Simplified Robust Design for Nonregenerative Multicasting MIMO Relay Systems

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***Abstract*—In this paper, we propose a robust transceiver design for nonregenerative multicasting multiple-input multiple-output (MIMO) relay systems where a transmitter broadcasts common message to multiple receivers with aid of a relay node and the transmitter, relay and receivers are all equipped with multiple antennas. In the proposed design, the actual channel state information (CSI) is assumed as a Gaussian random matrix with the estimated CSI as the mean value, and the channel estimation errors are derived from the well-known Kronecker model. In the proposed design scheme, the transmitter and relay precoding matrices are jointly optimized to minimize the maximal mean squared-error (MSE) of the estimated signal at all receivers. The optimization problem is highly nonconvex in nature. Hence, we propose a low complexity solution by exploiting the optimal structure of the relay precoding matrix. Numerical simulations demonstrate the improved robustness of the proposed transceiver design algorithm against the CSI mismatch.**

***Index Terms*—Nonregenerative MIMO relay, multicasting, minimum mean-squared error (MMSE), robustness.**

I. Introduction

In many practical wireless communication systems, one source transmits common information to multiple destination nodes simultaneously. These systems are also called multicast broadcasting or multicasting systems. Recently, multicasting systems have attracted much research interest, due to the increasing demand for mobile applications such as location based video broadcasting and streaming media.

The wireless channel has the multicast broadcasting nature, hence it is very suitable for multicasting applications. However, the wireless system performance may be degraded due to the channel fading and shadowing effects. By deploying multi-antenna and beamforming techniques at the transmitter and receiver, the channel shadowing effect can be mitigated [1]. Next generation wireless standards such as WiMAX 802.16m and 3GPP LTE-Advanced have already included technologies which enable better multicasting solutions based on multi-antenna and beamforming techniques [2].

Due to the nonconvex nature of the problem, designing the optimal beamforming vector for multicasting is difficult in general. Capacity limits of multi-antenna multicast channel have been studied in [3], and the channel spatial correlation effect on the channel capacity has been investigated in [4]. In [5], algorithms for designing transmit beamforming vectors for physical layer multicasting have been proposed with the assumption that the channel state information (CSI) is available at the transmitter. Recently [6], achievable information rate and relay precoder design of non-regenerative MIMO relay networks are investigated under imperfect channel state information (CSI) including channel estimation errors and feedback/feedforward delay errors, without considering the direct link from source to destination link.

In the proposed multicasting systems [2]–[5], single antenna has been assumed at receiver. Recently multi-antenna receiver design has been developed in [7]. In [8], the cooperative protocol for multicast systems with multiple transmit antennas is proposed with the assumption that the users are equipped with single antenna.

In the case of long distance between the transmitter and receivers, it is necessary to have a relay node between the transmitter and receivers to efficiently mitigate the pathloss of wireless channel. A two-hop MIMO relay multicasting system has been proposed in [9] where one transmitter multicasts common message to multiple receivers with the aid of a relay node. The authors of [9] assume that the transmitter, relay and receivers are all equipped with multiple antennas and the full CSI of all channels is available at the relay. However, in practical communication systems, the exact CSI is not available and has to be estimated. There is always mismatch between the true and estimated CSI. Hence, the performance of the algorithm in [9] will degrade due to such CSI mismatch. Robust transceiver design, which could mitigate such performance degradation by taking the channel estimation errors into account, is therefore of great importance and highly desirable for practical applications [10].

In this paper, we propose a transceiver design algorithm for nonregenerative multicasting MIMO relay systems which is robust against the CSI mismatch. Similar to [9], we assume in the proposed design that one transmitter broadcasts common message to multiple receivers with the aid of a relay node and the transmitter, relay and receivers are all equipped with multiple antennas. However, different to [9], the true channel matrices have Gaussian distribution, with the estimated channels as the mean value, and the channel estimation errors follow the well-known Kronecker model [10], [11]. Hence, we propose

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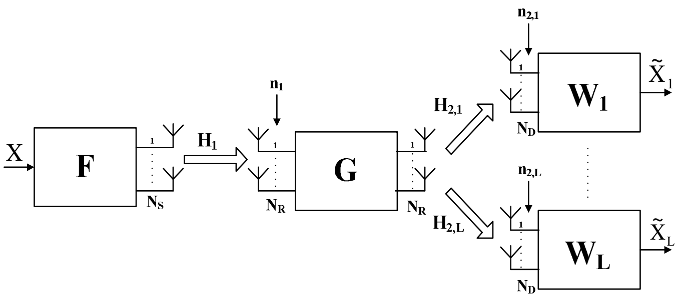

