Optimal Joint Source and Relay Beamforming for MIMO Relays with Direct Link

Yue Rong, *Member, IEEE*

***Abstract*—In this letter, we investigate the optimal structure of the source precoding matrix and the relay amplifying matrix for non-regenerative multiple-input multiple-output (MIMO) relay communication systems with the direct source-destination link. We show that both the optimal source precoding matrix and the optimal relay amplifying matrix have a beamforming structure. Based on this structure, an iterative joint source and relay beamforming algorithm is developed to minimize the mean-squared error (MSE) of the signal waveform estimation. Numerical example demonstrates an improved performance of the proposed algorithm.**

***Index Terms*—Beamforming, direct link, MIMO relay, linear non-regenerative relay.**

I. INTRODUCTION

RECENTLY, non-regenerative multiple-input multiple-output (MIMO) relay communications have attracted much research interest. For a MIMO relay system, there are two independent links between the source and the destination nodes: the source-relay-destination link and the direct source-destination link. Many works studied the optimal relay amplifying matrix for the source-relay-destination channel. In [1] and [2], the optimal relay amplifying matrix which maximizes the source-destination mutual information (MI) was derived. In [3] and [4], the relay amplifying matrix was designed to minimize the mean-squared error (MSE) of the signal waveform estimation at the destination. A unified framework was developed in [5] to jointly optimize the source precoding matrix and the relay amplifying matrix for a broad class of objective functions. All these works did not consider the direct source-destination link.

In practice, the direct source-destination link provides valuable spatial diversity to the MIMO relay system and should not be ignored. Obviously, the relay amplifying matrix designed for the source-relay-destination link only [1]-[4] is not optimal when the direct link is included. Recently, it was shown in [6] that with the direct link, the optimal relay amplifying matrix has a general beamforming structure. In [7], an alternating algorithm was proposed to optimize both the source and relay matrices. However, the algorithm in [7] is strictly suboptimal and has a high computational complexity, since the optimal beamforming structure of the relay amplifying matrix was not exploited. Moreover, the structure of the optimal source precoding matrix with the direct link was not investigated in [6] and [7].

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Y. Rong is with the Department of Electrical and Computer Engineering, Curtin University of Technology, Bentley, WA 6102, Australia (e-mail: y.rong@curtin.edu.au).

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In this letter, we derive the optimal structure of the source precoding matrix when the direct link is included. Interestingly, we prove that the source precoding matrix has a transmit beamforming structure. By exploiting the optimal structure of the source and relay matrices, we develop an iterative joint source and relay beamforming algorithm to minimize the MSE of the signal waveform estimation at the destination node. Numerical example demonstrates the effectiveness of our algorithm.

II. SYSTEM MODEL

We consider a three-node MIMO communication system where the source node transmits information to the destination node with the aid of one relay node. The source, relay, and destination nodes are equipped with N_s, N_r, and N_d antennas, respectively. Due to its merit of simplicity, a non-regenerative strategy is applied at the relay node to amplify and forward the received signals. The signal vector received at the destination node over two consecutive time slots is

$$\mathbf{y}(t) \triangleq \begin{bmatrix} \mathbf{y}_d(t+1) \\ \mathbf{y}_d(t) \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{rd}\mathbf{F}\mathbf{H}_{sr} \\ \mathbf{H}_{sd} \end{bmatrix} \mathbf{B}\mathbf{s}(t) + \begin{bmatrix} \mathbf{H}_{rd}\mathbf{F}\mathbf{v}_r(t) + \mathbf{v}_d(t+1) \\ \mathbf{v}_d(t) \end{bmatrix} \quad (1)$$

where $\mathbf{y}_d(t+1)$ and $\mathbf{y}_d(t)$ are $N_d \times 1$ signal vectors received at the destination through the source-relay-destination link and the direct source-destination link, respectively, $\mathbf{H}_{sd}$, $\mathbf{H}_{rd}$, $\mathbf{H}_{sr}$ are the channel matrices for the source-destination, relay-destination, and source-relay links with dimension $N_d \times N_s$, $N_d \times N_r$, $N_r \times N_s$, respectively, $\mathbf{s}(t)$ is the $N_b \times 1$ source signal vector, $\mathbf{F}$ is the $N_r \times N_r$ relay amplifying matrix, $\mathbf{B}$ is the $N_s \times N_b$ source precoding matrix, $\mathbf{v}_r(t)$ is the $N_r \times 1$ noise vector at the relay, $\mathbf{v}_d(t+1)$ and $\mathbf{v}_d(t)$ are the $N_d \times 1$ noise vectors at the destination at time $t+1$ and t, respectively.

