A Compressive Sensing Based Iterative Algorithm for Channel and Impulsive Noise Estimation in Underwater Acoustic OFDM Systems

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***Abstract*—Underwater acoustic (UA) channel is often affected by strong impulsive noise. In this paper, a compressive sensing based iterative algorithm is proposed to accurately estimate the channel state information and mitigate the impulsive noise, which is important to ensure high-speed data transmission in UA orthogonal frequency-division multiplexing communication systems. By exploiting the sparsity of the impulsive noise and channel impulse response in the time domain, we adopt the orthogonal matching pursuit algorithm to improve the accuracy of channel estimation with relatively low computational complexity. The proposed algorithm is evaluated through numerical simulations and real data collected during a UA communication experiment conducted in December 2015 in the estuary of the Swan River, Western Australia. The results show that the proposed algorithm has a better performance than existing approaches.**

I. Introduction

It is well-known that UA channels are much more challenging for wireless communication than terrestrial radio channels because of significant delay, large Doppler shift, strong multi-path fading, and limited bandwidth [1]. To meet the increasing demand for high-rate underwater communication systems in both defence and civil applications, orthogonal frequency-division multiplexing (OFDM) technique has been applied in UA communication systems to eliminate the inter-symbol interference (ISI) caused by multipath propagation [2]. It has been shown that the noise in UA communication systems consists of additive white Gaussian noise (AWGN) and impulsive noise [3]. Due to the non-Gaussianity of noise, it is difficult to theoretically analyze the performance of a UA communication system [4]-[6]. Moreover, detection and estimation of transmitted signals are much more challenging under impulsive noise than under Gaussian noise. In fact, impulsive noise can significantly degrade the performance of UA OFDM systems [7].

Recently, by exploiting the sparsity of impulsive noise, the compressive sensing (CS) technology [8] has been adopted in joint channel and impulsive noise estimation [9]-[11], which has been demonstrated to outperform traditional blanking and clipping methods [12]. While the algorithm in [9] uses only pilot subcarriers, additional null subcarriers are required in [10] and [11] to estimate the impulsive noise. A convex optimization based impulsive noise reduction method has been proposed in [13]. However, the method in [13] has a high computational complexity when solving the convex optimization problem.

In this paper, we propose a CS based iterative channel and impulsive noise estimation algorithm. Since the amplitude of impulsive noise is much higher than that of the received signal of interest, we first estimate the impulsive noise using the least-squares (LS) method. Then, we subtract the estimated impulsive noise from the received signals and perform channel estimation using the pilot subcarriers. An iterative method is proposed to further improve the accuracy of channel estimation and impulsive noise cancelation.

The effectiveness of the proposed algorithm is verified through numerical simulations and on real data collected during a UA communication experiment conducted in December 2015 in the estuary of the Swan River, Western Australia. The simulation and experiment results show that the proposed algorithm outperforms the existing CS based joint channel and impulsive noise estimation method [9] in terms of both bit-error-rate (BER) and frame-error-rate (FER).

II. System Model

We consider an OFDM system with N subcarriers, where N_d subcarriers are used for data transmission, N_p pilot subcarriers are uniformly distributed, N_{null} null subcarriers are placed at both edges of the passband to avoid interference caused by out-of-band signals, and the remaining N_{cfo} subcarriers are used for carrier frequency offset compensation. Each transmitted data frame consists of L_s OFDM symbols. The baseband discrete time signal of one OFDM symbol can be represented as

$$\mathbf{x} = \mathbf{F}^H \mathbf{d} \tag{1}$$

where $\mathbf{F}$ is the $N \times N$ normalized discrete Fourier transform (DFT) matrix, $(\cdot)^H$ denotes conjugate transpose, and $\mathbf{d}$ is a vector containing the transmitted symbols of all N subcarriers.

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To combat the ISI caused by the multipath fading, a cyclic prefix (CP) with a length of T_{cp} is inserted in front of each OFDM symbol. The obtained signal is then modulated at a carrier frequency of f_c and transmitted through the UA channel. In this paper, we adopt the following commonly used assumptions [10] on the UA channel:

- The path gains, delays, and the Doppler scaling factors remain constant over one OFDM symbol.
- All paths share the same Doppler scaling factor a.

Based on these assumptions, the UA channel can be modeled as

$$h(n) = \sum_{l=0}^{L-1} A_l \delta(n + an - \tau_l) \tag{2}$$

where L is the maximal number of paths, A_l and τ_l represent the amplitude and time delay of the lth path, respectively. It is notable that for UA channels, only $N_h \ll L$ paths have non-zero amplitude, which indicates the sparsity of the channel impulse response. Therefore, we can adopt CS recovery algorithms to estimate the channel state information.

