Joint Transceiver Design for Amplify-and-Forward Multiuser MIMO Relay Communication Systems with Source-Destination Links

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Abstract —In this paper, we generalize the iterative algorithms proposed by Khandaker to amplify-and-forward multiuser Multiple-Input Multiple-Output (MIMO) relay communication systems with the source-destination links. Compared with the conventional algorithms, where the source-destination links are ignored, the evolved Tri-Step method and Bi-Step method are derived for the new system model with the source-destination links for more spatial diversity gain. From the theoretical derivation of the Minimum Mean-Squared Error (MMSE) of the signal waveform estimation at the destination node, we show that the existence of the source-destination links bring benefits no matter how weak they are. Numerical examples demonstrate that the evolved algorithms perform much better than the original algorithms and other existing ones in terms of both MSE and Bit-Error-Rate (BER).

Index Terms—MIMO relay, MMSE, multiuser, direct links

I. Introduction

Multiple-Input Multiple-Output (MIMO) relay communication has attracted much research interest in recent years for its advantages in increasing the coverage and the capacity of wireless networks. In particular, Amplify-and-Forward (AF) relay has been extensively investigated thanks to its simplicity and short processing delay.

A joint transceiver design has been proposed in [1] to minimize the Mean-Squared Error (MSE) of the signal waveform estimation. A unified framework has been introduced in [2] to jointly optimize the source and relay precoding matrices for a broad class of frequently used objective functions in MIMO relay system design. In [3], the source and relay precoding matrices are designed for multiuser MIMO relay networks. Considering the capacity criteria, the relay precoding matrix is developed in [4] and [5] for the multiuser relay networks with only one receiving antenna at each user. In [6], two iterative algorithms have been proposed for the transceiver design of multiuser relay networks. In [7], robust design for the imperfect CSI is studied without considering the direct link. In [8], source and relay precoding matrices design based on a Tomlinson-Harashima precoder has been studied considering the direct source-destination link. A closed-form design of the relay precoding matrix has been proposed in [9]. It has been proven in [10] that the optimal relay precoding matrix has a general beamforming structure for most commonly used objective functions. In [11], a joint source and relay precoding matrices design based on the MSE criterion has been proposed. A single user MIMO relay with direct link is considered in [12], where only one single transmission stream is allowed. Source and relay design are investigated in [13] for single user case considering both the direct link and the CSI mismatch, which supports for multiple transmission streams. Relay precoding matrix design with Successive Interference Cancelation (SIC) receiver has been proposed in [14].

In [1]-[7], the direct source-destination link has been ignored. However, the direct signal transmission from source to destination provides a spatial copy of the source signals, and thus, should be considered in the MIMO relay system design, the performance gain of which is already proved in [8], [12] and [13]. Meanwhile, multiuser MIMO relay is very common in practical relay systems. But in [8]-[13], all the source precoding and relay processing matrices are designed for the single-user MIMO relay network.

In this paper, we study the AF multiuser MIMO relay communication systems with direct source-destination links. Compared with [6], where the direct links are ignored, the system model is more complex to deal with for the increased transmitting signal links. Two iterative algorithms, namely the evolved Tri-Step method and the evolved Bi-Step method are derived for the new system model, where the source-destination links are taken into consideration for more spatial diversity gain. The optimal source, relay and receiving matrices are obtained by solving new optimization problems with direct links. From the theoretical derivation of the MMSE of the signal waveform estimation at the destination node, we show that the existence of the source-destination links bring benefits no matter how weak they are. Numerical examples demonstrate that the evolved algorithms perform much better than the original algorithms in [6] and other existing ones in terms of both MSE and BER.

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The remainder parts of this paper are organized as follows. In Section II, the model of an AF multiuser MIMO relay communication system with source-destination links is introduced. The evolved Tri-Step and Bi-Step algorithms are proposed in Section III. Numerical simulation results are displayed in Section IV to validate the advantages of the proposed algorithms under different scenarios. Conclusions are drawn in Section V.

II. System Model

We consider a two-hop MIMO communication system as shown in Fig. 1, where K users (user $i, i=1,2...K$) transmit information to the destination node with the aid of a relay node. The ith user is equipped with N_i, $i=1,2...K$ antennas. The relay and the destination nodes are equipped with N_r and N_d antennas, respectively.

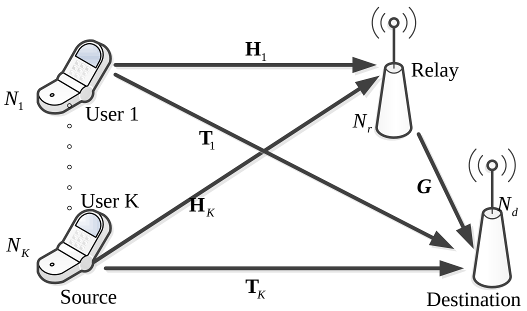

Fig. 1. Amplify-and-forward multiuser MIMO relay communication system with direct links.

Using a half-duplex relay, the communication processing is completed in two time slots. During the first time slot, the received signal vectors at the relay node and the destination node can be written as

$$\mathbf{y}_r = \sum_{i=1}^{K} \mathbf{H}_i \mathbf{B}_i \mathbf{s}_i + \mathbf{n}_r = \hat{\mathbf{H}}\mathbf{s} + \mathbf{n}_r \tag{1}$$

$$\mathbf{y}_{d,1} = \sum_{i=1}^{K} \mathbf{T}_i \mathbf{B}_i \mathbf{s}_i + \mathbf{n}_{d,1} = \hat{\mathbf{T}}\mathbf{s} + \mathbf{n}_{d,1} \tag{2}$$

where $\mathbf{s}_i$ is the $N_i \times 1$ source signal vector and $N_b = \sum_{i=1}^{K} N_i$ ($N_b \le \min(N_r, N_d)$) denotes the total number of different streams. $\mathbf{B}_i$ is the $N_i \times N_i$ source precoding matrix. $\mathbf{H}_i \in \mathbb{C}^{N_r \times N_i}$ and $\mathbf{T}_i \in \mathbb{C}^{N_d \times N_i}$ are the MIMO fading channel matrix of the ith user-relay link and user-destination link, respectively. $\mathbf{n}_r \in \mathbb{C}^{N_r \times 1}$ and $\mathbf{n}_{d1} \in \mathbb{C}^{N_d \times 1}$ are the noise vectors at the relay and the destination node at time slot one. $\hat{\mathbf{H}} \triangleq [\mathbf{H}_1\mathbf{B}_1, \cdots, \mathbf{H}_K\mathbf{B}_K]$ is the equivalent MIMO channel matrix of the users-relay link, $\hat{\mathbf{T}} \triangleq [\mathbf{T}_1\mathbf{B}_1, \cdots, \mathbf{T}_K\mathbf{B}_K]$ is the equivalent MIMO channel matrix of the users-destination link, $\mathbf{s} \triangleq [\mathbf{s}_1^T, \cdots, \mathbf{s}_K^T]^T$ is the equivalent transmitted signal vector from all users, and $(\cdot)^T$ denotes matrix (vector) transpose. We assume that $E[\mathbf{s}\mathbf{s}^H] = \mathbf{I}_{N_b}$, where $\mathbf{I}_{N_b}$ is an $N_b \times N_b$ identity matrix, $(\cdot)^H$ denotes matrix (vector) Hermitian transpose, and $E[\cdot]$ stands for the statistical expectation.