Fig. 1. Block diagram of a two-hop nonregenerative multicasting MIMO relay system.

a robust algorithm to jointly design the transmitter, relay, and receiver matrices to minimize the maximal mean-squared error (MSE) of the signal waveform estimation among all receivers. We would like to mention that although robust transceiver design has been studied for single-user MIMO relay systems [10]–[12], and multiuser MIMO relay systems [13], to the best of our knowledge, robust transceiver design for multicasting MIMO relay systems has not been investigated in existing works.

II. System Model

We consider a two-hop nonregenerative multicasting MIMO relay system with L receivers as shown in Fig.1, where the transmitter and relay have N_S and N_R antennas, respectively. For simplicity, we assume that each receiver has N_D antennas. It is assumed that there is no direct link between the transmitter and receivers. The data transmission takes place over two time slots. The received signal at the relay during the first time slot is given by

$$\mathbf{y}_r = \mathbf{H}_1\mathbf{F}\mathbf{x} + \mathbf{n}_1 \tag{1}$$

where $\mathbf{x} \in \mathbb{C}^{N_B\times 1}$ is the transmitted signal vector which satisfies $E\{\mathbf{x}\mathbf{x}^H\} = \mathbf{I}_{N_B}$, N_B is chosen to satisfy $N_B \leq \min(N_S, N_R, N_D)$, $\mathbf{H}_1 \in \mathbb{C}^{N_R\times N_S}$ is the MIMO channel matrix between the transmitter and relay nodes, $\mathbf{F} \in \mathbb{C}^{N_S\times N_B}$ is the transmitter precoding matrix, $\mathbf{n}_1 \in \mathbb{C}^{N_R\times 1}$ is the additive noise vector at the relay. Here $E\{.\}$ denotes the statistical expectation and $(.)^H$ stands for the matrix Hermitian transpose, and $\mathbf{I}_n$ denotes the $n\times n$ identity matrix.

In the second time slot, the relay node linearly precodes $\mathbf{y}_r$ with the relay precoding matrix $\mathbf{G} \in \mathbb{C}^{N_R\times N_R}$, and broadcasts the linearly precoded signal vector $\mathbf{x}_r = \mathbf{G}\mathbf{y}_r$ to all receivers. The received signal at the ith receiver in the second time slot is given by

$$\mathbf{y}_{d,i} = \mathbf{H}_{2,i}\mathbf{G}\mathbf{H}_1\mathbf{F}\mathbf{x} + \mathbf{H}_{2,i}\mathbf{G}\mathbf{n}_1 + \mathbf{n}_{2,i},\ i = 1,\cdots,L \tag{2}$$

where $\mathbf{H}_{2,i} \in \mathbb{C}^{N_D\times N_R}$ is the MIMO channel matrix between the relay and the ith receiver, $\mathbf{n}_{2,i} \in \mathbb{C}^{N_D\times 1}$ is the additive noise vector at the ith receiver. We assume that all noises are i.i.d with zero mean and unit variance. In general, channel state information is required for optimal design of precoders. However, in practice, the perfect CSI is not available at relay or receivers due to channel mismatch. With this assumption, the channel matrices $\mathbf{H}_1$ and $\mathbf{H}_{2,i}$ can be modeled as [10]

$$\mathbf{H}_1 = \widehat{\mathbf{H}}_1 + \mathbf{\Delta}_1 \tag{3}$$

$$\mathbf{H}_{2,i} = \widehat{\mathbf{H}}_{2,i} + \mathbf{\Delta}_{2,i},\ i = 1,\cdots,L \tag{4}$$

where $\widehat{\mathbf{H}}_1$ and $\widehat{\mathbf{H}}_{2,i}$ are the estimated transmitter-relay and relay-receiver channels matrices, $\mathbf{\Delta}_1$ and $\mathbf{\Delta}_{2,i}$ are the corresponding channel estimation errors whose elements are zero mean Gaussian random variables. In general, the channel estimation error matrices, $\mathbf{\Delta}_1$ and $\mathbf{\Delta}_{2,i}$, depend on specific channel estimation algorithms. In this paper, the channel estimation algorithm proposed in [14] is used. The probability density function (PDF) of $\mathbf{\Delta}_1$ and $\mathbf{\Delta}_{2,i}$ can be modeled as [15]

