \mathbf{s})^H\big]\big) \\ &= \mathrm{tr}\big((\mathbf{W}_i^H\mathbf{A}_i - \mathbf{I}_{N_\mathrm{b}})(\mathbf{W}_i^H\mathbf{A}_i - \mathbf{I}_{N_\mathrm{b}})^H + \mathbf{W}_i^H\mathbf{C}_i\mathbf{W}_i\big)\end{aligned} \tag{4}$$

where $\mathrm{tr}(\cdot)$ denotes matrix trace and

$$\mathbf{C}_i \triangleq \mathrm{E}[\mathbf{n}_i\mathbf{n}_i^H] = \mathbf{G}_i\mathbf{F}\mathbf{F}^H\mathbf{G}_i^H + \mathbf{I}_{N_\mathrm{d}}$$

is the covariance matrix of $\mathbf{n}_i$.

We aim at minimizing the maximal MSE of the signal waveform estimations among all destination nodes, given the power constraints at the source and the relay nodes. Such problem formulation is important when the power constraint is a strict system restriction that cannot be relaxed. Since the source and relay transmit powers are given, respectively, by $\mathrm{tr}(\mathbf{F}(\mathbf{H}\mathbf{B}\mathbf{B}^H\mathbf{H}^H + \mathbf{I}_{N_\mathrm{r}})\mathbf{F}^H)$ and $\mathrm{tr}(\mathbf{B}\mathbf{B}^H)$, the optimization problem can be written as

$$\min_{\mathbf{B},\mathbf{F},\{\mathbf{W}_i\}} \quad \max_i \quad E_i \tag{5a}$$
$$\text{s.t.} \quad \mathrm{tr}(\mathbf{F}(\mathbf{H}\mathbf{B}\mathbf{B}^H\mathbf{H}^H + \mathbf{I}_{N_\mathrm{r}})\mathbf{F}^H) \le P_\mathrm{r} \tag{5b}$$
$$\mathrm{tr}(\mathbf{B}\mathbf{B}^H) \le P_\mathrm{s} \tag{5c}$$

where $\{\mathbf{W}_i\} \triangleq \{\mathbf{W}_i, i = 1, \cdots, L\}$, (5b) and (5c) are the transmission power constraints at the relay and the source nodes, respectively, and $P_\mathrm{r} > 0$, $P_\mathrm{s} > 0$ are the corresponding power budgets.

Obviously, for any given $\mathbf{B}$ and $\mathbf{F}$, the weight matrix $\mathbf{W}_i$ minimizing (4) is the Wiener filter and given by

$$\mathbf{W}_i = \left(\mathbf{A}_i\mathbf{A}_i^H + \mathbf{C}_i\right)^{-1}\mathbf{A}_i, \quad i = 1, \cdots, L \tag{6}$$

where $(\cdot)^{-1}$ denotes matrix inversion. By substituting (6) back into (4), we have

$$E_i = \mathrm{tr}\left(\left[\mathbf{I}_{N_\mathrm{b}} + \mathbf{A}_i^H\mathbf{C}_i^{-1}\mathbf{A}_i\right]^{-1}\right). \tag{7}$$

Therefore, we can equivalently rewrite the problem (5) as

$$\min_{\mathbf{B},\mathbf{F}} \quad \max_i \quad \mathrm{tr}\left(\left[\mathbf{I}_{N_\mathrm{b}} + \mathbf{A}_i^H\mathbf{C}_i^{-1}\mathbf{A}_i\right]^{-1}\right) \tag{8a}$$
$$\text{s.t.} \quad \mathrm{tr}(\mathbf{F}(\mathbf{H}\mathbf{B}\mathbf{B}^H\mathbf{H}^H + \mathbf{I}_{N_\mathrm{r}})\mathbf{F}^H) \le P_\mathrm{r} \tag{8b}$$
$$\mathrm{tr}(\mathbf{B}\mathbf{B}^H) \le P_\mathrm{s}. \tag{8c}$$

The min-max problem (8) is highly nonconvex with matrix variables, and a globally optimal solution is hard to obtain with a reasonable computational complexity (non-exhaustive searching). In the following, we propose a low complexity solution to the problem (8).