During the second time slot, the source node keeps silent, and the relay node linearly precodes $\mathbf{y}_r$ and forwards it to the destination node. The received signal vector at the destination node is given by

$$\begin{aligned} \mathbf{y}_{d,2} &= \mathbf{G}\mathbf{F}\mathbf{y}_r + \mathbf{n}_{d,2} \\ &= \mathbf{G}\mathbf{F}\textstyle\sum_{i=1}^{K} \mathbf{H}_i\mathbf{B}_i\mathbf{s}_i + \mathbf{G}\mathbf{F}\mathbf{n}_r + \mathbf{n}_{d,2} \\ &= [\mathbf{G}\mathbf{F}\mathbf{H}_1\mathbf{B}_1, \cdots, \mathbf{G}\mathbf{F}\mathbf{H}_K\mathbf{B}_K]\mathbf{s} + \mathbf{G}\mathbf{F}\mathbf{n}_r + \mathbf{n}_{d,2} \\ &= \tilde{\mathbf{H}}\mathbf{s} + \mathbf{n} \end{aligned} \tag{3}$$

where $\mathbf{F} \in \mathbb{C}^{N_r \times N_r}$ is the relay precoding matrix, $\mathbf{G} \in \mathbb{C}^{N_d \times N_r}$ is the MIMO fading channel matrix between the relay and destination nodes and $\mathbf{n}_{d,2} \in \mathbb{C}^{N_d \times 1}$ is the noise vector at the destination node at time slot two. $\tilde{\mathbf{H}} \triangleq [\mathbf{G}\mathbf{F}\mathbf{H}_1\mathbf{B}_1, \cdots, \mathbf{G}\mathbf{F}\mathbf{H}_K\mathbf{B}_K] = \mathbf{G}\mathbf{F}\hat{\mathbf{H}}$ is the equivalent MIMO channel matrix of the users-relay-destination link, and $\mathbf{n} \triangleq \mathbf{G}\mathbf{F}\mathbf{n}_r + \mathbf{n}_{d,2}$ is the equivalent noise vector at the destination node at the second time slot.

Combining (1)-(3), the received signals can be written as

$$\begin{aligned} \mathbf{y} &= \begin{bmatrix} \mathbf{y}_{d,2} \\ \mathbf{y}_{d,1} \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{G}\mathbf{F}\mathbf{H}_1\mathbf{B}_1, \cdots, \mathbf{G}\mathbf{F}\mathbf{H}_K\mathbf{B}_K \\ \mathbf{T}_1\mathbf{B}_1, \cdots, \mathbf{T}_K\mathbf{B}_K \end{bmatrix}\mathbf{s} + \begin{bmatrix} \mathbf{G}\mathbf{F}\mathbf{n}_r + \mathbf{n}_{d,2} \\ \mathbf{n}_{d,1} \end{bmatrix} \\ &= \bar{\mathbf{H}}\mathbf{s} + \bar{\mathbf{v}} \end{aligned} \tag{4}$$

where $\bar{\mathbf{H}}$ is the equivalent channel matrix between the users and the destination node over two time slots given by

$$\bar{\mathbf{H}} = \begin{bmatrix} \mathbf{G}\mathbf{F}\mathbf{H}_1\mathbf{B}_1, \cdots, \mathbf{G}\mathbf{F}\mathbf{H}_K\mathbf{B}_K \\ \mathbf{T}_1\mathbf{B}_1, \cdots, \mathbf{T}_K\mathbf{B}_K \end{bmatrix} \tag{5}$$

and

$$\bar{\mathbf{v}} = \begin{bmatrix} \mathbf{G}\mathbf{F}\mathbf{n}_r + \mathbf{n}_{d,2} \\ \mathbf{n}_{d,1} \end{bmatrix} \tag{6}$$

represents the equivalent noise vector at the destination node over two time slots.

We assume that the channel matrices $\mathbf{H}_i, \mathbf{T}_i, i = 1, \cdots, K$, and $\mathbf{G}$ are all quasi-static throughout a block of transmission. All noises are assumed to be independent and identically distributed (i.i.d.) complex circularly symmetric Gaussian noise with zero mean and unit variance.

Using a linear receiver matrix $\mathbf{W}$ at the destination node, the MSE of the signal waveform estimation is given by

$$\begin{aligned}\mathrm{MSE} &= \mathrm{tr}\left\{E\left[(\mathbf{s}-\hat{\mathbf{s}})(\mathbf{s}-\hat{\mathbf{s}})^H\right]\right\} \\ &= \mathrm{tr}\left\{\left(\mathbf{W}^H\bar{\mathbf{H}}-\mathbf{I}_{N_b}\right)\left(\mathbf{W}^H\bar{\mathbf{H}}-\mathbf{I}_{N_b}\right)^H+\mathbf{W}^H\mathbf{C}\mathbf{W}\right\}\end{aligned} \quad (7)$$

where $\mathrm{tr}\{\cdot\}$ denotes matrix trace and $\mathbf{C}$ is the equivalent noise covariance matrix given by

$$\begin{aligned}\mathbf{C} &= E\left[\bar{\mathbf{v}}\bar{\mathbf{v}}^H\right] \\ &= E\left[\begin{bmatrix}\mathbf{G}\mathbf{F}\mathbf{n}_r+\mathbf{n}_{d,2} \\ \mathbf{n}_{d,1}\end{bmatrix}\begin{bmatrix}\mathbf{G}\mathbf{F}\mathbf{n}_r+\mathbf{n}_{d,2} \\ \mathbf{n}_{d,1}\end{bmatrix}^H\right] \\ &= \begin{bmatrix}\mathbf{G}\mathbf{F}\mathbf{F}^H\mathbf{G}^H+\mathbf{I}_{N_d} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{N_d}\end{bmatrix}\end{aligned} \quad (8)$$

From (5)-(8), we see the influence of the direct links is reflected in the received signal and MSE. Then the joint source, relay, and receiver matrices optimization problem can be formulated as

$$\min_{\{\mathbf{B}_i\},\mathbf{W},\mathbf{F}} \mathrm{MSE} \quad (9)$$

$$\mathrm{s.t.}\,\mathrm{tr}\left\{\mathbf{F}\left(\sum_{i=1}^{K}\mathbf{H}_i\mathbf{B}_i\mathbf{B}_i^H\mathbf{H}_i^H+\mathbf{I}_{N_r}\right)\mathbf{F}^H\right\}\le P_r \quad (10)$$

$$\mathrm{tr}\left\{\mathbf{B}_i\mathbf{B}_i^H\right\}\le P_i,\ i=1,\cdots,K \quad (11)$$

where $P_r>0$, $P_i>0, i=1,\cdots,K$, are the power constraints at the relay and the ith user, respectively. The optimization problem (9)-(11) is nonconvex with matrix variables and a globally optimal solution to this problem is computationally intractable.

Remark: From the optimization problem (9)-(11), we can see that the introduction of the direct links doesn't change the power constraints of the source nodes and the relay. But the influence of the direct links is introduced in the expression of MSE by the equivalent channel matrix $\bar{\mathbf{H}}$ and the equivalent noise vector $\bar{\mathbf{v}}$. In the following derivations, we will show the effects of the source-destination links on the optimal source, relay matrices and the MMSE of the signal waveform estimation.