$$\mathbf{\Delta}_1 \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Sigma}_1 \otimes \mathbf{\Psi}_1^T) \tag{5}$$

$$\mathbf{\Delta}_{2,i} \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Sigma}_{2,i} \otimes \mathbf{\Psi}_{2,i}^T) \tag{6}$$

where $\otimes$ denotes the matrix Kronecker product, $(\cdot)^T$ stands for the matrix transpose, $\mathbf{\Sigma}_1$ and $\mathbf{\Psi}_1$ are the row and column covariance matrices of $\mathbf{\Delta}_1$, respectively, and $\mathbf{\Sigma}_{2,i}$ and $\mathbf{\Psi}_{2,i}$ are the row and column matrices of $\mathbf{\Delta}_{2,i}$, respectively. Here we assume that $\mathbf{\Delta}_1$ and $\mathbf{\Delta}_{2,i}$ are multivariate complex Gaussian distributed with zero mean.

At the ith receiver, linear receiver $\mathbf{W}_i$ is applied to retrieve the transmitted signal vector $\mathbf{x}$. Hence, the estimated signal at the ith receiver can be expressed as

$$\tilde{\mathbf{x}}_i = \mathbf{W}_i\mathbf{y}_{d,i},\ i = 1,\cdots,L. \tag{7}$$

Let us assume that P_s and P_r are the upper bound of the transmitter and relay powers. Hence, the power constraints on the transmitter and relay node can be expressed as

$$p(\mathbf{F}) = tr\Big\{\mathbf{F}\mathbf{F}^H\Big\} \leq P_s \tag{8}$$

$$p(\mathbf{G},\mathbf{F}) = tr\Big\{\mathbf{G}(\mathbf{H}_1\mathbf{F}\mathbf{F}^H\mathbf{H}_1^H + \mathbf{I}_{N_R})\mathbf{G}^H\Big\} \leq P_r \tag{9}$$

where $tr\{.\}$ is the trace of a matrix. In our proposed transceiver design, our main aim is to minimize the maximum MSE over all receivers. In the proposed desigh algorithm, we derive the optimal transmitter and relay precoder matrices $\mathbf{F}$, $\mathbf{G}$ and ith receiver matrix $\mathbf{W}_i$ to minimized the maximum MSE of the signal waveform estimation. Using (2) and (7), the MSE of the signal waveform estimation at the ith receiver is given by

$$\begin{aligned} &J_i(\mathbf{W}_i,\mathbf{G},\mathbf{F}) \\ &= tr\big\{\big(\mathbf{W}_i\mathbf{H}_{2,i}\mathbf{G}\mathbf{H}_1\mathbf{F} - \mathbf{I}_{N_B}\big)\big(\mathbf{W}_i\mathbf{H}_{2,i}\mathbf{G}\mathbf{H}_1\mathbf{F} - \mathbf{I}_{N_B}\big)^H \\ &+ \mathbf{W}_i\mathbf{R}_{n,i}\mathbf{W}_i^H\big\},\ i = 1,\cdots,L \end{aligned} \tag{10}$$

where $\mathbf{R}_{n,i}$ is the equivalent noise covariance matrix given by

$$\mathbf{R}_{n,i} = \mathbf{H}_{2,i}\mathbf{G}\mathbf{G}^H\mathbf{H}_{2,i}^H + \mathbf{I}_{N_D}. \tag{11}$$

III. Proposed Robust Transceiver Design Algorithm

For any given precoding matrices $\mathbf{F}$ and $\mathbf{G}$ which satisfy the power constraints at the transmitter and relay node (8) and (9), the weight matrix $\mathbf{W}_i$ minimizing (10) is the well known MMSE filter which is given by [16]

$$\mathbf{W}_i = \mathbf{F}^H\mathbf{H}_1^H\mathbf{G}^H\mathbf{H}_{2,i}^H \times (\mathbf{H}_{2,i}\mathbf{G}\mathbf{H}_1\mathbf{F}\mathbf{F}^H\mathbf{H}_1^H\mathbf{G}^H\mathbf{H}_{2,i}^H + \mathbf{R}_{n,i})^{-1} \quad (12)$$

where $(\cdot)^{-1}$ stands for the matrix inverse. After substituting (12) into (10) and using the matrix inversion lemma [17], the linear transceiver design problem can be formulated as