It can be shown similar to [14] that for any $\mathbf{B}$, the optimal $\mathbf{F}$ for *each user* has the generic structure of $\mathbf{F} = \mathbf{T}\mathbf{D}^H$, where $\mathbf{D} = (\mathbf{H}\mathbf{B}\mathbf{B}^H\mathbf{H}^H + \mathbf{I}_{N_\mathrm{r}})^{-1}\mathbf{H}\mathbf{B}$. Interestingly, $\mathbf{D}$ can be viewed as the weight matrix of the MMSE receiver for the first-hop MIMO channel at the relay node given by (1) and $\mathbf{T}$ can be viewed as the transmit precoding matrix for the effective second-hop MIMO multicasting system $\mathbf{y}_i = \mathbf{G}_i\mathbf{T}\mathbf{x} + \mathbf{v}_i$, where $\mathbf{x}$ is the transmitted signal vector and $\mathbf{v}_i$ is the noise vector. Using such $\mathbf{F}$, the MSE of signal estimation at the ith receiver in (7) can be equivalently rewritten as the sum of two individual MSEs

$$E_i = \mathrm{tr}\left(\left[\mathbf{I}_{N_\mathrm{b}} + \mathbf{B}^H\mathbf{H}^H\mathbf{H}\mathbf{B}\right]^{-1}\right) + \mathrm{tr}\left(\left[\mathbf{R}^{-1} + \mathbf{T}^H\mathbf{G}_i^H\mathbf{G}_i\mathbf{T}\right]^{-1}\right), \; i = 1, \cdots, L \tag{9}$$

where $\mathbf{R} = \mathbf{B}^H\mathbf{H}^H(\mathbf{H}\mathbf{B}\mathbf{B}^H\mathbf{H}^H + \mathbf{I}_{N_\mathrm{r}})^{-1}\mathbf{H}\mathbf{B}$. Note that the first term $\mathrm{tr}([\mathbf{I}_{N_\mathrm{b}} + \mathbf{B}^H\mathbf{H}^H\mathbf{H}\mathbf{B}]^{-1})$ in (9) is actually the MSE of signal waveform estimation at the relay node and $\mathrm{tr}([\mathbf{R}^{-1} + \mathbf{T}^H\mathbf{G}_i^H\mathbf{G}_i\mathbf{T}]^{-1}), i = 1, \cdots, L$, can be viewed as the increment of the MSE introduced by the second-hop. Here $\mathbf{R}$ is in fact the covariance matrix of $\mathbf{D}^H\mathbf{y}_\mathrm{r}$ as $\mathbf{D}^H\mathrm{E}[\mathbf{y}_\mathrm{r}\mathbf{y}_\mathrm{r}^H]\mathbf{D}$ and $\mathbf{R}^{-1}$ can be viewed as the covariance matrix of the amplified noise at the relay node. Using the optimal structure of $\mathbf{F}$, the relay power consumption is equivalent to $\mathrm{tr}(\mathbf{T}\mathbf{R}\mathbf{T}^H)$. Therefore, the problem (8) can be equivalently rewritten as

$$\min_{\mathbf{B},\mathbf{T}} \quad \max_i \quad \mathrm{tr}\left(\left[\mathbf{I}_{N_\mathrm{b}} + \mathbf{B}^H\mathbf{H}^H\mathbf{H}\mathbf{B}\right]^{-1}\right) + \mathrm{tr}\left(\left[\mathbf{R}^{-1} + \mathbf{T}^H\mathbf{G}_i^H\mathbf{G}_i\mathbf{T}\right]^{-1}\right) \tag{10a}$$
$$\text{s.t.} \quad \mathrm{tr}(\mathbf{T}\mathbf{R}\mathbf{T}^H) \le P_\mathrm{r} \tag{10b}$$
$$\mathrm{tr}(\mathbf{B}\mathbf{B}^H) \le P_\mathrm{s}. \tag{10c}$$