III. Proposed Source, Relay, and Receiver Matrices Design

Although the optimization problem (9)-(11) is more complicated than the problem in [6] for the introduction of direct links, we can still try to solve the problem using iterative algorithms. In this section, we derive the two iterative algorithms namely the evolved Tri-Step and Bi-Step algorithms to optimize the source, relay, and receiver matrices taking into account the direct links. The evolved Tri-Step and Bi-Step algorithms can be treated as generalizations of the original algorithms in [6]. Following the similar steps, the solution of problem (9)-(11) is obtained. For simplicity, we mainly focus on the difference of the derivation caused by the direct links. In the solving process, we use variable substitutions for a simple form of the equations and to show the relations between the evolved and the original algorithms, that is the influence of the direct links.

A. The Evolved Tri-Step Algorithm

Firstly, with given $\mathbf{F}$ and $\{\mathbf{B}_i\}$, the optimal $\mathbf{W}$ is the MMSE receiver given by

$$\mathbf{W}=(\bar{\mathbf{H}}\bar{\mathbf{H}}^H+\mathbf{C})^{-1}\bar{\mathbf{H}} \quad (12)$$

where $(\cdot)^{-1}$ denotes matrix inversion. The optimal $\mathbf{W}$ seems to have the same form with that in [6], however the influence of direct links is already considered in $\bar{\mathbf{H}}$ and $\mathbf{C}$.

Secondly, with given $\mathbf{W}$ and $\{\mathbf{B}_i\}$, we try to update $\mathbf{F}$. Since both $\bar{\mathbf{H}}$ and $\mathbf{C}$ are partitioned matrices with $\mathbf{F}$ as their elements, we consider to substitute (5) and (8) into (9) to get the expansion form. In order to do this, we let $\mathbf{W}=[\mathbf{W}_1^T,\mathbf{W}_2^T]^T$, where $\mathbf{W}_1$ and $\mathbf{W}_2$ are $N_d\times N_b$ matrices. Let $\bar{\mathbf{G}}\triangleq\mathbf{W}_1^H\mathbf{G}$ and $\bar{\mathbf{T}}\triangleq\mathbf{W}_2^H\hat{\mathbf{T}}$, $\mathbf{F}$ can be updated by solving the following problem

$$\begin{aligned}\min_{\mathbf{F}}\,\mathrm{tr}&\left\{\left(\bar{\mathbf{G}}\mathbf{F}\hat{\mathbf{H}}+\bar{\mathbf{T}}-\mathbf{I}_{N_b}\right)\left(\bar{\mathbf{G}}\mathbf{F}\hat{\mathbf{H}}+\bar{\mathbf{T}}-\mathbf{I}_{N_b}\right)^H\right. \\ &\left.+\bar{\mathbf{G}}\mathbf{F}\mathbf{F}^H\bar{\mathbf{G}}^H+\mathbf{W}^H\mathbf{W}\right\}\end{aligned} \quad (13)$$

$$\mathrm{s.t.}\,\mathrm{tr}\left\{\mathbf{F}(\hat{\mathbf{H}}\hat{\mathbf{H}}^H+\mathbf{I}_{N_r})\mathbf{F}^H\right\}\le P_r \quad (14)$$

Problem (13)-(14) can be solved using the Lagrange multiplier method, the optimal $\mathbf{F}$ is

$$\mathbf{F}=\bar{\mathbf{G}}^H(\bar{\mathbf{G}}\bar{\mathbf{G}}^H+\mu\mathbf{I}_{N_b})^{-1}(\mathbf{I}_{N_b}-\bar{\mathbf{T}})\hat{\mathbf{H}}^H(\hat{\mathbf{H}}\hat{\mathbf{H}}^H+\mathbf{I}_{N_r})^{-1}. \quad (15)$$

The Lagrangian multiplier μ in (15) can be found from the following complementary slackness condition

$$\mu(\mathrm{tr}\{\mathbf{F}(\hat{\mathbf{H}}\hat{\mathbf{H}}^H+\mathbf{I}_{N_r})\mathbf{F}^H\}-P_r)=0. \quad (16)$$

For the case that $\mu=0$, $\mathbf{F}$ can be directly obtained from (15) as (17), if only $\mathbf{F}$ in (17) satisfies the constraint (14).

$$\mathbf{F}=\bar{\mathbf{G}}^H(\bar{\mathbf{G}}\bar{\mathbf{G}}^H)^{-1}(\mathbf{I}_{N_b}-\bar{\mathbf{T}})\hat{\mathbf{H}}^H(\hat{\mathbf{H}}\hat{\mathbf{H}}^H+\mathbf{I}_{N_r})^{-1}. \quad (17)$$

Otherwise, there must be $\mu>0$ and $tr\{\mathbf{F}(\hat{\mathbf{H}}\hat{\mathbf{H}}^H+\mathbf{I}_{N_r})\mathbf{F}^H\}=P_r$ in (16). In this case, substituting (15) into the constraint (14), we get the following equation

$$\begin{aligned}\mathrm{tr}&\left\{\bar{\mathbf{G}}^H(\bar{\mathbf{G}}\bar{\mathbf{G}}^H+\mu\mathbf{I}_{N_b})^{-1}(\mathbf{I}_{N_b}-\bar{\mathbf{T}})\hat{\mathbf{H}}^H(\hat{\mathbf{H}}\hat{\mathbf{H}}^H+\mathbf{I}_{N_r})^{-1}\right. \\ &\left.\times\mathbf{H}(\mathbf{I}_{N_b}-\bar{\mathbf{T}}^H)(\bar{\mathbf{G}}\bar{\mathbf{G}}^H+\mu\mathbf{I}_{N_b})^{-1}\bar{\mathbf{G}}\right\}=P_r.\end{aligned} \quad (18)$$

Using the Singular Value Decomposition (SVD) of $\bar{\mathbf{G}}\triangleq\mathbf{U}\mathbf{\Lambda}\mathbf{V}^H$ in (18), where $\mathbf{U}$ is an $N_b\times N_b$ unitary matrix, $\mathbf{\Lambda}$ is an $N_b\times N_b$ diagonal matrix, and $\mathbf{V}$ is an $N_r\times N_b$ semi-unitary matrix with $\mathbf{V}^H\mathbf{V}=\mathbf{I}_{N_b}$, we have

$$\text{tr}\left\{\mathbf{\Lambda}(\mathbf{\Lambda}^2+\mu\mathbf{I}_{N_b})^{-1}\mathbf{U}^H(\mathbf{I}_{N_b}-\bar{\mathbf{T}})\hat{\mathbf{H}}^H(\hat{\mathbf{H}}\hat{\mathbf{H}}^H+\mathbf{I}_{N_r})^{-1} \times\hat{\mathbf{H}}(\mathbf{I}_{N_b}-\bar{\mathbf{T}}^H)\mathbf{U}(\mathbf{\Lambda}^2+\mu\mathbf{I}_{N_b})^{-1}\mathbf{\Lambda}\right\}=P_r. \quad (19)$$

Denoting $\mathbf{\Theta}\triangleq\mathbf{U}^H(\mathbf{I}_{N_b}-\bar{\mathbf{T}})\hat{\mathbf{H}}^H(\hat{\mathbf{H}}\hat{\mathbf{H}}^H+\mathbf{I}_{N_r})^{-1}\hat{\mathbf{H}}(\mathbf{I}_{N_b}-\bar{\mathbf{T}}^H)\mathbf{U}$, (19) can be equivalently written as

$$\sum_{i=1}^{N_b}\frac{\lambda_i^2\theta_{i,i}}{(\lambda_i^2+\mu)^2}=P_r. \quad (20)$$

where λ_i and $\theta_{i,i}$ are the ith main diagonal elements of $\mathbf{\Lambda}$ and $\mathbf{\Theta}$ respectively. Since the left-hand side of (20) is a monotonically decreasing function of $\mu>0$, μ can be obtained, for example, using the bisection method.