$$\begin{aligned} \min_{\mathbf{G},\mathbf{F}} \ \max_i \ & J_i(\mathbf{G},\mathbf{F}) = tr\Big\{\Big[\mathbf{I}_{N_B} + \bar{\mathbf{H}}_i^H\mathbf{R}_{n,i}^{-1}\bar{\mathbf{H}}_i\Big]^{-1}\Big\} \\ s.t. \ & tr\Big\{\mathbf{G}(\mathbf{H}_1\mathbf{F}\mathbf{F}^H\mathbf{H}_1^H + \mathbf{I}_{N_R})\mathbf{G}^H\Big\} \le P_r \\ & tr\Big\{\mathbf{F}\mathbf{F}^H\Big\} \le P_s \end{aligned} \quad (13)$$

where $\bar{\mathbf{H}}_i = \mathbf{H}_{2,i}\mathbf{G}\mathbf{H}_1\mathbf{F}$.

Note that directly solving the min-max problem (13) is difficult due to the complicated function of $J_i(\mathbf{G},\mathbf{F})$. In the following, we propose a low computational complexity approach to solve the problem (13). It can be shown similar to [18] that the optimal relay precoding matrix $\mathbf{G}$ for each link can be expressed as

$$\mathbf{G} = \mathbf{T}\mathbf{D}^H \quad (14)$$

where $\mathbf{D} = (\mathbf{H}_1\mathbf{F}\mathbf{F}^H\mathbf{H}_1 + \mathbf{I}_{N_R})^{-1}\mathbf{H}_1\mathbf{F}$ and $\mathbf{T}$ can be considered as the precoding matrix at the transmit side of the second-hop MIMO multicasting channel.

Using the relay precoding matrix $\mathbf{G}$ (14), the MSE of the estimated signal at the ith receiver can be reformulated as the sum of two individual MSE [18] functions

$$\begin{aligned} J_i(\mathbf{T},\mathbf{F}) = & tr\Big\{\Big[\mathbf{I}_{N_B} + \mathbf{F}^H\mathbf{H}_1^H\mathbf{H}_1\mathbf{F}\Big]^{-1}\Big\} \\ & + tr\Big\{\Big[\mathbf{R}^{-1} + \mathbf{T}^H\mathbf{H}_{2,i}^H\mathbf{H}_{2,i}\mathbf{T}\Big]^{-1}\Big\}, \\ & i = 1, \cdots, L \end{aligned} \quad (15)$$

where

$$\mathbf{R} = \mathbf{F}^H\mathbf{H}_1^H\big(\mathbf{H}_1\mathbf{F}\mathbf{F}^H\mathbf{H}_1^H + \mathbf{I}_{N_R}\big)^{-1}\mathbf{H}_1\mathbf{F}. \quad (16)$$

Interestingly, the first term in (15) is the MSE of estimating $\mathbf{x}$ from the signal vector (1) received at the relay node using the MMSE receiver with the weight matrix $\mathbf{D}$, while the second term in (15) can be viewed as the increment of the MSE introduced by the second-hop.

Using the relay precoding matrix $\mathbf{G}$ in (14), the power consumption at the relay power can be rewritten as $tr(\mathbf{T}\mathbf{R}\mathbf{T}^H)$ and using the matrix inversion lemma [17], the matrix $\mathbf{R}$ (16) can be expressed as

$$\begin{aligned} \mathbf{R} = & \mathbf{F}^H\mathbf{H}_1^H\Big(\mathbf{I}_{N_R} - \mathbf{H}_1\mathbf{F} \\ & \times\Big(\mathbf{F}^H\mathbf{H}_1^H\mathbf{H}_1\mathbf{F} + \mathbf{I}_{N_B}\Big)^{-1}\mathbf{F}^H\mathbf{H}_1^H\Big)\mathbf{H}_1\mathbf{F} \\ = & \mathbf{F}^H\mathbf{H}_1^H\mathbf{H}_1\mathbf{F}\Big(\mathbf{F}^H\mathbf{H}_1^H\mathbf{H}_1\mathbf{F} + \mathbf{I}_{N_B}\Big)^{-1} \end{aligned} \quad (17)$$