By applying the matrix inversion lemma, the matrix $\mathbf{R}$ can be rewritten as

$$\mathbf{R} = \mathbf{B}^H\mathbf{H}^H\left(\mathbf{I}_{N_\mathrm{r}} - \mathbf{H}\mathbf{B}\left(\mathbf{B}^H\mathbf{H}^H\mathbf{H}\mathbf{B} + \mathbf{I}_{N_\mathrm{b}}\right)^{-1}\mathbf{B}^H\mathbf{H}^H\right)\mathbf{H}\mathbf{B}$$
$$= \mathbf{B}^H\mathbf{H}^H\mathbf{H}\mathbf{B}\left(\mathbf{B}^H\mathbf{H}^H\mathbf{H}\mathbf{B} + \mathbf{I}_{N_\mathrm{b}}\right)^{-1}. \tag{11}$$

An interesting observation from (11) is that with the increase in the first-hop SNR, $\mathbf{B}^H\mathbf{H}^H\mathbf{H}\mathbf{B}$ approaches to infinity and at (moderately) high SNR level $\mathbf{B}^H\mathbf{H}^H\mathbf{H}\mathbf{B} \gg \mathbf{I}_{N_\mathrm{b}}$. Thus we can approximate $\mathbf{R}$ as $\mathbf{I}_{N_\mathrm{b}}$ for the high SNR case [15]. As a consequence, $\mathrm{tr}([\mathbf{R}^{-1} + \mathbf{T}^H\mathbf{G}_i^H\mathbf{G}_i\mathbf{T}]^{-1})$ in (10a) is upper-bounded by $\mathrm{tr}([\mathbf{I}_{N_\mathrm{b}} + \mathbf{T}^H\mathbf{G}_i^H\mathbf{G}_i\mathbf{T}]^{-1})$, for $i = 1, \cdots, L$. Thus, the problem (10) can be approximated as

$$\min_{\mathbf{B},\mathbf{T}} \quad \max_i \quad \mathrm{tr}\left(\left[\mathbf{I}_{N_\mathrm{b}} + \mathbf{B}^H\mathbf{H}^H\mathbf{H}\mathbf{B}\right]^{-1}\right) + \mathrm{tr}\left(\left[\mathbf{I}_{N_\mathrm{b}} + \mathbf{T}^H\mathbf{G}_i^H\mathbf{G}_i\mathbf{T}\right]^{-1}\right) \tag{12a}$$
$$\text{s.t.} \quad \mathrm{tr}(\mathbf{T}\mathbf{T}^H) \le P_\mathrm{r} \tag{12b}$$
$$\mathrm{tr}(\mathbf{B}\mathbf{B}^H) \le P_\mathrm{s}. \tag{12c}$$

Note that such approximation may result in some power loss at the relay node for the low SNR case. We can simply scale the relay matrix obtained from the optimal solution of (12) to compensate the loss and make the best use of the available power budget at the relay node.

Interestingly, it can be seen from (12) that $\mathbf{T}$ has no effect on the first term of the objective function (12a) and $\mathbf{B}$ has no effect on the second term as well. This fact implies that the objective function (12a) and the constraints (12b)-(12c) are decoupled with respect to the optimization variables $\mathbf{B}$ and $\mathbf{T}$. In this case, the source precoding matrix $\mathbf{B}$ can be determined independent of $\mathbf{T}$, and vice-versa, which greatly simplifies the source and relay matrices design. Therefore, the problem (12) can be decomposed into the following source precoding matrix optimization problem

$$\min_{\mathbf{B}} \quad \mathrm{tr}\left(\left[\mathbf{I}_{N_\mathrm{b}} + \mathbf{B}^H\mathbf{H}^H\mathbf{H}\mathbf{B}\right]^{-1}\right) \tag{13a}$$
$$\text{s.t.} \quad \mathrm{tr}(\mathbf{B}\mathbf{B}^H) \le P_\mathrm{s} \tag{13b}$$

and the relay amplifying matrix optimization problem

$$\min_{\mathbf{T}} \quad \max_{i} \quad \mathrm{tr}\Big(\big[\mathbf{I}_{N_\mathrm{b}} + \mathbf{T}^H\mathbf{G}_i^H\mathbf{G}_i\mathbf{T}\big]^{-1}\Big) \tag{14a}$$

$$\text{s.t.} \quad \mathrm{tr}(\mathbf{T}\mathbf{T}^H) \leq P_\mathrm{r} \tag{14b}$$

with the high SNR assumption.