From (15), (17) and (19), we can see clearly how the channel matrices of the direct links, which are implicit in $\bar{\mathbf{T}}$, have effects on the optimal relay matrix $\mathbf{F}$.

Then with given $\mathbf{W}$ and $\mathbf{F}$, we show in the following that the problem (9)-(11) can be cast as a QCQP problem in a similar way as it is in [6]. By introducing $\mathbf{A}_i\triangleq\mathbf{W}_1^H\mathbf{G}\mathbf{F}\mathbf{H}_i+\mathbf{W}_2^H\mathbf{T}_i$, $i=1,\cdots,K$, and $\mathbf{A}_{ii}$ as a matrix containing the $(\sum_{j=1}^{i-1}N_j+1)$-th to the $(\sum_{j=1}^{i}N_j)$-th rows of $\mathbf{A}_i$, we can rewrite the MSE in (7) as

$$\begin{aligned}\text{MSE}&=\text{tr}\left\{(\mathbf{W}_1^H\mathbf{G}\mathbf{F}\hat{\mathbf{H}}+\mathbf{W}_2^H\hat{\mathbf{T}}-\mathbf{I}_{N_b})(\mathbf{W}_1^H\mathbf{G}\mathbf{F}\hat{\mathbf{H}}+\mathbf{W}_2^H\hat{\mathbf{T}}-\mathbf{I}_{N_b})^H+\mathbf{W}^H\mathbf{C}\mathbf{W}\right\}\\&=\text{tr}\{([\mathbf{A}_1\mathbf{B}_1,\cdots,\mathbf{A}_K\mathbf{B}_K]-\mathbf{I}_{N_b})\times([\mathbf{A}_1\mathbf{B}_1,\cdots,\mathbf{A}_K\mathbf{B}_K]-\mathbf{I}_{N_b})^H+\mathbf{W}^H\mathbf{C}\mathbf{W}\}\\&=\sum_{k=1}^{K}\left(\text{tr}\{\mathbf{A}_i\mathbf{B}_i\mathbf{B}_i^H\mathbf{A}_i^H\}-\text{tr}\{\mathbf{A}_{ii}\mathbf{B}_i\}-\text{tr}\{\mathbf{B}_i^H\mathbf{A}_{ii}^H\}\right)+t_1\end{aligned} \quad (21)$$

where $t_1\triangleq\text{tr}\left\{\mathbf{I}_{N_b}+\mathbf{W}^H\mathbf{C}\mathbf{W}\right\}$.

Let us introduce $\mathbf{b}_i\triangleq vec(\mathbf{B}_i)$, where for a matrix $\mathbf{A}$, $vec(\mathbf{A})$ stands for a vector obtained by stacking all column vectors of $\mathbf{A}$ on top of each other. Using the identity of $\text{tr}(\mathbf{A}^T\mathbf{D})=(vec(\mathbf{A}))^T vec(\mathbf{D})$ and $vec(\mathbf{AB})=(\mathbf{I}\otimes\mathbf{A})vec(\mathbf{B})$, where $\otimes$ denotes the matrix Kronecker product, we can rewrite (21) as

$$\begin{aligned}\text{MSE}&=\sum_{i=1}^{K}\left(\mathbf{b}_i^H(\mathbf{I}_{N_i}\otimes(\mathbf{A}_i^H\mathbf{A}_i))\mathbf{b}_i-\left(vec(\mathbf{A}_{ii}^T)\right)^T\mathbf{b}_i-\mathbf{b}_i^H vec(\mathbf{A}_{ii}^H)\right)+t_1\\&=\mathbf{b}^H\mathbf{A}\mathbf{b}-\mathbf{c}^H\mathbf{b}-\mathbf{b}^H\mathbf{c}+t_1\\&=(\mathbf{b}^H\mathbf{A}^{\frac{1}{2}}-\mathbf{c}^H\mathbf{A}^{-\frac{1}{2}})(\mathbf{A}^{\frac{1}{2}}\mathbf{b}-\mathbf{A}^{-\frac{1}{2}}\mathbf{c})+t_2\end{aligned} \quad (22)$$

where

$$\mathbf{b}\triangleq[\mathbf{b}_1^T,\cdots,\mathbf{b}_K^T]^T$$
$$\mathbf{A}\triangleq bd\left(\mathbf{I}_{N_1}\otimes(\mathbf{A}_1^H\mathbf{A}_1),\cdots,\mathbf{I}_{N_K}\otimes(\mathbf{A}_K^H\mathbf{A}_K)\right)$$
$$\mathbf{c}\triangleq\left[\left(vec(\mathbf{A}_{11}^H)\right)^T,\cdots,\left(vec(\mathbf{A}_{KK}^H)\right)^T\right]^T$$

and $bd(\cdot)$ forms a block-diagonal matrix. Note that we can ignore the term $t_2\triangleq t_1-\mathbf{c}^H\mathbf{A}^{-1}\mathbf{c}$ while optimizing $\mathbf{b}$ with given $\mathbf{W}$ and $\mathbf{F}$, since it is free of the optimization variable $\mathbf{b}$.

By introducing $\mathbf{D}_i\triangleq\mathbf{F}\mathbf{H}_i$, $i=1,\cdots,K$, the relay transmit power constraint in (10) can be rewritten as

$$\mathbf{b}^H\mathbf{D}\mathbf{b}\le P_r-\text{tr}\left\{\mathbf{F}\mathbf{F}^H\right\} \quad (23)$$

where $\mathbf{D}\triangleq bd\left(\mathbf{I}_{N_1}\otimes(\mathbf{D}_1^H\mathbf{D}_1),\cdots,\mathbf{I}_{N_K}\otimes(\mathbf{D}_K^H\mathbf{D}_K)\right)$. Using (22) and (23), the optimization problem (9)-(11) can be equivalently rewritten as the following QCQP problem

$$\min_{\mathbf{b}}\left(\mathbf{b}^H\mathbf{A}^{\frac{1}{2}}-\mathbf{c}^H\mathbf{A}^{-\frac{1}{2}}\right)\left(\mathbf{A}^{\frac{1}{2}}\mathbf{b}-\mathbf{A}^{-\frac{1}{2}}\mathbf{c}\right) \quad (24)$$

$$\text{s.t.}\,\mathbf{b}^H\mathbf{D}\mathbf{b}\le P_r-\text{tr}\left\{\mathbf{F}\mathbf{F}^H\right\} \quad (25)$$

$$\mathbf{b}^H\mathbf{E}_i\mathbf{b}\le P_i,\ i=1,\cdots,K \quad (26)$$

where $\mathbf{E}_i\triangleq bd(\tilde{\mathbf{E}}_{i1},\cdots,\tilde{\mathbf{E}}_{iK})$, with $\tilde{\mathbf{E}}_{ii}=\mathbf{I}_{N_i^2}$ and $\tilde{\mathbf{E}}_{ij}=\mathbf{0},\ j=1,\cdots,K,\ j\ne i$. The QCQP problem (24)-(26) can be efficiently solved by the disciplined convex programming toolbox CVX [15].