We can observe from (17) that with increase in the first-hop SNR, the term $\mathbf{F}^H\mathbf{H}_1^H\mathbf{H}_1\mathbf{F}$ approaches infinity and at a (moderately) high SNR level, $\mathbf{F}^H\mathbf{H}_1^H\mathbf{H}_1\mathbf{F} \gg \mathbf{I}_{N_B}$. Hence, $\mathbf{R}$ can be approximated as $\mathbf{I}_{N_B}$ for high SNR value [9], [18]. Using (15) and (17), the optimization problem (13) can be reformulated as

$$\begin{aligned} \min_{\mathbf{F},\mathbf{T}} \ \max_i \ & tr\Big\{\Big[\mathbf{I}_{N_B} + \mathbf{F}^H\mathbf{H}_1^H\mathbf{H}_1\mathbf{F}\Big]^{-1}\Big\} \\ & + tr\Big\{\Big[\mathbf{I}_{N_B} + \mathbf{T}^H\mathbf{H}_{2,i}^H\mathbf{H}_{2,i}\mathbf{T}\Big]^{-1}\Big\} \\ s.t. \ & tr\Big\{\mathbf{T}\mathbf{T}^H\Big\} \le P_r, \\ & tr\Big\{\mathbf{F}\mathbf{F}^H\Big\} \le P_s \end{aligned} \quad (18)$$

It can be noticed from (18) that $\mathbf{T}$ has no influence on the first term of the objective function (18) and $\mathbf{F}$ has no influence on the second term as well. Hence, the optimization problem (18) can be divided into the following transmitter precoding matrix optimization problem

$$\begin{aligned} \min_{\mathbf{F}} \ & tr\Big\{\Big[\mathbf{I}_{N_B} + \mathbf{F}^H\mathbf{H}_1^H\mathbf{H}_1\mathbf{F}\Big]^{-1}\Big\} \\ s.t. \ & tr\big\{\mathbf{F}\mathbf{F}^H\big\} \le P_s \end{aligned} \quad (19)$$

and the relay precoding matrix optimization problem can be expressed as

$$\begin{aligned} \min_{\mathbf{T}} \ \max_i \ & tr\Big\{\Big[\mathbf{I}_{N_B} + \mathbf{T}^H\mathbf{H}_{2,i}^H\mathbf{H}_{2,i}\mathbf{T}\Big]^{-1}\Big\} \\ s.t. \ & tr\big\{\mathbf{T}\mathbf{T}^H\big\} \le P_r. \end{aligned} \quad (20)$$

Lemma 1: Let $f(\mathbf{X})$ be a function of random matrix $\mathbf{X}$ having finite expectation $E(\mathbf{X})$. If f is a matrix-convex function, then $E[f(\mathbf{X})] \succeq f(E[\mathbf{X}])$ [19].

A. Optimization of $\mathbf{F}$

It can be noticed from (19) that the problem is reduced to find the optimal precoding matrix $\mathbf{F}$ to minimize the MSE of the received signal at the relay node. However, as the exact $\mathbf{H}_1$ is unknown, we cannot solve the problem (19). If we optimize $\mathbf{F}$ based on $\widehat{\mathbf{H}}_1$, there might be great performance degradation due to the mismatch between $\mathbf{H}_1$ and $\widehat{\mathbf{H}}_1$. Thus, instead of minimizing $M(\mathbf{F}) = tr\{(\mathbf{I}_{N_B} + \mathbf{F}^H\mathbf{H}_1^H\mathbf{H}_1\mathbf{F})^{-1}\}$, we consider minimizing $E_{\Delta_1}\{M(\mathbf{F})\}$, where the expectation is over the distribution of $\boldsymbol{\Delta}_1$.

However, the exact expression of $E_{\Delta_1}\{M(\mathbf{F})\}$ is difficult to obtain. Using the channel estimation error model (3) and Lemma 1, the lower bound of $E_{\Delta_1}\{M(\mathbf{F})\}$ can be written as

$$\begin{aligned} E_{\Delta_1}\{M(\mathbf{F})\} \succeq & tr\big\{\big(\mathbf{I}_{N_B} + \mathbf{F}^H E_{\Delta_1}\{\mathbf{H}_1^H\mathbf{H}_1\}\mathbf{F}\big)^{-1}\big\} \\ = & tr\big\{\big(\mathbf{I}_{N_B} + \mathbf{F}^H\mathbf{A}\mathbf{F}\big)^{-1}\big\} \end{aligned} \quad (21)$$

where $\mathbf{A} = \widehat{\mathbf{H}}_1^H\widehat{\mathbf{H}}_1 + tr\{\boldsymbol{\Sigma}_1\}\boldsymbol{\Psi}_1$. Using (21), the source precoding matrix optimization problem can be written as

$$\begin{aligned} \min_{\mathbf{F}} \ & tr\big\{\big(\mathbf{I}_{N_B} + \mathbf{F}^H\mathbf{A}\mathbf{F}\big)^{-1}\big\} \\ s.t. \ & tr\big\{\mathbf{F}\mathbf{F}^H\big\} \le P_s. \end{aligned} \quad (22)$$