Using a resampling factor $\hat{a}$ and considering the impulsive noise, the received baseband signal after removing the CP can be expressed as

$$\mathbf{r}_t \approx \mathbf{\Xi}(\mathbf{h}_t \circledast \mathbf{x}) + \mathbf{v}_t + \mathbf{n}_t \tag{3}$$

where $\mathbf{\Xi} = \mathrm{diag}([1, e^{j2\pi\xi/B}, \ldots, e^{j2\pi\xi(N-1)/B}])$ is a diagonal matrix consisting of the phase rotations, $\xi = f_c(a-\hat{a})/(1+\hat{a})$ is the residual frequency offset caused by resampling error, B is the bandwidth of the transmitted OFDM signal, $\mathbf{h}_t$ is the channel impulse response vector, $\mathbf{v}_t$ and $\mathbf{n}_t$ are impulsive noise and AWGN vectors, respectively, and $\circledast$ denotes the circular convolution with the length of N.

After removing the frequency offset from (3) and conducting the DFT, the received signal of each OFDM symbol can be written as

$$\begin{aligned} \mathbf{r}_f &= \mathbf{D}\mathbf{h}_f + \mathbf{v}_f + \mathbf{n}_f \\ &= \mathbf{D}\mathbf{F}\mathbf{h}_t + \mathbf{F}\mathbf{v}_t + \mathbf{F}\mathbf{n}_t \end{aligned} \tag{4}$$

where $\mathbf{D} = \mathrm{diag}(\mathbf{d})$ is a diagonal matrix containing signals of all N subcarriers, $\mathbf{h}_f = \mathbf{F}\mathbf{h}_t$, $\mathbf{v}_f = \mathbf{F}\mathbf{v}_t$, and $\mathbf{n}_f = \mathbf{F}\mathbf{n}_t$.

III. The Proposed CS based Iterative Algorithm

In this section, we propose a CS based iterative algorithm to estimate the channel impulse response and impulsive noise. To mitigate the impact of impulsive noise on channel estimation, we first estimate the impulsive noise and subtract its estimated value from the received signal, based on the fact that the amplitude of the impulsive noise is much larger than the received signal of interest and other noise. Then we perform channel estimation using the pilot symbols. To improve the accuracy of the channel and impulsive noise estimation, we adopt an iterative structure as shown in Fig. 1. In the following, we show the details of the proposed algorithm.

Let us introduce an $N_p \times N$ matrix $\mathbf{P}$ which selects N_p pilot subcarriers out of the total N subcarrier, i.e., $\mathbf{P}$ has unit entries at the $(i, \mathcal{I}_p[i])$-th position, $i = 1, \ldots, N_p$, and zero elsewhere, where $\mathcal{I}_p$ contains the indices of subcarriers with pilot symbols. Let us also introduce $\mathbf{v}_I$ as a vector which contains all the N_I samples of impulsive noise in one OFDM block, and an $N \times N_I$ matrix indicating the position of the impulsive noise given by

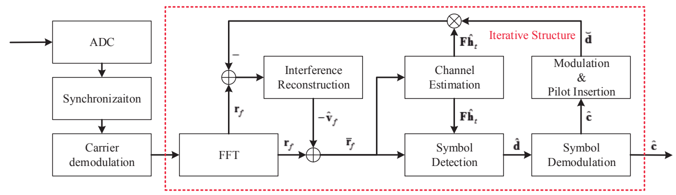

Fig. 1. Block diagram of the proposed CS based iterative algorithm.

$$\mathbf{P}_I[i,k] = \begin{cases} 1, & i = \mathcal{I}_I[k], k = 1, \ldots, N_I \\ 0, & \text{otherwise} \end{cases}$$

where $\mathcal{I}_I$ contains the indices of samples with impulsive noise. The received signal vector on the pilot subcarriers can be written based on (4) as

$$\mathbf{r}_p = \mathbf{P}\mathbf{D}\mathbf{F}\mathbf{h}_t + \mathbf{P}\mathbf{F}\mathbf{P}_I\mathbf{v}_I + \mathbf{P}\mathbf{F}\mathbf{n}_t. \tag{5}$$