The QCQP problem for optimizing $\mathbf{B}_i$ seems to have the same form as it is in [6]. However, the optimal $\mathbf{B}_i$ has been impacted by the direct links. In (21), the noise of the direct links is implicit in the term $\mathbf{W}^H\mathbf{C}\mathbf{W}$, which can be treated as a constant. The channel matrices of the direct links are implicit in $\mathbf{A}_i$.

TABLE I: PROCEDURE OF THE EVOLVED TRI-STEP ALGORITHM

1. Initialize the algorithm with $\mathbf{B}_i^{(0)}=\sqrt{\frac{P_i}{N_i}}\mathbf{I}_{N_i}$, $i=1,\cdots,K$, and $\mathbf{F}^{(0)}=\sqrt{P_r/\text{tr}\{\hat{\mathbf{H}}\hat{\mathbf{H}}^H+\mathbf{I}_{N_r}\}}\mathbf{I}_{N_r}$; Set $n=0$.
2. Update $\mathbf{W}^{(n)}$ using $\{\mathbf{B}_i^{(n)}\}$ and $\mathbf{F}^{(n)}$ as in (12).
3. Update $\mathbf{F}^{(n+1)}$ as in (15) using given $\{\mathbf{B}_i^{(n)}\}$ and $\mathbf{W}^{(n)}$.
4. Solve the subproblem (24)-(26}) using known $\mathbf{F}^{(n+1)}$ and $\mathbf{W}^{(n)}$ to obtain $\mathbf{B}_i^{(n+1)}$, $i=1,\cdots,K$.
5. Calculate $\text{MSE}^{(n+1)}$ using $\mathbf{W}^{(n)}$, $\mathbf{F}^{(n+1)}$, and $\{\mathbf{B}_i^{(n+1)}\}$, If $\frac{\text{MSE}^{(n)}-\text{MSE}^{(n+1)}}{\text{MSE}^{(n+1)}}\le\varepsilon$, then end.
   Otherwise, let $n:=n+1$ and go to step 2.

The procedure of applying the evolved Tri-Step algorithm to solve the original source, relay, and receiver matrices optimization problem (9)-(11) is listed in Table I, where ε is a small positive number close to zero and the superscript (n) denotes the number of iterations. Since the update of $\mathbf{W}$, $\mathbf{F}$, and $\{\mathbf{B}_i\}$ at each iteration may decrease or maintain, but can not increase the value of the objective function (7), and (7) is lower bounded by zero, a monotonic convergence of the evolved Tri-Step algorithm toward (at least) a stationary point follows directly from this observation.

B. The Evolved Bi-Step Algorithm

The joint source and relay optimization algorithm, also named the evolved Bi-Step algorithm, is proposed in this subsection. In this algorithm, the source and relay matrices are updated iteratively, and the receiver matrix is calculated after the convergence of the algorithm. It will be shown later that the E-Bi-Step algorithm converges faster than the Tri-Step algorithm.

By substituting (12) back into (7), the MSE becomes a function of $\{\mathbf{B}_i\}$ and $\mathbf{F}$ as

$$\text{MMSE} = \text{tr}\left\{\left[\mathbf{I}_{N_b} + \bar{\mathbf{H}}^H \mathbf{C}^{-1}\bar{\mathbf{H}}\right]^{-1}\right\}. \qquad (27)$$

Based on (27), the joint source and relay matrices optimization problem is given by

$$\min_{\{\mathbf{B}_i\},\mathbf{F}} \text{tr}\left\{\left[\mathbf{I}_{N_b} + \bar{\mathbf{H}}^H \mathbf{C}^{-1}\bar{\mathbf{H}}\right]^{-1}\right\} \qquad (28)$$

$$\text{s.t.tr}\left\{\mathbf{F}\left(\sum_{i=1}^{K}\mathbf{H}_i\mathbf{B}_i\mathbf{B}_i^H\mathbf{H}_i^H + \mathbf{I}_{N_r}\right)\mathbf{F}^H\right\} \le P_r \qquad (29)$$

$$\text{tr}\left\{\mathbf{B}_i\mathbf{B}_i^H\right\} \le P_i,\ i = 1,\cdots,K. \qquad (30)$$

The problem (28)-(30) seems to be the same with that of [6]. However, the influence of the direct links is implicit in $\bar{\mathbf{H}}$. To get an inside view of the problem (28)-(30), (28) can be written as

$$\begin{aligned}&(\mathbf{I}_{N_b} + \bar{\mathbf{H}}^H\mathbf{C}^{-1}\bar{\mathbf{H}})^{-1}\\ &= \Big(\mathbf{I}_{N_b} + \sum_{i=1}^{K}\mathbf{B}_i^H\mathbf{T}_i^H\mathbf{T}_i\mathbf{B}_i + \sum_{i=1}^{K}\mathbf{B}_i^H\mathbf{H}_i^H\mathbf{F}^H\mathbf{G}^H\\ &\times(\mathbf{G}\mathbf{F}\mathbf{F}^H\mathbf{G}^H + \mathbf{I}_{N_d})^{-1}\mathbf{G}\mathbf{F}\mathbf{H}_i\mathbf{B}_i\Big)^{-1}\end{aligned} \qquad (31)$$

, which shows that the MSE is decreased due to the existence of the direct links no matter how weak they are. From (31), the difference between the problem (28)-(30) and that in [6] is obvious.

Firstly, with given source matrices $\{\mathbf{B}_i\}$ satisfying the source power constraints in (30), we update the relay matrix $\mathbf{F}$ by solving the following problem

$$\min_{\mathbf{F}} \text{tr}\left\{\left[\mathbf{I}_{N_b} + \bar{\mathbf{H}}^H \mathbf{C}^{-1}\bar{\mathbf{H}}\right]^{-1}\right\} \qquad (32)$$

$$\text{s.t.tr}\left\{\mathbf{F}\left(\sum_{i=1}^{K}\mathbf{H}_i\mathbf{B}_i\mathbf{B}_i^H\mathbf{H}_i^H + \mathbf{I}_{N_r}\right)\mathbf{F}^H\right\} \le P_r. \qquad (33)$$

It can be shown similar to [9] that the optimal $\mathbf{F}$ as the solution to the problem (32)-(33) has the structure of

$$\mathbf{F} = \mathbf{P}\mathbf{L} \qquad (34)$$

where $\mathbf{P}$ is an $N_r \times N_b$ matrix that remains to be optimized, and

$$\mathbf{L} = (\hat{\mathbf{H}}^H\hat{\mathbf{H}} + \mathbf{R}_T^{-1})^{-1}\hat{\mathbf{H}}^H,\ \mathbf{R}_T = (\hat{\mathbf{T}}^H\hat{\mathbf{T}} + \mathbf{I}_{N_b})^{-1}.$$

Let us introduce a positive semi-definite (PSD) matrix $\mathbf{\Omega} = \mathbf{L}(\hat{\mathbf{H}}\mathbf{R}_T\hat{\mathbf{H}}^H + \mathbf{I}_{N_r})\mathbf{L}^H$ and its eigenvalue decomposition (EVD) $\mathbf{\Omega} = \mathbf{U}_\omega\mathbf{\Lambda}_\omega\mathbf{U}_\omega^H$, where $\mathbf{\Lambda}_\omega$ is the diagonal eigenvalues matrix with eigenvalues $\lambda_{\omega,k}$, $k = 1,\cdots,N_b$, arranged in descending order. Let us also introduce the EVD of $\mathbf{G}^H\mathbf{G} = \mathbf{U}_g\mathbf{\Lambda}_g\mathbf{U}_g^H$, where $\mathbf{\Lambda}_g$ is the diagonal eigenvalue matrix with eigenvalues $\lambda_{g,k}$, $k = 1,\cdots,N_r$, arranged in descending order.