Let us introduce the eigenvalue decomposition (EVD) of the matrix $\mathbf{A}$

$$\mathbf{A} = \mathbf{U}_A \mathbf{\Lambda}_A \mathbf{U}_A^H \tag{23}$$

where the diagonal elements of $\mathbf{A}$ are sorted in a decreasing order. It can be shown that the solution to the problem (22) is given by

$$\mathbf{F} = \mathbf{U}_{A,1} \mathbf{\Lambda}_F^{\frac{1}{2}} \tag{24}$$

where $\mathbf{U}_{A,1}$ contains the leftmost N_B columns of $\mathbf{U}_A$ associated with the largest N_B eigenvalues and $\mathbf{\Lambda}_F$ is a diagonal matrix. After substituting (23) and (24) into (22), the problem (22) can be written as the following optimization problem with scalar variables

$$\min_{\{\lambda_{F,i}\}} \quad \sum_{i=1}^{N_B} \frac{1}{1+\lambda_{F,i}\lambda_{A,i}} \tag{25}$$

$$s.t. \quad \sum_{i=1}^{N_B} \lambda_{F,i} \leq P_s \tag{26}$$

$$\lambda_{F,i} \geq 0, \qquad i = 1, \cdots, N_B \tag{27}$$

where $\lambda_{F,i}$ and $\lambda_{A,i}$, $i = 1, \cdots, N_B$, are the ith diagonal elements of $\mathbf{\Lambda}_F$ and $\mathbf{\Lambda}_A$, respectively, and $\{\lambda_{F,i}\} = \{\lambda_{F,1}, \cdots, \lambda_{F,N_B}\}$. The problem (25)-(27) has the well-known water-filling solution as [20]

$$\lambda_{F,i} = \frac{1}{\lambda_{A,i}} \left(\sqrt{\frac{\lambda_{A,i}}{\mu}} - 1 \right)^+, \qquad i = 1, \cdots, N_B$$

where $(x)^+ = \max(x, 0)$, and $\mu > 0$ satisfies the nonlinear equation of $\sum_{i=1}^{N_B} \frac{1}{\lambda_{A,i}} \big(\sqrt{\frac{\lambda_{A,i}}{\mu}} - 1\big)^+ = P_s$.

B. Optimization of $\mathbf{T}$

It can be seen from (20) that the problem is reduced to find the optimal precoding matrix $\mathbf{T}$ to minimize the maximal MSE of the received signal at the receiver. Similar to the approach we used to optimize $\mathbf{F}$, using the channel estimation error model (4) and Lemma 1, we have

$$\begin{aligned} &E_{\Delta_{2,i}}\{tr\{(\mathbf{I}_{N_B} + \mathbf{T}^H \mathbf{H}_{2,i}^H \mathbf{H}_{2,i} \mathbf{T})^{-1}\}\} \\ &\succeq tr\Big\{ \Big[\mathbf{I}_{N_B} + \mathbf{T}^H E_{\Delta_{2,i}}\{\mathbf{H}_{2,i}^H \mathbf{H}_{2,i}\} \mathbf{T} \Big]^{-1} \Big\} \\ &= tr\{(\mathbf{I}_{N_B} + \mathbf{T}^H \mathbf{B}_i \mathbf{T})^{-1}\} \end{aligned} \tag{28}$$

where $\mathbf{B}_i = \widehat{\mathbf{H}}_{2,i}^H \widehat{\mathbf{H}}_{2,i} + tr\{\mathbf{\Sigma}_{2,i}\}\mathbf{\Psi}_{2,i}$. Using (28), the problem of optimizing $\mathbf{T}$ can be written as

$$\begin{aligned} \min_{\mathbf{T}} \ \max_{i} \quad & tr\Big\{ \Big[\mathbf{I}_{N_B} + \mathbf{T}^H \mathbf{B}_i \mathbf{T} \Big]^{-1} \Big\} \\ s.t. \quad & tr\big(\mathbf{T}\mathbf{T}^H\big) \leq P_r. \end{aligned} \tag{29}$$