We detect the positions of the impulsive noise by introducing a threshold parameter β in the time domain. The possible positions of the impulsive noise which are denoted by indices $\hat{\mathcal{I}}_I[i]$ can be obtained from

$$|r_t(\hat{\mathcal{I}}_I[i])|^2 > G\beta, \quad i = 1, \ldots, \hat{N}_I \tag{6}$$

where G is the average power of the received OFDM symbol and $\hat{N}_I$ is the number of possible positions of impulsive noise. An initial estimation of the impulsive noise can be obtained from (5) using the LS method as

$$\hat{\mathbf{v}}_I = (\mathbf{M}_v^H \mathbf{M}_v)^{-1} \mathbf{M}_v^H \mathbf{r}_p \tag{7}$$

where $\mathbf{M}_v = \mathbf{P}\mathbf{F}\hat{\mathbf{P}}_I$, $(\cdot)^{-1}$ stands for the matrix inversion, and $\hat{\mathbf{P}}_I$ is the estimated value of $\mathbf{P}_I$. The effect of the impulsive noise can be mitigated in the frequency domain from the received signal as

$$\bar{\mathbf{r}}_f = \mathbf{r}_f - \hat{\mathbf{v}}_f. \tag{8}$$

where $\hat{\mathbf{v}}_f = \mathbf{F}\hat{\mathbf{P}}_I\hat{\mathbf{v}}_I$.

Assuming a perfect impulsive noise cancelation, the received pilot symbols $\bar{\mathbf{r}}_p$ can be expressed as

$$\bar{\mathbf{r}}_p = \mathbf{P}\mathbf{D}\mathbf{F}\mathbf{h}_t + \mathbf{P}\mathbf{F}\mathbf{n}_t. \tag{9}$$

As $\mathbf{h}_t$ contains $N_h \ll N$ non-zero elements, the recovery of $\mathbf{h}_t$ can be regarded as a sparse signal estimation problem. We can adopt various CS recovery algorithms to estimate $\mathbf{h}_t$ from (9). Among them, the orthogonal matching pursuit (OMP) algorithm [14] is widely used because of its low complexity and easy implementation. After obtaining $\hat{\mathbf{h}}_t$, the channel frequency response $\mathbf{h}_f$ can be calculated by an N-point fast Fourier transform (FFT). The transmitted bits can then be estimated after channel equalization and demodulation.

The estimated bits $\hat{\mathbf{c}}$ are not sent to the channel decoder directly. Instead, we reconstruct the transmitted OFDM symbol $\breve{\mathbf{d}}$ after modulation and pilot insertion, and then subtract $\breve{\mathbf{d}}$ from the received signal $\mathbf{r}_f$ to obtain

$$\breve{\mathbf{r}}_t = \mathbf{F}^H(\mathbf{r}_f - \breve{\mathbf{D}}\mathbf{F}\hat{\mathbf{h}}_t) \quad (10)$$

$$\breve{\mathbf{r}}_p = \mathbf{r}_p - \mathbf{P}\breve{\mathbf{D}}\mathbf{F}\hat{\mathbf{h}}_t \quad (11)$$

where $\breve{\mathbf{D}} = \mathrm{diag}(\breve{\mathbf{d}})$ is the diagonal matrix containing the reconstructed OFDM symbol. Then we use (6) to estimate the positions of the impulsive noise, where $\breve{\mathbf{r}}_t$ in (10) is used instead of $\mathbf{r}_t$. Using $\breve{\mathbf{r}}_p$ in (11) instead of $\mathbf{r}_p$, We apply (7) to estimate $\mathbf{v}_I$ again. As the interference caused by the transmitted signals is removed in (10) and (11), the estimation of the impulsive noise is more accurate than the initial estimation.

By using $\breve{\mathbf{d}}$, (9) can be extended from pilot subcarriers to both pilot and data subcarriers as

$$\bar{\mathbf{r}}_{pd} = \mathbf{P}_{pd}\mathbf{D}\mathbf{F}\mathbf{h}_t + \mathbf{P}_{pd}\mathbf{F}\mathbf{n}_t \quad (12)$$

where $\mathbf{P}_{pd}$ is an $(N_p + N_d) \times N$ matrix having unit entries at the $(i, \mathcal{I}_{pd}[i])$-th position, $i = 1, \ldots, N_p + N_d$, and zeros elsewhere. Here $\mathcal{I}_{pd}$ contains the indices of subcarriers with pilot and data symbols. It is worth noting that compared with (9), the channel estimation from (12) has a higher accuracy as the number of observations increases.

To further improve the performance of channel estimation and impulsive noise suppression, we carry out the steps above in an iterative fashion till the convergence of the uncoded BER or for a predetermined number of iterations. Finally, the estimated bits $\hat{\mathbf{c}}$ are sent to the channel decoder.