We assume that the source signal vector satisfies $\mathrm{E}[\mathbf{s}(t)(\mathbf{s}(t))^H] = \mathbf{I}_{N_b}$ and all noises are independent and identically distributed (i.i.d.) additive white Gaussian noise (AWGN) with zero mean and unit variance. Here $\mathrm{E}[\cdot]$ stands for the statistical expectation, $\mathbf{I}_n$ is an $n \times n$ identity matrix, and $(\cdot)^H$ denotes the Hermitian transpose.

When a linear receiver is used at the destination node, the estimated signal waveform is given by

$$\hat{\mathbf{s}}(t) = \mathbf{W}^H \mathbf{y}(t) \quad (2)$$

where $\mathbf{W}$ is a $2N_d \times N_b$ weight matrix. The receiver weight matrix which minimizes the signal waveform estimation error is the Wiener filter given by [8]

$$\mathbf{W} = \left(\bar{\mathbf{H}}\bar{\mathbf{H}}^H + \bar{\mathbf{C}}\right)^{-1} \bar{\mathbf{H}} \quad (3)$$

where

$$\bar{\mathbf{H}} \triangleq \begin{bmatrix} \mathbf{H}_{rd}\mathbf{F}\mathbf{H}_{sr} \\ \mathbf{H}_{sd} \end{bmatrix} \mathbf{B}, \quad \bar{\mathbf{C}} \triangleq \begin{bmatrix} \mathbf{H}_{rd}\mathbf{F}\mathbf{F}^H\mathbf{H}_{rd}^H + \mathbf{I}_{N_d} & \mathbf{0}_{N_d\times N_d} \\ \mathbf{0}_{N_d\times N_d} & \mathbf{I}_{N_d} \end{bmatrix}.$$

Here $\mathbf{0}_{m\times n}$ denotes an $m \times n$ matrix with all zeros entries, and $(\cdot)^{-1}$ denotes the matrix inversion. Using (1)-(3), the MSE matrix $\mathbf{E}$ of the signal waveform estimation is given by

$$\begin{aligned} \mathbf{E} &= \mathrm{E}\big[(\hat{\mathbf{s}}(t) - \mathbf{s}(t))(\hat{\mathbf{s}}(t) - \mathbf{s}(t))^H\big] \\ &= \big[\mathbf{I}_{N_b} + \mathbf{B}^H\mathbf{H}_{sd}^H\mathbf{H}_{sd}\mathbf{B} + \mathbf{B}^H\mathbf{H}_{sr}^H\mathbf{F}^H\mathbf{H}_{rd}^H \\ &\quad \times(\mathbf{H}_{rd}\mathbf{F}\mathbf{F}^H\mathbf{H}_{rd}^H + \mathbf{I}_{N_d})^{-1}\mathbf{H}_{rd}\mathbf{F}\mathbf{H}_{sr}\mathbf{B}\big]^{-1}. \end{aligned} \quad (4)$$

III. Optimal Joint Source And Relay Beamforming

With the direct link, the relay amplifying matrix and source precoding matrix optimization problem is written as

$$\min_{\mathbf{F},\mathbf{B}} \quad \mathrm{tr}(\mathbf{E}) \quad (5)$$

$$\text{s.t.} \quad \mathrm{tr}\big(\mathbf{F}(\mathbf{H}_{sr}\mathbf{B}\mathbf{B}^H\mathbf{H}_{sr}^H + \mathbf{I}_{N_r})\mathbf{F}^H\big) \le P_r \quad (6)$$

$$\mathrm{tr}\big(\mathbf{B}\mathbf{B}^H\big) \le P_s \quad (7)$$

where $\mathrm{tr}(\cdot)$ denotes the trace of a matrix, (6) and (7) is the power constraint at the relay node and the source node, respectively, and $P_r > 0$, $P_s > 0$ is the power budget available at the relay and source node, respectively. The problem (5)-(7) is highly nonconvex and a closed-form expression of the optimal $\mathbf{F}$ and $\mathbf{B}$ is intractable. In this letter, we develop an iterative algorithm to optimize $\mathbf{F}$ and $\mathbf{B}$.