Let $\mathbf{H} = \mathbf{U}_\mathrm{h}\mathbf{\Lambda}_\mathrm{h}\mathbf{V}_\mathrm{h}^H$ denote the singular value decomposition (SVD) of $\mathbf{H}$, where the dimensions of $\mathbf{U}_\mathrm{h}, \mathbf{\Lambda}_\mathrm{h}, \mathbf{V}_\mathrm{h}$ are $N_\mathrm{r} \times N_\mathrm{r}, N_\mathrm{r} \times N_\mathrm{s}, N_\mathrm{s} \times N_\mathrm{s}$, respectively. We assume that the main diagonal elements of $\mathbf{\Lambda}_\mathrm{h}$ are arranged in decreasing order. According to Lemma 2 in [14], the source optimization problem (13) has a closed form solution with the optimal structure of $\mathbf{B}$ given by $\mathbf{B} = \mathbf{V}_{\mathrm{h},1}\mathbf{\Lambda}_\mathrm{b}$, where $\mathbf{V}_{\mathrm{h},1}$ contains the leftmost N_b columns of $\mathbf{V}_\mathrm{h}$ and $\mathbf{\Lambda}_\mathrm{b}$ is an $N_\mathrm{b} \times N_\mathrm{b}$ diagonal power loading matrix. Substituting the optimal $\mathbf{B}$ back into (13) and using the Lagrangian multiplier method, we find that the ith diagonal element of $\mathbf{\Lambda}_\mathrm{b}$ is given by $\lambda_{\mathrm{b},i} = \big[\frac{1}{\lambda_{\mathrm{h},i}}\big(\sqrt{\frac{\lambda_{\mathrm{h},i}}{\mu}} - 1\big)^+\big]^{\frac{1}{2}}, i = 1, \cdots, N_\mathrm{b}$. Here, $(x)^+ \triangleq \max(x, 0)$, $\lambda_{\mathrm{h},i}$ is the ith diagonal element of $\mathbf{\Lambda}_\mathrm{h}$, and $\mu > 0$ is the Lagrangian multiplier which is the solution to the nonlinear equation of $\sum_{i=1}^{N_\mathrm{b}} \frac{1}{\lambda_{\mathrm{h},i}}\big(\sqrt{\frac{\lambda_{\mathrm{h},i}}{\mu}} - 1\big)^+ = P_\mathrm{s}$.

By introducing $\mathbf{T}\mathbf{T}^H \triangleq \mathbf{Q}$, the problem (14) can be rewritten as

$$\min_{\mathbf{Q}} \quad \max_{i} \quad \mathrm{tr}\Big(\big[\mathbf{I}_{N_\mathrm{d}} + \mathbf{G}_i\mathbf{Q}\mathbf{G}_i^H\big]^{-1}\Big) + N_\mathrm{b} - N_\mathrm{d} \tag{15a}$$

$$\text{s.t.} \quad \mathrm{tr}(\mathbf{Q}) \leq P_\mathrm{r} \tag{15b}$$

$$\mathbf{Q} \succcurlyeq 0. \tag{15c}$$

Here $\mathbf{A} \succcurlyeq 0$ indicates that the matrix $\mathbf{A}$ is positive semidefinite (PSD). By introducing $\big[\mathbf{I}_{N_\mathrm{d}} + \mathbf{G}_i\mathbf{Q}\mathbf{G}_i^H\big]^{-1} \preccurlyeq \mathbf{Y}_i, i = 1, \cdots, L$, and a real-valued slack variable t, the problem (15) can be transformed to

$$\min_{t, \mathbf{Q}, \{\mathbf{Y}_i\}} \quad t \tag{16a}$$

$$\text{s.t.} \quad \mathrm{tr}(\mathbf{Y}_i) \leq t, \qquad i = 1, \cdots, L \tag{16b}$$

$$\mathrm{tr}(\mathbf{Q}) \leq P_\mathrm{r} \tag{16c}$$

$$\begin{pmatrix} \mathbf{Y}_i & \mathbf{I}_{N_\mathrm{d}} \\ \mathbf{I}_{N_\mathrm{d}} & \mathbf{I}_{N_\mathrm{d}} + \mathbf{G}_i\mathbf{Q}\mathbf{G}_i^H \end{pmatrix} \succcurlyeq 0, \quad i = 1, \cdots, L \tag{16d}$$