Using the matrix identity $tr\big\{\big[\mathbf{I}_m + \mathbf{A}_{m\times n}\mathbf{B}_{n\times m}\big]^{-1}\big\} = tr\big\{\big[\mathbf{I}_n + \mathbf{B}_{n\times m}\mathbf{A}_{m\times n}\big]^{-1}\big\} + m - n$ the min-max problem (29) can be written as

$$\begin{aligned} \min_{\mathbf{Q}} \ \max_{i} \quad & tr\Big\{ \Big[\mathbf{I}_{N_D} + \mathbf{B}_i^{\frac{1}{2}} \mathbf{Q} \mathbf{B}_i^{\frac{1}{2}} \Big]^{-1} \Big\} + N_B - N_D \\ s.t. \quad & tr\big(\mathbf{Q}\big) \leq P_r \\ & \mathbf{Q} \succeq 0 \end{aligned} \tag{30}$$

where $\mathbf{Q} = \mathbf{T}\mathbf{T}^H$ and $\mathbf{Q} \succeq 0$ denotes that $\mathbf{Q}$ is a positive semidefinite (PSD) matrix. Let us introduce a PSD matrix $\mathbf{Z}_i$ with $\big[\mathbf{I}_{N_D} + \mathbf{B}_i^{\frac{1}{2}} \mathbf{Q} \mathbf{B}_i^{\frac{1}{2}}\big]^{-1} \preceq \mathbf{Z}_i, i = 1, \cdots, L$ and a real valued slack variable ρ. By using the Schur complement [19], the optimization problem (30) can be reformulated as

$$\begin{aligned} \min_{\rho,\, \mathbf{Q},\, \mathbf{Z}_i} \quad & \rho \\ s.t. \quad & tr\big(\mathbf{Z}_i\big) \leq \rho, \quad i = 1, \cdots, L \\ & tr\big(\mathbf{Q}\big) \leq P_r \\ & \begin{pmatrix} \mathbf{Z}_i & \mathbf{I}_{N_D} \\ \mathbf{I}_{N_D} & \mathbf{I}_{N_D} + \mathbf{B}_i^{\frac{1}{2}} \mathbf{Q} \mathbf{B}_i^{\frac{1}{2}} \end{pmatrix} \succeq 0, \quad i = 1, \cdots, L \\ & \mathbf{Q} \succeq 0. \end{aligned} \tag{31}$$

The optimization problem (31) is a convex semidefinite programming (SDP) problem and the convex programming toolbox CVX [21] can be used to solve the SDP problem.

IV. Simulation Results

In this section, we investigate the performance of the proposed robust transceiver optimization algorithm for MIMO relay multicasting systems through numerical simulations. We simulate a two-hop nonregenerative MIMO relay multicasting system with $L = 2$ and $N_S = N_R = N_D = 4$. The information-carrying symbols are generated from QPSK constellations. The signal-to-noise ratios (SNRs) of the first-hop and second-hop channels are defined as $\text{SNR}_1 = P_s/N_S$ and $\text{SNR}_2 = P_r/N_R$, respectively. We set $\text{SNR}_1 = 30$dB. In the simulations, the correlation matrices of the channel estimation errors are modeled as [10]

$$\mathbf{\Psi}_1 = \mathbf{\Psi}_{2,i} = \begin{pmatrix} 1 & \alpha & \alpha^2 & \alpha^3 \\ \alpha & 1 & \alpha & \alpha^2 \\ \alpha^2 & \alpha & 1 & \alpha \\ \alpha^3 & \alpha^2 & \alpha & 1 \end{pmatrix}, \quad i = 1, \cdots, L$$

$$\mathbf{\Sigma}_1 = \mathbf{\Sigma}_{2,i} = \sigma_e^2 \begin{pmatrix} 1 & \beta & \beta^2 & \beta^3 \\ \beta & 1 & \beta & \beta^2 \\ \beta^2 & \beta & 1 & \beta \\ \beta^3 & \beta^2 & \beta & 1 \end{pmatrix}, \ i = 1, \cdots, L$$

where $0 \leq \alpha, \beta \leq 1$ are correlation coefficients, and σ_e^2 measures the variance of the estimated error.