First we derive the optimal structure of $\mathbf{F}$ and $\mathbf{B}$. For a given $\mathbf{B}$, the relay matrix $\mathbf{F}$ is optimized by solving the following problem

$$\min_{\mathbf{F}} \quad \mathrm{tr}(\mathbf{E}) \quad (8)$$

$$\text{s.t.} \quad \mathrm{tr}\Big(\mathbf{F}(\mathbf{H}_{sr}\mathbf{B}\mathbf{B}^H\mathbf{H}_{sr}^H + \mathbf{I}_{N_r})\mathbf{F}^H\Big) \le P_r\,. \quad (9)$$

Let us introduce the following singular value decomposition

$$\mathbf{H}_{sr}\mathbf{B} = \mathbf{U}_s\mathbf{\Lambda}_s\mathbf{V}_s^H, \qquad \mathbf{H}_{rd} = \mathbf{U}_r\mathbf{\Lambda}_r\mathbf{V}_r^H$$

where $\mathbf{\Lambda}_s$ and $\mathbf{\Lambda}_r$ are $R_s \times R_s$ and $R_r \times R_r$ square diagonal matrices (i.e., zero singularvalues are excluded). Here $R_s \triangleq \mathrm{rank}(\mathbf{H}_{sr}\mathbf{B})$, $R_r \triangleq \mathrm{rank}(\mathbf{H}_{rd})$, $\mathrm{rank}(\cdot)$ denotes the rank of a matrix. Based on the theorem in [6], the optimal $\mathbf{F}$ is given by

$$\mathbf{F} = \mathbf{V}_r\mathbf{A}\mathbf{U}_s^H\,. \quad (10)$$

It can be seen from (10) that $\mathbf{F}$ has a generalized beamforming structure. The relay first performs receive beamforming using the Hermitian transpose of the left singular matrix of the effective source-relay channel $\mathbf{H}_{sr}\mathbf{B}$. Then the relay conducts a linear precoding operation using $\mathbf{A}$. Finally, a transmit beamforming is performed by the relay using the right singular matrix of the relay-destination channel $\mathbf{H}_{rd}$.

Substituting (10) back into (8) and (9), the optimal $\mathbf{A}$ can be obtained by solving the following optimization problem

$$\min_{\mathbf{A}} \quad \mathrm{tr}\Big(\big[\mathbf{I}_{N_b} + \mathbf{B}^H(\mathbf{H}_{sd}^H\mathbf{H}_{sd} + \mathbf{H}_{sr}^H\mathbf{H}_{sr})\mathbf{B} - \mathbf{V}_s\mathbf{\Lambda}_s\left(\mathbf{A}^H\mathbf{\Lambda}_r^2\mathbf{A} + \mathbf{I}_{R_s}\right)^{-1}\mathbf{\Lambda}_s\mathbf{V}_s^H\big]^{-1}\Big) \quad (11)$$

$$\text{s.t.} \quad \mathrm{tr}\big(\mathbf{A}(\mathbf{\Lambda}_s^2 + \mathbf{I}_{R_s})\mathbf{A}^H\big) \le P_r \quad (12)$$

where the matrix inversion lemma is applied to obtain the objective function in (11) from (4). For $N_b \ge 2$, the problem (11)-(12) does not have a closed-form solution for general $\mathbf{H}_{sd}$. We should resort to numerical methods, such as the projected gradient method [9] to solve (11)-(12).

For a fixed $\mathbf{F}$, the source precoding matrix $\mathbf{B}$ is optimized by solving the following problem

$$\min_{\mathbf{B}} \quad \mathrm{tr}\big([\mathbf{I}_{N_b} + \mathbf{B}^H\mathbf{\Psi}_1\mathbf{B}]^{-1}\big) \quad (13)$$

$$\text{s.t.} \quad \mathrm{tr}(\mathbf{B}^H\mathbf{\Psi}_2\mathbf{B}) \le \bar{P}_r \quad (14)$$