$$t \geq 0, \qquad \mathbf{Q} \succcurlyeq 0 \tag{16e}$$

where $\{\mathbf{Y}_i\} \triangleq \{\mathbf{Y}_i, i = 1, \cdots, L\}$ and we use the Schur complement to obtain (16d). Note that in the above formulation, t provides an MSE upper bound (UB) for the relay-destination channels. The problem (16) is a semidefinite programming (SDP) problem which can be efficiently solved by the disciplined convex programming toolbox CVX [16] at a maximum complexity cost of $\mathcal{O}\big((N_\mathrm{r}^2 + L + 1)^{3.5}\big)$. While most of the computation task in solving problem (13) involves performing SVD and calculating the power loading parameters, the computation overhead is negligible compared to that of problem (16). Note that the problem (12) can also be formulated as an SDP problem which can be solved using interior point-based solvers at a complexity cost that is at most $\mathcal{O}\big((N_\mathrm{s}^2 + N_\mathrm{r}^2 + L + 2)^{3.5}\big)$ [4]. Thus the decoupled source and relay optimization problems have much less computational overhead compared with the problem (12).

We would like to mention that although a high SNR approximation is employed in the derivation of the proposed solution, it has been shown in [14] and [15] by numerical examples that it provides negligible performance loss in all SNR range in comparison to the optimal designs, with a significantly reduced computational complexity.

IV. Numerical Examples

In this section, we study the performance of the proposed multicasting MIMO relay optimization algorithm through numerical simulations. The source, relay, and destination nodes are equipped with N_s, N_r, and N_d antennas, respectively. We simulate a flat Rayleigh fading environment where the channel matrices have entries with zero mean and variances $1/N_\mathrm{s}$ and $1/N_\mathrm{r}$, for $\mathbf{H}$ and $\mathbf{G}_i, i = 1, \cdots, L$, respectively. All simulation results are averaged over 500 independent channel realizations.

We compare the performance of the proposed min-max MSE algorithm with the naive amplify-and-forward (NAF) algorithm and the pseudo match-and-forward (PMF) algorithm in terms of both MSE and BER. For the NAF scheme, we use

$$\mathbf{B} = \sqrt{P_\mathrm{s}/N_\mathrm{s}}\,\mathbf{I}_{N_\mathrm{s}}, \quad \mathbf{F} = \sqrt{P_r/\mathrm{tr}(\mathbf{H}\mathbf{B}\mathbf{B}^H\mathbf{H}^H + \mathbf{I}_{N_\mathrm{r}})}\,\mathbf{I}_{N_\mathrm{r}}.$$

For the PMF algorithm, the same $\mathbf{B}$ in the NAF algorithm is taken and

$$\mathbf{F} = \sqrt{P_r/\mathrm{tr}((\mathbf{H}\mathbf{G})^H(\mathbf{H}\mathbf{B}\mathbf{B}^H\mathbf{H}^H + \mathbf{I}_{N_\mathrm{r}})\mathbf{H}\mathbf{G})}\,(\mathbf{H}\mathbf{G})^H$$

where we randomly pick $\mathbf{G}$ from among the relay-destination channels $\mathbf{G}_i, i = 1, \cdots, L$. Both the NAF and the PMF algorithms use the MMSE receiver at the destination nodes.

In the first example, we compare the performance of the proposed algorithm with the other two approaches in terms of MSE normalized by the number of data streams (NMSE) for $L = 4$, $N_\mathrm{b} = N_\mathrm{s} = N_\mathrm{r} = N_\mathrm{d} = 3$. Fig. 2 shows the MSE performance of all tested algorithms versus P_s with $P_\mathrm{r} =$ 20dB. For the proposed algorithm, we plot the NMSE of the user with the worst channel and the average of all the users. Our results clearly demonstrate the better performance of the proposed joint source and relay optimization algorithm. It can be seen that the proposed optimal algorithm consistently yields the lowest average MSE over the entire P_s region. The worst-user MSE is always better than the MSE upper bound. The NAF and PMF algorithms have much higher MSE compared with the proposed scheme even at very high P_s level.