The estimated channel matrices $\widehat{\mathbf{H}}_1$ and $\widehat{\mathbf{H}}_{2,i}$ are generated based on the following distributions

$$\widehat{\mathbf{H}}_1 \sim \mathcal{CN}\Big(\mathbf{0}, \frac{1-\sigma_e^2}{\sigma_e^2} \mathbf{\Sigma}_1 \otimes \mathbf{\Psi}_1^T\Big)$$

$$\widehat{\mathbf{H}}_{2,i} \sim \mathcal{CN}\Big(\mathbf{0}, \frac{1-\sigma_e^2}{\sigma_e^2} \mathbf{\Sigma}_{2,i} \otimes \mathbf{\Psi}_{2,i}^T\Big), \quad i = 1, \cdots, L.$$

We compare the performance of the proposed robust min-max MSE algorithm, namely the robust algorithm with the non-robust min-max MSE [9] algorithm in terms of both MSE and BER.

In the first simulation example, we investigate the BER performance of the proposed algorithm at different levels of σ_e^2. Fig. 2 shows the BER performance of the proposed robust algorithm versus SNR_2 while fixing $\text{SNR}_1 = 30$dB,

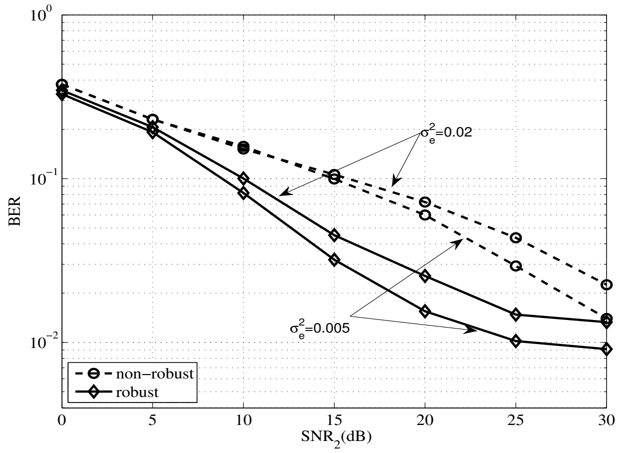

Fig. 2. Example 1: BER versus SNR_2 while fixing $L = 2, N_B = N_S = N_R = N_D = 4, \mathrm{SNR}_1 = 30$dB.

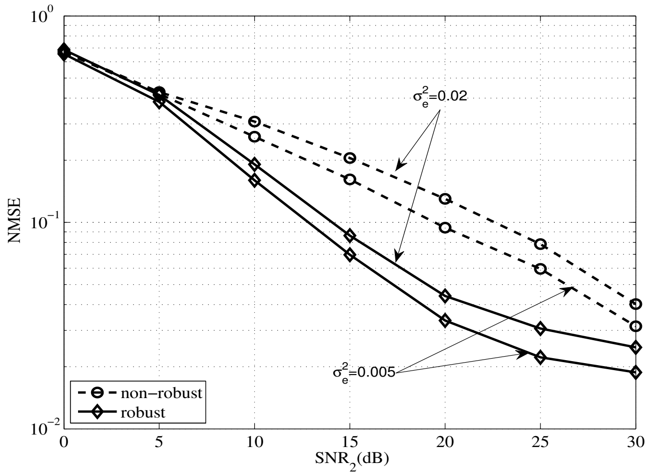

Fig. 3. Example 2: NMSE versus SNR_2 while fixing $L = 2, N_B = N_S = N_R = N_D = 4, \mathrm{SNR}_1 = 30$dB.

$L = 2,\ N_B = N_S = N_R = N_D = 4$. It can be seen from Fig. 2 that over the whole range of SNR_2, the proposed robust algorithm significantly outperforms the non-robust algorithm in terms of BER.

In the second simulation example, we study the MSE performance of the proposed algorithm at different levels of σ_e^2. In Fig. 3, we compare the performance of the proposed algorithm in terms of MSE versus SNR_2 while fixing $\mathrm{SNR}_1 = 30$dB, $L = 2,\ N_B = N_S = N_R = N_D = 4$. It can be noted from Fig. 3 that the proposed robust algorithm shows better MSE performance over the whole range of SNR_2 than the existing non-robust algorithm.

V. Conclusions

We have addressed the challenging issue of precoding matrices optimization for a MIMO relay multicasting system where the actual CSI is assumed as a Gaussian random matrix with the estimated CSI as the mean value, and estimated error of the channels is derived from the well-known Kronecker model. In the proposed design scheme, the transmitter and relay precoding matrices are jointly optimized to minimize the maximal MSE of the estimated signal at all receivers. Numerical simulations demonstrate that the proposed robust transceiver design algorithm outperforms the non-robust transceiver design algorithm.

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