$$\mathrm{tr}(\mathbf{B}^H\mathbf{B}) \le P_s \quad (15)$$

where $\mathbf{\Psi}_1 \triangleq \mathbf{H}_{sd}^H\mathbf{H}_{sd} + \mathbf{H}_{sr}^H\mathbf{F}^H\mathbf{H}_{rd}^H(\mathbf{H}_{rd}\mathbf{F}\mathbf{F}^H\mathbf{H}_{rd}^H + \mathbf{I}_{N_d})^{-1}\mathbf{H}_{rd}\mathbf{F}\mathbf{H}_{sr}$, $\mathbf{\Psi}_2 \triangleq \mathbf{H}_{sr}^H\mathbf{F}^H\mathbf{F}\mathbf{H}_{sr}$, $\bar{P}_r \triangleq P_r - \mathrm{tr}(\mathbf{F}\mathbf{F}^H)$. The following Theorem establishes the structure of the optimal $\mathbf{B}$.

Theorem 1: The optimal $\mathbf{B}$ as the solution to the problem (13)-(15) is given by

$$\mathbf{B} = \mathbf{M}^{-H}\mathbf{V}_1\mathbf{D} \quad (16)$$

where $\mathbf{M}\mathbf{M}^H = \mu_1\mathbf{I}_{N_s} + \mu_2\mathbf{\Psi}_2$, $\mathbf{D}$ is an $N_b \times N_b$ diagonal matrix, $\mathbf{M}^{-1}\mathbf{\Psi}_1\mathbf{M}^{-H} = \mathbf{V}\mathbf{\Sigma}\mathbf{V}^H$, and $\mathbf{V}_1$ contains N_b columns of $\mathbf{V}$ associated with eigenvalues that are greater than one. Here $\mathbf{\Sigma}$ is the diagonal eigenvalue matrix, and $\mu_1 \ge 0$, $\mu_2 \ge 0$ are the Lagrange multipliers.

Proof: The Lagrangian function associated with the problem (13)-(15) is given by

$$\begin{aligned} \mathcal{L} &= \mathrm{tr}([\mathbf{I}_{N_b} + \mathbf{B}^H\mathbf{\Psi}_1\mathbf{B}]^{-1}) + \mu_1(\mathrm{tr}(\mathbf{B}^H\mathbf{B}) - P_s) \\ &\quad + \mu_2(\mathrm{tr}(\mathbf{B}^H\mathbf{\Psi}_2\mathbf{B}) - \bar{P}_r) \\ &= \mathrm{tr}([\mathbf{I}_{N_b} + \mathbf{B}^H\mathbf{\Psi}_1\mathbf{B}]^{-1}) + \mathrm{tr}(\mathbf{B}^H\mathbf{M}\mathbf{M}^H\mathbf{B}) \\ &\quad - \mu_1 P_s - \mu_2\bar{P}_r. \end{aligned} \quad (17)$$

Making the derivative of $\mathcal{L}$ with respect to $\mathbf{B}$ be zero, we obtain

$$\frac{\partial\mathcal{L}}{\partial\mathbf{B}} = -\big[\mathbf{I}_{N_b} + \mathbf{B}^H\mathbf{\Psi}_1\mathbf{B}\big]^{-2}\mathbf{B}^H\mathbf{\Psi}_1 + \mathbf{B}^H\mathbf{M}\mathbf{M}^H = 0 \quad (18)$$

where the derivatives of $\partial\mathrm{tr}(\mathbf{\Theta}\mathbf{X}^{-1})/\partial\mathbf{X} = -(\mathbf{X}^{-1}\mathbf{\Theta}\mathbf{X}^{-1})^T$ and $\partial\mathrm{tr}(\mathbf{\Theta}\mathbf{X})/\partial\mathbf{X} = \mathbf{\Theta}^T$ are used. Since $\mathbf{M}$ is non-singular, (18) can be equivalently written as

$$\begin{aligned} &-\big[\mathbf{I}_{N_b} + \mathbf{B}^H\mathbf{M}\mathbf{M}^{-1}\mathbf{\Psi}_1\mathbf{M}^{-H}\mathbf{M}^H\mathbf{B}\big]^{-2} \\ &\quad \times\mathbf{B}^H\mathbf{M}\mathbf{M}^{-1}\mathbf{\Psi}_1\mathbf{M}^{-H} + \mathbf{B}^H\mathbf{M} = 0. \end{aligned} \quad (19)$$