Based on the result in [9], $\mathbf{P}$ has the structure of

$$\mathbf{P} = \mathbf{U}_{g,1}\mathbf{\Delta}\mathbf{U}_\omega^H \qquad (35)$$

, where $\mathbf{U}_{g,1}$ contains the leftmost N_b columns of $\mathbf{U}_g$, $\mathbf{\Delta}$ is a diagonal matrix and the solution to the following problem

$$\min_{\mathbf{\Delta}} \text{tr}\left\{\left(\mathbf{\Delta}^H\mathbf{\Lambda}_{g,1}\mathbf{\Delta} + \mathbf{\Lambda}_\omega^{-1}\right)^{-1}\right\} \qquad (36)$$

$$\text{s.t.tr}\left\{\mathbf{\Delta}\mathbf{R}_\omega\mathbf{\Delta}^H\right\} \le P_r. \qquad (37)$$

Here $\mathbf{R}_\omega \triangleq \mathbf{U}_\omega^H\mathbf{L}(\hat{\mathbf{H}}\hat{\mathbf{H}}^H + \mathbf{I}_{N_r})\mathbf{L}^H\mathbf{U}_\omega$, and $\mathbf{\Lambda}_{g,1}$ contains the largest N_b diagonal elements of $\mathbf{\Lambda}_g$. The solution to the problem (36)-(37) can be efficiently obtained by using the Lagrange multiplier method as

$$|\delta_k|^2 = \frac{1}{\lambda_{\omega,k}\lambda_{g,k}}\left(\sqrt{\frac{\lambda_{\omega,k}^2\lambda_{g,k}}{\gamma R_k}} - 1\right)^+,\ k = 1,\cdots,N_b$$

where δ_k is the kth main diagonal element of $\mathbf{\Delta}$, $(x)^+ \triangleq \max(x,0)$, $R_k = [\mathbf{R}_\omega]_{k,k}$, and $\gamma > 0$ is the Lagrangian multiplier and chosen to satisfy the power constraint in (37).

Secondly, we optimize source matrices $\{\mathbf{B}_i\}$ with updated $\mathbf{F}$. Using the identity of

$$\begin{aligned}&\text{tr}\left\{\left[\mathbf{I}_m + \mathbf{A}_{m\times n}\mathbf{B}_{n\times m}\right]^{-1}\right\}\\ &= \text{tr}\left\{\left[\mathbf{I}_n + \mathbf{B}_{n\times m}\mathbf{A}_{m\times n}\right]^{-1}\right\} + m - n\end{aligned}$$

, for a given feasible $\mathbf{F}$, the objective function in (27) can be rewritten as

$$\begin{aligned}\text{MMSE} &= \text{tr}\left\{\left[\mathbf{I}_{2N_d} + \bar{\mathbf{H}}\bar{\mathbf{H}}^H\mathbf{C}^{-1}\right]^{-1}\right\} + N_b - 2N_d\\ &= \text{tr}\left\{\left[\mathbf{I}_{2N_d} + \sum_{i=1}^{K}\tilde{\mathbf{H}}_i\mathbf{Q}_i\tilde{\mathbf{H}}_i^H\right]^{-1}\right\} + N_b - 2N_d\end{aligned} \qquad (38)$$

where $\tilde{\mathbf{H}}_i \triangleq \mathbf{C}^{-\frac{1}{2}}\begin{bmatrix}\mathbf{G}\mathbf{F}\mathbf{H}_i\\ \mathbf{T}_i\end{bmatrix}$ and $\mathbf{Q}_i = \mathbf{B}_i\mathbf{B}_i^H$ is the source covariance matrix of the ith user.

From (38), we can see that only $\tilde{\mathbf{H}}_i$ is involved with the direct links. $\mathbf{Q}_i$ constructed by $\mathbf{B}_i$ is relatively independent. So the optimization of the optimal $\mathbf{B}_i$ is equivalent to the optimization of the optimal $\mathbf{Q}_i$.

Based on (38) and the power constraints in (29)-(30), we can get the final optimization problem in the similar way as [6]. For the sake of simplicity, the derivation is omitted. The equivalent optimization problem is the following semidefinite programming (SDP) problem

$$\min_{\{\mathbf{Q}_i\},\mathbf{X}} \operatorname{tr}\{\mathbf{X}\} \tag{39}$$

$$\text{s.t.} \begin{bmatrix} \mathbf{X} & \mathbf{I}_{2N_d} \\ \mathbf{I}_{2N_d} & \mathbf{I}_{2N_d} + \sum_{i=1}^{K} \tilde{\mathbf{H}}_i \mathbf{Q}_i \tilde{\mathbf{H}}_i^H \end{bmatrix} \succeq 0 \tag{40}$$

$$\sum_{i=1}^{K} \operatorname{tr}\{\mathbf{Q}_i \boldsymbol{\Psi}_i\} \le \bar{P}_r \tag{41}$$

$$\operatorname{tr}\{\mathbf{Q}_i\} \le P_i,\ \mathbf{Q}_i \succeq 0,\ i = 1,\cdots,K. \tag{42}$$

We use the CVX software package [15] to solve the problem (39)-(42). The procedure of using the evolved Bi-Step algorithm to solve the source and relay matrices optimization problem (28)-(30) is shown in Table II. It is noted that the iteration order of $\mathbf{F}$ and $\{\mathbf{B}_i\}$ can be exchanged. Because the performances of the two iteration orders are almost identical, we use the iteration order as shown in Table II. Since the update of $\mathbf{F}$ and $\{\mathbf{B}_i\}$ at each iteration may decrease or maintain, but can not increase the value of the objective function (27), and (27) is lower bounded by zero, a monotonic convergence of the evolved Bi-Step algorithm toward (at least) a stationary point follows directly from this observation.

TABLE II: PROCEDURE OF THE EVOLVED BI-STEP ALGORITHM

1. Initialize the algorithm with $\mathbf{Q}_i^{(0)} = \frac{P_i}{N_i}$, $i = 1,\cdots,K$; Set $n = 0$.
2. Solve the subproblem (36)-(37) using given $\{\mathbf{B}_i^{(n)}\}$ to obtain $\mathbf{F}^{(n)}$.
3. Solve the subproblem (39)-(42) using known $\mathbf{F}^{(n)}$ to obtain $\mathbf{Q}_i^{(n+1)}$, $i = 1,\cdots,K$.
4. Calculate $\text{MSE}^{(n+1)}$ using $\mathbf{F}^{(n)}$ and $\{\mathbf{B}_i^{(n+1)}\}$, If $\frac{\text{MSE}^{(n)} - \text{MSE}^{(n+1)}}{\text{MSE}^{(n+1)}} \le \varepsilon$, then end. Otherwise, let $n := n+1$ and go to step 2.