In the second example, we compare the MSE performance of the proposed algorithm for different number of receiving nodes. Fig. 3 illustrates the NMSE performance versus P_r with $P_\mathrm{s} =$ 20dB for $L = 2, 4$, and 6. It can be clearly seen from Fig. 3 that as the number of receivers increases, the MSE upper bound and the worst-user MSE keep increasing. This is quite reasonable since it is more likely to find a worse relay-destination channel among the increased number of users and we choose the worst-user MSE as the objective function.

In the last example, we compare the performance of the min-max MSE algorithm with that of the NAF and the PMF

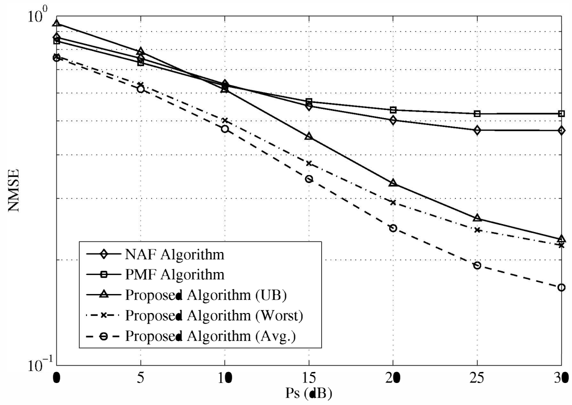

Fig. 2. Example 1: Normalized MSE versus P_{s}. $L = 4$, $N_{\mathrm{b}} = N_{\mathrm{s}} = N_{\mathrm{r}} = N_{\mathrm{d}} = 3$, $P_{\mathrm{r}} = 20$dB.

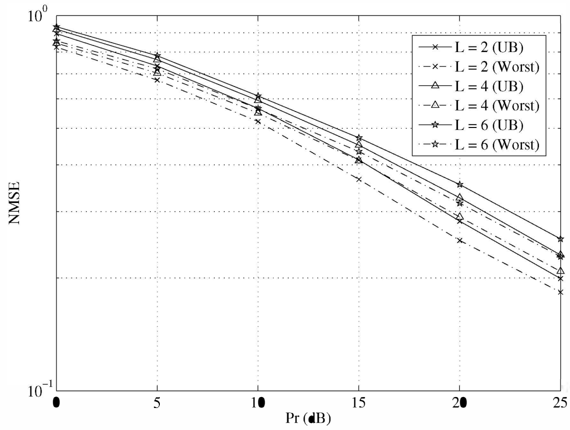

Fig. 3. Example 2: Normalized MSE versus P_{r}. $N_{\mathrm{b}} = N_{\mathrm{s}} = N_{\mathrm{r}} = N_{\mathrm{d}} = 3$, $P_{\mathrm{s}} = 20$dB.

schemes in terms of BER. QPSK signal constellations are used to modulate the transmitted signals. We set $L = 2$, $N_{\mathrm{b}} = 2$, $N_{\mathrm{s}} = 4$, $N_{\mathrm{r}} = 2$, $N_{\mathrm{d}} = 4$, and multicast $N_{\mathrm{b}} \times 1000$ randomly generated bits from the transmitter in each channel realization. Fig. 4 shows the BER performance of all tested algorithms versus P_{s} with $P_{\mathrm{r}} = 20$dB. It can be seen from Fig. 4 that on the average the proposed joint source and relay optimization algorithm obtains the lowest BER compared with the other approaches. Even the worst-user BER is always much better than that of the NAF and the PMF schemes.

V. Conclusions

We considered a two-hop multicasting MIMO relay system with multi-antenna nodes and developed the optimal source and relay precoding matrices under some mild approximation which results in significantly smaller computational complexity. The worst case MSE is minimized subject to power constraints at the source and the relay nodes. Simulation results demonstrate that the jointly optimal source and relay design algorithm outperforms the existing techniques.

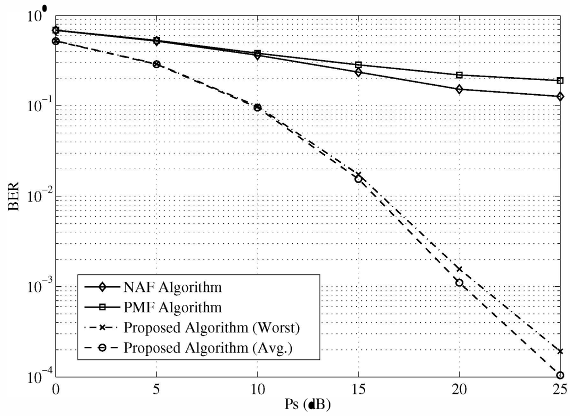

Fig. 4. Example 3: BER versus P_{s}. $L = 2$, $N_{\mathrm{b}} = 2$, $N_{\mathrm{s}} = 4$, $N_{\mathrm{r}} = 2$, $N_{\mathrm{d}} = 4$, $P_{\mathrm{r}} = 20$dB.