The computational complexity of the proposed algorithm is analyzed as follows. It can be seen from Fig. 1 that the major operations required are mainly in channel and impulsive noise estimation, and the reconstruction of the transmitted symbol. Let us denote the number of iterations as N_i. The complexity order of calculating the weight matrix in (7) is $\mathcal{O}(N_i N_p N_I^2)$. From (10), the complexity of reconstructing the transmitted symbols can be written as $\mathcal{O}(N_i N^2)$. The OMP algorithm used in channel estimation has a complexity of $\mathcal{O}(N_h N_p L)$ in the first iteration. But in other iterations, it increases to $\mathcal{O}(N_h(N_p + N_d)L)$ as shown in (12). Therefore, the complexity order of the proposed algorithm is $\mathcal{O}(N_i N_p N_I^2 + N_h N_p L + (N_i - 1)N_h(N_p + N_d)L + N_i N^2)$.

IV. Results and Discussions

In this section, we evaluate the performance of the proposed algorithm through both numerical simulations and on data collected from a UA communication experiment. In the numerical simulations, we assume that the UA channel has 15 paths. The time delay between two adjacent paths follows the exponential distribution with a mean value of 1 ms. The gain of each path is an independent complex Gaussian variable, whose variance decreases exponentially with the time delay. The ratio of the channel variances between the start and the end of the CP is 20dB.

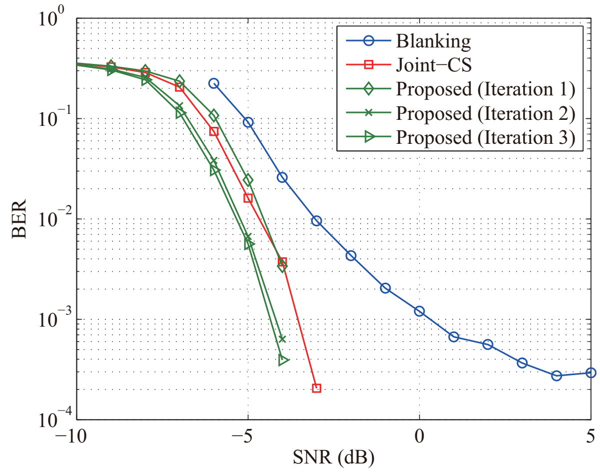

Fig. 2. BER performance of various algorithms versus SNR.

For simplicity, we use the two-component Gaussian mixture model [10] to generate the additive noise with a probability density function of

$$(1-q)\mathcal{N}(0, \sigma^2) + q\mathcal{N}(0, \sigma_I^2) \quad (13)$$

where $\mathcal{N}(0, \cdot)$ denotes a zero-mean complex Gaussian probability density function, σ^2 is the variance of the background (non-impulsive) noise, σ_I^2 is the variance of the impulsive noise, and q is the probability of occurrence of the impulsive noise. We define the signal-to-noise ratio (SNR) as

$$\mathrm{SNR} = \frac{P_s}{q\delta_I^2 + (1-q)\delta^2} \quad (14)$$

where P_s is the average power of the transmitted signal. In the simulations, we set $q = 0.02$, $\delta_I^2/\delta^2 = 400$, and normalize the power of the modulation constellation to $P_s = 1$.

The BER performance of various algorithms versus the SNR averaged over 10^4 channel realizations is shown in Fig. 2. It can be seen that the blanking method suffers from an error floor as the SNR increases, because this method destroys the structure of the transmitted signal. Compared with the CS based joint channel and impulsive noise estimation (Joint-CS) algorithm [9], the proposed algorithm has a gain of approximately 1dB after the first iteration. As the number of iterations increases, the amount of BER performance improvement reduces but the computational complexity increases. Thus, in practice, it is sufficient to run the proposed algorithm for 2 or 3 iterations.

The proposed algorithm is applied to process the data collected during a UA communication experiment conducted in December 2015 in the estuary of the Swan River, Western Australia. The transmitter and receiver locations during the experiment is shown in Fig. 3. The parameters of the experimental UA communication system are listed in Table I. The information bits are encoded through turbo codes with a 1/2 or 1/3 coding rate. The coded data stream is modulated by quadrature phase-shift keying (QPSK) constellations.

TABLE I
PARAMETERS OF THE EXPERIMENTAL UA COMMUNICATION SYSTEM

Bandwidth	4 kHz	Carrier frequency	12 kHz
N	512	T_{cp}	25 ms
N_d	325	N_{null}	128
N_{null}	36	N_{cfo}	23
L_s	5	β	5

Fig. 3. Transmitter and receiver locations during the experiment.