Remark: From Table I and Table II, we can find that the procedures of the evolved Tri-Step and Bi-Step algorithm seem to be the same with the original ones of [6]. However, the influence of the direct links is already reflected in the update of the source, relay and receiver matrices. Although the optimization problem is different from that in [6], the idea of converting the optimization problem into convex subproblems is useful. By variable substitution, we get similar forms of subproblems with [6], such as the QCQP problem, the SDP problem, etc. So the evolved Tri-Step and Bi-Step algorithms, which take the direct links into consideration can be seen as the generalization of the original ones.

IV. NUMERICAL EXAMPLES

In this section, simulation results are carried out to verify the performance superiority of the evolved Tri-Step and Bi-Step algorithms with the existing algorithms. Five other schemes are compared with the evolved Tri-Step and Bi-Step algorithms in terms of MSE and BER. The alternative schemes are: 1) the Naive Amplify-and-Forward (NAF) algorithm without direct links (referred to as NAF without DL); 2) the Naïve Amplify-and-Forward algorithm with direct links (referred to as NAF with DL); 3) the Pseudo Match-and-Forward (PMF) algorithm without direct links (referred to as PMF without DL); 4) the Pseudo Match-and-Forward algorithm with direct links (referred to as PMF with DL); 5) the original Tri-Step algorithm without direct links in [6] (referred to as O-Tri-Step). Since the original Bi-Step algorithm has similar performance with the original Tri-Step algorithm, its performance is not shown.

For the convenience of comparison with the above algorithms, we choose the same simulation parameters as those in [6]. We consider a two user AF MIMO relay system with direct links. All the channels are flat Rayleigh fading with zero mean and variances σ_g^2 / N_r, $\sigma_{h,i}^2 / N_i$, $\sigma_{t,i}^2 / N_i$, $i = 1,2$, for $\mathbf{G}$, $\mathbf{H}_i$, $\mathbf{T}_i$, $i = 1,2$, respectively. We define

$$\text{SNR}_{\text{r-d}} \triangleq \frac{\sigma_g^2 P_r N_d}{N_r},\ \ \text{SNR}_{\text{i-r}} \triangleq \frac{\sigma_{h,i}^2 P_i N_r}{N_i},$$
$$\text{SNR}_{\text{i-d}} \triangleq \frac{\sigma_{t,i}^2 P_i N_d}{N_i},\quad i = 1,2$$

as the Signal-to-Noise Ratio (SNR) of the relay-destination, user-i-relay, and user-i-destination links, respectively. For simplicity, we assume $N_1 = N_2 = N_s$, $\text{SNR}_{\text{1-r}} = \text{SNR}_{\text{2-r}} = \text{SNR}_{\text{s-r}}$, and $\text{SNR}_{\text{1-d}} = \text{SNR}_{\text{2-d}} = \text{SNR}_{\text{s-d}}$ throughout the simulations. In particular, $\text{SNR}_{\text{s-d}}$ varies with either the position of the source nodes when the relay and destination nodes are fixed, or the position of the destination node when the source and relay nodes are fixed. Due to a larger pass loss, we assume that $\text{SNR}_{\text{s-d}}$ is 10dB lower than either $\text{SNR}_{\text{s-r}}$ or $\text{SNR}_{\text{r-d}}$. All simulation results are averaged over 1000 independent channel realizations. QPSK signal constellations are used for transmitting.

In the first simulation example, we set $N_s = 2$ and $N_r = N_d = 4$. The MSE normalized by the number of data streams (denoted as the NMSE) of all algorithms tested versus $\text{SNR}_{\text{s-r}}$ with $\text{SNR}_{\text{r-d}} = 20$ dB is shown in Fig. 2. The NMSE of all algorithms versus $\text{SNR}_{\text{r-d}}$ with $\text{SNR}_{\text{s-r}} = 20$ dB is illustrated in Fig. 3. It can be seen from Fig. 2 and Fig. 3 that the NMSE of the AF multiuser MIMO relay system can be greatly reduced by considering the source-destination links. The evolved Tri-

Step and Bi-Step algorithms consistently have a better MSE performance than all the other schemes over the whole $\text{SNR}_{\text{s-r}}$ and $\text{SNR}_{\text{r-d}}$ range. This indicates that the source-destination links are properly exploited in the newly derived source, relay and receiver matrices, which are obtained from the evolved algorithms.

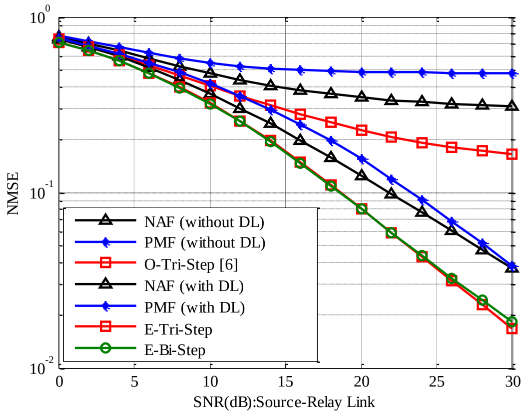

Fig. 2. Example 1: Normalized MSE versus $\text{SNR}_{\text{s-r}}$. $N_s = 2$, $N_r = N_d = 4$. $\text{SNR}_{\text{r-d}} = 20$ dB, $\text{SNR}_{\text{s-d}} = \text{SNR}_{\text{s-r}} - 10$ dB.

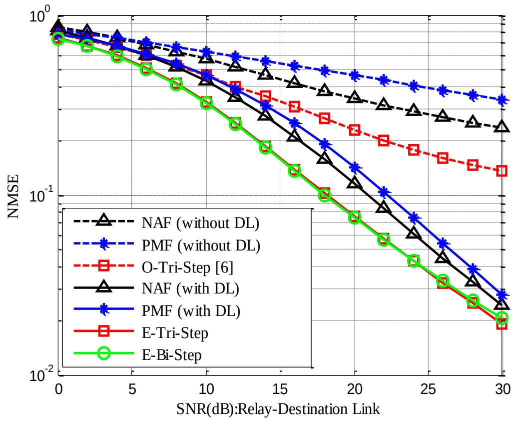

Fig. 3 Example 1: Normalized MSE versus $\text{SNR}_{\text{r-d}}$. $N_s = 2$, $N_r = N_d = 4$. $\text{SNR}_{\text{s-r}} = 20$ dB, $\text{SNR}_{\text{s-d}} = \text{SNR}_{\text{r-d}} - 10$ dB.

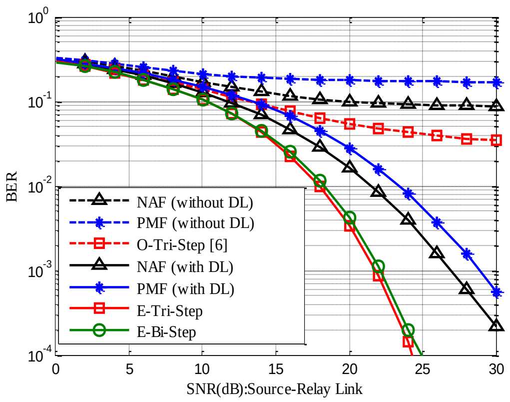

Fig. 4 Example 2: BER versus $\text{SNR}_{\text{s-r}}$. $N_s = 3$, $N_r = N_d = 6$. $\text{SNR}_{\text{r-d}} = 20$ dB, $\text{SNR}_{\text{s-d}} = \text{SNR}_{\text{s-r}} - 10$ dB.