Substituting $\mathbf{M}^{-1}\mathbf{\Psi}_1\mathbf{M}^{-H} = \mathbf{V}\mathbf{\Sigma}\mathbf{V}^H$ back into (19) and assuming $\mathbf{M}^H\mathbf{B} = \mathbf{V}_1\mathbf{T}$, where $\mathbf{T}$ is non-singular, we have

$$-\big[\mathbf{I}_{N_b} + \mathbf{T}^H\mathbf{\Sigma}_1\mathbf{T}\big]^{-2}\mathbf{T}^H\mathbf{\Sigma}_1\mathbf{V}_1^H + \mathbf{T}^H\mathbf{V}_1^H = 0 \quad (20)$$

where $\mathbf{\Sigma}_1$ contains the N_b eigenvalues that are greater than 1. Applying the matrix identity of $(\mathbf{I}_m + \mathbf{X}\mathbf{X}^H)^{-1}\mathbf{X} = \mathbf{X}(\mathbf{I}_n + \mathbf{X}^H\mathbf{X})^{-1}$ for any $m \times n$ matrix $\mathbf{X}$, we have

$$\big[\mathbf{I}_{N_b} + \mathbf{T}^H\mathbf{\Sigma}_1\mathbf{T}\big]^{-2}\mathbf{T}^H\mathbf{\Sigma}_1^{\frac{1}{2}} = \mathbf{T}^H\mathbf{\Sigma}_1^{\frac{1}{2}}\Big[\mathbf{I}_{N_b} + \mathbf{\Sigma}_1^{\frac{1}{2}}\mathbf{T}\mathbf{T}^H\mathbf{\Sigma}_1^{\frac{1}{2}}\Big]^{-2}. \quad (21)$$

Using (21), (20) is equivalent to

$$-\mathbf{T}^H\mathbf{\Sigma}_1^{\frac{1}{2}}\Big[\mathbf{I}_{N_b} + \mathbf{\Sigma}_1^{\frac{1}{2}}\mathbf{T}\mathbf{T}^H\mathbf{\Sigma}_1^{\frac{1}{2}}\Big]^{-2}\mathbf{\Sigma}^{\frac{1}{2}} + \mathbf{T}^H = 0. \quad (22)$$

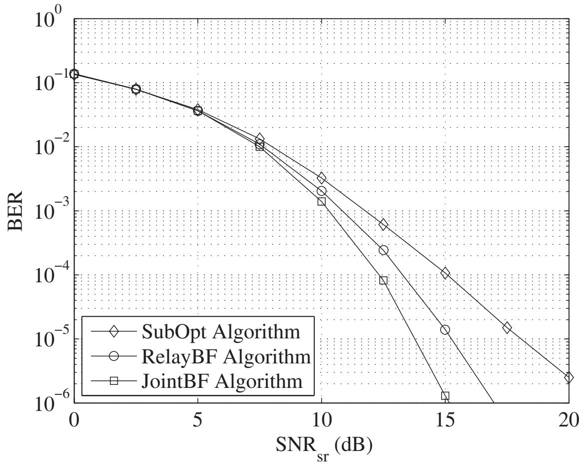

Fig. 1. BER versus $\mathrm{SNR}_{\mathrm{sr}}$. $N_s = N_d = 2$, $N_r = 6$, $\mathrm{SNR}_{\mathrm{rd}}$ = 20dB, $\mathrm{SNR}_{\mathrm{sd}} = \mathrm{SNR}_{\mathrm{sr}} - 10$dB.

Solving (22) for $\mathbf{T}$, we obtain $\mathbf{T}\mathbf{T}^H = \mathbf{\Sigma}_1^{-\frac{1}{2}} - \mathbf{\Sigma}_1^{-1}$, which is valid since the diagonal elements of $\mathbf{\Sigma}_1$ are greater than 1. Consequently, we have $\mathbf{T} = (\mathbf{\Sigma}_1^{-\frac{1}{2}} - \mathbf{\Sigma}_1^{-1})^{\frac{1}{2}}$, and $\mathbf{B} = \mathbf{M}^{-H}\mathbf{V}_1(\mathbf{\Sigma}_1^{-\frac{1}{2}} - \mathbf{\Sigma}_1^{-1})^{\frac{1}{2}}$. Thus, we prove Theorem 1 with $\mathbf{D} = (\mathbf{\Sigma}_1^{-\frac{1}{2}} - \mathbf{\Sigma}_1^{-1})^{\frac{1}{2}}$. $\square$