In each transmission, 500 data frames were sent through a transducer mounted on a steel frame which was 0.5 meter above the river bed and cabled to shore. The receiver hydrophone was located at 936 meters away from the transducer and was also firmly attached to a steel frame. Thus, the Doppler shift of the channel is small during the experiment. Each of the first 250 data frames conveyed 1088 information bits (1/3 rate code), and each of the remaining 250 data frames contained 1632 information bits (1/2 rate code). The data files recorded at the receiver during three transmissions were named T83, T84, and T85, respectively.

The proposed algorithm is applied to process the received data. The performance of the three algorithms tested in terms of the raw (uncoded) BER, coded BER, and FER is shown in Tables II, III, IV for the T83, T84, and T85 files, respectively.

The results in Table II show that the signals in the T83 file are least affected by impulsive noise and channel fading during the experiment. In this case, all three algorithms tested including the blanking method can successfully recover the information bits. Nevertheless, the proposed algorithm achieves a lower raw BER than the joint-CS method after the second iteration.

TABLE II
PERFORMANCE COMPARISON FOR THE T83 FILE

Coding rate	Method	Raw BER	Coded BER	FER
1/3	Blanking	5.2%	0	0
	Joint-CS	3.5%	0	0
	Proposed (1 It.)	5.2%	0	0
	Proposed (2 It.)	3.1%	0	0
	Proposed (3 It.)	2.7%	0	0
1/2	Blanking	4.7%	0	0
	Joint-CS	3.3%	0	0
	Proposed (1 It.)	4.8%	0	0
	Proposed (2 It.)	3.1%	0	0
	Proposed (3 It.)	2.8%	0	0

TABLE III
PERFORMANCE COMPARISON FOR THE T84 FILE

Coding rate	Method	Raw BER	Coded BER	FER
1/3	Blanking	15.4%	1.2%	6.2%
	Joint-CS	14.0%	0.6%	2.9%
	Proposed (1 It.)	15.1%	1.0%	5.4%
	Proposed (2 It.)	12.9%	0.09%	0.4%
	Proposed (3 It.)	12.3%	0.05%	0.4%
1/2	Blanking	14.6%	15.9%	84.6%
	Joint-CS	13.5%	10.9%	61.4%
	Proposed (1 It.)	14.1%	14.5%	76.8%
	Proposed (2 It.)	12.1%	6.5%	39.0%
	Proposed (3 It.)	11.5%	4.9%	31.7%

TABLE IV
PERFORMANCE COMPARISON FOR THE T85 FILE

Coding rate	Method	Raw BER	Coded BER	FER
1/3	Blanking	11.3%	0	0
	Joint-CS	9.1%	0	0
	Proposed (1 It.)	11.2%	0	0
	Proposed (2 It.)	8.5%	0	0
	Proposed (3 It.)	7.8%	0	0
1/2	Blanking	11.6%	3.8%	24.2%
	Joint-CS	9.8%	0.6%	5.7%
	Proposed (1 It.)	11.1%	2.3%	14.5%
	Proposed (2 It.)	9.0%	0.2%	2.4%
	Proposed (3 It.)	8.4%	0.1%	1.2%

From Table III, we can note that the signals in the T84 file are most heavily affected by the impulsive noise. In this environment, the blanking method does not work properly with the 1/2 rate code, as the FER reaches 84.6%. Compared with the blanking method, after the third iteration, the proposed algorithm reduces the FER from 6.2% to 0.4% for the 1/3 rate code, and from 84.6% to 31.7% for the 1/2 rate code.

It can be seen from the results in Table IV that the signals in the T85 file are affected moderately by the impulsive noise. Similar to Tables II and III, the proposed algorithm yields the lowest BER and FER after the third iteration. The results in Tables II-IV demonstrate that the proposed algorithm consistently has the best performance among three algorithms tested under various noise conditions.

V. CONCLUSIONS

In this paper, a compressive sensing based iterative algorithm is proposed for accurate channel estimation and impulsive noise mitigation in UA OFDM communication systems. Compared with the existing algorithms, the proposed algorithm does not need additional null subcarriers. Both simulation and experiment results show that the proposed algorithm has a better performance than the blanking method after the first iteration. After the second iteration, the proposed algorithm yields a lower BER and FER than an existing joint channel estimation and impulsive noise suppression algorithm under various noise and channel conditions.

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