In the second example, the BER performance of all algorithms tested is compared for $N_s = 3$ and $N_r = N_d = 6$. Fig. 4 shows the BER performance of all algorithms versus $\text{SNR}_{\text{s-r}}$ for $\text{SNR}_{\text{r-d}} = 20$ dB, whereas Fig. 5 demonstrates the BER performance of all algorithms versus $\text{SNR}_{\text{r-d}}$ for $\text{SNR}_{\text{s-r}} = 20$ dB. From Fig. 4 and Fig. 5, we can see that the BER performance has a large improvement after considering the source-destination links. The evolved Tri-Step and Bi-Step algorithms perform better than the other algorithms over the entire $\text{SNR}_{\text{s-r}}$ and $\text{SNR}_{\text{r-d}}$ range.

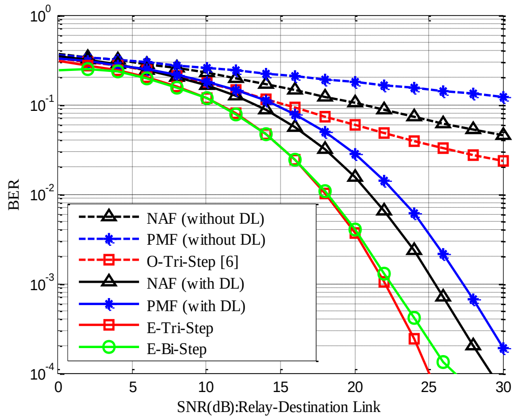

Fig. 5 Example 2: BER versus $\text{SNR}_{\text{r-d}}$. $N_s = 3$, $N_r = N_d = 6$. $\text{SNR}_{\text{s-r}} = 20$ dB, $\text{SNR}_{\text{s-d}} = \text{SNR}_{\text{r-d}} - 10$ dB.

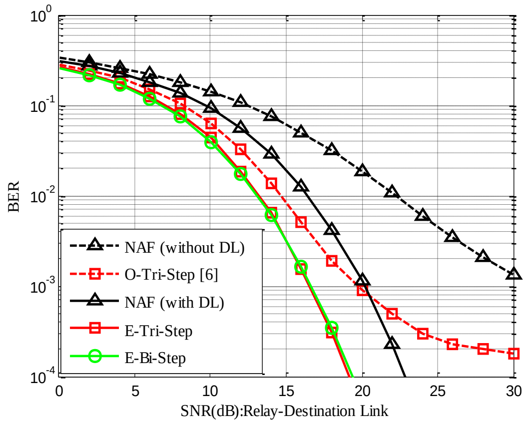

Fig. 6 Example 3: BER versus $\text{SNR}_{\text{r-d}}$. $N_s = 2$, $N_r = 6$, $N_d = 8$. $\text{SNR}_{\text{s-r}} = 20$ dB, $\text{SNR}_{\text{s-d}} = \text{SNR}_{\text{r-d}} - 10$ dB.

In the third example, we change the set of antennas and compare the BER performance. Let $N_s = 2$, $N_r = 6$, and $N_d = 8$. Note that the number of relay and destination antennas is larger than the sum of source antennas. The PMF algorithm is not included because it requires $N_b = N_d$. All the algorithms benefit significantly from the increase of the antenna numbers. The evolved Tri-Step and Bi-Step algorithms still perform better than the other algorithms. From all the three examples, the results clearly demonstrate the better performance of the proposed algorithms.

The E-Tri-Step algorithm performs slightly better than the E-Bi-Step algorithm. The reason is as follows. To get the exact solution for the relay matrix in (35), $\mathbf{\Delta}$ should be considered as a general matrix. But, by assuming $\mathbf{\Delta}$ is diagonal, the closed-form solution of the relay matrix is obtained. Hence, due to the assumption, the E-Bi-Step algorithm loses the optimality for the relay matrix.

Finally, we compare the computational complexity of the two algorithms. For simplicity, we assume the antenna number at each node is N. In each iteration of the evolved Tri-Step algorithm, the operation complexity of updating $\mathbf{W}$ and $\mathbf{F}$, such as matrix inversion and SVD, is $O(N^3)$. As for the updating of $\{\mathbf{B}_i\}$, the complexity order is about $O(N^6)$ for the QCQP problem. For each iteration of the evolved Bi-Step algorithm, the main operation for optimizing $\mathbf{F}$ is matrix EVD, whose complexity order is $O(N^3)$, while the SDP problem for updating $\{\mathbf{B}_i\}$ is $O(N^7)$.

TABLE III: AVERAGE NUMBER OF ITERATIONS

$\mathrm{SNR}_{\text{r-d}}$ (dB)	0	6	12	18	24	30
E-Tri-Step	5	5	8	13	21	31
E-Bi-Step	2	2	3	4	5	5

Table III shows the average number of iterations required by the evolved Tri-Step and Bi-Step algorithms till convergence with $N_s = 2$, $N_r = N_d = 6$, and $\mathrm{SNR}_{\text{s-r}} = 20$ dB. Both algorithms are required to converge up to $\varepsilon = 10^{-3}$. It can be seen from Table III that the number of iterations required by the evolved Tri-Step algorithm increases with $\mathrm{SNR}_{\text{r-d}}$, whereas that of the evolved Bi-Step algorithm is almost unchanged.

With the above analysis, we can get the conclusion that with small antenna number and high SNR, the evolved Bi-Step algorithm has a lower computational complexity. On the other hand, when N is large and the SNR is low, the evolved Tri-Step algorithm has a smaller complexity.

We can find that the performance of the two algorithms is in accordance with the complexity of the two algorithms. In other words, at low SNR, the evolved Bi-Step algorithm has higher complexity because its complexity of one iteration is higher, and the difference of the total iteration number between the two algorithms is small. So the evolved Bi-Step algorithm has slightly better performance than the evolved Tri-Step algorithm at low SNR such as in Fig. 5. However, as SNR increases, the iteration number of the evolved Tri-Step algorithm increases significantly as shown in Table III, which means higher complexity at higher SNR, while that of the evolved Bi-Step algorithm is almost unchanged. This explains why the gap between the E-Bi-Step and the E-Tri-Step methods becomes large as SNR increases. Compared with the original algorithms in [6], the computational complexity of the evolved algorithms is almost the same.

V. CONCLUSIONS

We have investigated the optimal source, relay, and receiver matrices design for AF multiuser MIMO relay communication systems when the source-destination links are considered. The evolved Tri-Step method and Bi-Step method are derived for the new system model to exploit the spatial diversity gain brought by the source-destination links. The optimal source, relay and receiver matrices taking into consideration of the direct links are obtained. From the theoretical derivation of MSE of the signal waveform estimation at the destination node, we show that the existence of the source-destination links bring benefits no matter how weak they are. Numerical examples demonstrate that the proposed algorithms perform much better than the existing ones in terms of both MSE and BER.

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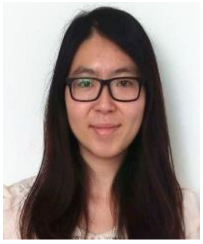

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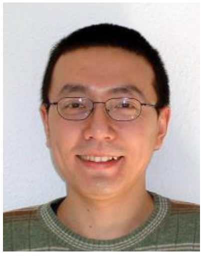

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