References

[1] M. A. Khojastepour, A. Salehi-Golsefidi, and S. Rangarajan, "Towards an optimal beamforming algorithm for physical layer multicasting," in *Proc. IEEE Inf. Theory Workshop*, Oct. 16-20, 2011, pp. 395–399.

[2] N. Jindal and Z.-Q. Luo, "Capacity limits of multiple antenna multicast," in *Proc. IEEE ISIT*, Seattle, USA, Jul. 09-14, 2006, pp. 1841–1845.

[3] S. Y. Park, D. J. Love, and D. H. Kim, "Capacity limits of multi-antenna multicasting under correlated fading channels," *IEEE Trans. Commun.*, vol. 58, pp. 2002–2013, Jul. 2010.

[4] N. D. Sidiropoulos, T. N. Davidson, and Z.-Q. Luo, "Transmit beamforming for physical-layer multicasting," *IEEE Trans. Signal Process.*, vol. 54, pp. 2239–2251, Jun. 2006.

[5] E. Chiu and V. Lau, "Precoding design for multi-antenna multicast broadcast services with limited feedback," *IEEE Systems Journal*, vol. 4, pp. 550–560, Dec. 2010.

[6] S. X. Wu and W.-K. Ma, "Multicast transmit beamforming using a randomize-in-time strategy," in *Proc. IEEE ICASSP*, Prague, Czech Republic, May 22-27, 2011, pp. 3376–3379.

[7] E. Matskani, N. D. Sidiropoulos, Z.-Q. Luo, and L. Tassiulas, "Efficient batch and adaptive approximation algorithms for joint multicast beamforming and admission control," *IEEE Trans. Signal Process.*, vol. 57, pp. 4882–4894, Dec. 2009.

[8] D. Senaratne and C. Tellambura, "Beamforming for physical layer multicasting," in *Proc. IEEE WCNC*, Cancun, Mexico, Mar. 28-31, 2011, pp. 1776–1781.

[9] M. A. Khojastepour, A. Khajehnejad, K. Sundaresan, and S. Rangarajan, "Adaptive beamforming algorithms for wireless link layer multicasting," in *Proc. IEEE PIMRC*, Toronto, Canada, Sep. 11-14, 2011, pp. 1994–1998.

[10] M. Kaliszan, E. Pollakis, and S. Stańczak, "Efficient beamforming algorithms for MIMO multicast with application-layer coding," in *Proc. IEEE ISIT*, St. Petersburg, Russia, Jul. 31-Aug. 5, 2011, pp. 928–932.

[11] H. Xu, J. Jin, and B. Li, "YMMV: Multiple session multicast with MIMO," in *Proc. IEEE GLOBECOM*, Texas, USA, Dec. 5-9, 2011.

[12] H. Zhang, X. You, G. Wu, and H. Wang, "Cooperative multi-antenna multicasting for wireless networks," in *IEEE GLOBECOM*, Miami, FL, USA, Dec. 6-10, 2010.

[13] D. P. Palomar and Y. Jiang, *MIMO Transceiver Design via Majorization Theory.* now Publishers, 2007.

[14] Y. Rong, "Simplified algorithms for optimizing multiuser multi-hop MIMO relay systems," *IEEE Trans. Commun.*, vol. 59, pp. 2896–2904, Oct. 2011.

[15] C. Song, K.-J. Lee, and I. Lee, "MMSE based transceiver designs in closed-loop non-regenerative MIMO relaying systems," *IEEE Trans. Wireless Commun.*, vol. 9, pp. 2310–2319, Jul. 2010.

[16] M. Grant and S. Boyd, "Cvx: Matlab software for disciplined convex programming (web page and software)." *http://cvxr.com/cvx*, April, 2010.