From (16) we see that $\mathbf{B}$ has a transmit beamforming structure, where the directions of the beams are given by $\mathbf{M}^{-H}\mathbf{V}_1$, while $\mathbf{D}$ represents the power allocation at each beam. The unknown Lagrange multipliers μ_1 and μ_2 in (16) can be obtained by solving the dual optimization problem associated with the problem (13)-(15) as explained in the following. Substituting (16) into (17) we have $\mathcal{L} = \mathrm{tr}(2\mathbf{\Sigma}_1^{-\frac{1}{2}} - \mathbf{\Sigma}_1^{-1}) - \mu_1 P_s - \mu_2 \bar{P}_r$. The dual optimization problem of the original problem (13)-(15) is given by

$$\max_{\mu_1 \geq 0, \mu_2 \geq 0} \quad \mathrm{tr}(2\mathbf{\Sigma}_1^{-\frac{1}{2}} - \mathbf{\Sigma}_1^{-1}) - \mu_1 P_s - \mu_2 \bar{P}_r. \qquad (23)$$

Since the dual problem is convex, the problem (23) can be efficiently solved by the interior-point method [10].

We have proved that in order to jointly minimize $\mathrm{tr}(\mathbf{E})$, both the optimal $\mathbf{B}$ and $\mathbf{F}$ have a general beamforming structure as given in (10) and (16), respectively. Exploiting this optimal structure, the problem (5)-(7) can be solved by an iterative algorithm. This algorithm is first initialized at $\mathbf{B} = \sqrt{P_s/N_s}\mathbf{I}_{N_s}$. Then $\mathbf{F}$ is updated by solving the problem (11)-(12) with a fixed $\mathbf{B}$, and $\mathbf{B}$ is updated by solving the problem (13)-(15) with a given $\mathbf{F}$. The updating of $\mathbf{F}$ and $\mathbf{B}$ is operated in an alternating fashion. Note that the conditional updates of $\mathbf{F}$ and $\mathbf{B}$ may either decrease or maintain but cannot increase the objective function $\mathrm{tr}(\mathbf{E})$. Monotonic convergence of $\mathbf{F}$ and $\mathbf{B}$ follows directly from this observation.

IV. Numerical Example

We simulate a MIMO relay system with $N_s = N_d = 2$ and $N_r = 6$. All channel matrices have Gaussian entries with zero-mean and variances σ_s^2/N_s, σ_r^2/N_r, σ_d^2/N_s for $\mathbf{H}_{sr}$, $\mathbf{H}_{rd}$, and $\mathbf{H}_{sd}$, respectively. Consequently, the signal-to-noise ratios (SNRs) are defined as $\mathrm{SNR}_{\mathrm{sr}} \triangleq \sigma_s^2 P_s/N_s$, $\mathrm{SNR}_{\mathrm{rd}} \triangleq \sigma_r^2 P_r/N_r$, $\mathrm{SNR}_{\mathrm{sd}} \triangleq \sigma_d^2 P_s/N_s$ for the source-relay, relay-destination, and source-destination links, respectively. We simulate a scenario where the distance between the relay and destination nodes is fixed, while the source-relay distance (and thus also the source-destination distance) are varying. In particular, we set $\mathrm{SNR}_{\mathrm{rd}} = 20$dB and $\mathrm{SNR}_{\mathrm{sd}} = \mathrm{SNR}_{\mathrm{sr}} - 10$dB. All simulation results are averaged over 1000 independent channel realizations.

We compare the performance of the proposed iterative joint beamforming (JointBF) algorithm with the suboptimal (SubOpt) algorithm in [5] which optimizes only the source-relay-destination link, and the relay-only beamforming (RelayBF) algorithm in [6]. For both the JointBF and RelayBF algorithms, the projected gradient method is applied to optimize $\mathbf{A}$ in the relay amplifying matrix.

Fig. 1 shows the performance of three algorithms in terms of bit-error-rate (BER) versus $\mathrm{SNR}_{\mathrm{sr}}$ using the QPSK constellation. It can be seen that the SubOpt algorithm has the worst performance, since it does not consider the direct link. The JointBF algorithm outperforms the other two algorithms in the whole $\mathrm{SNR}_{\mathrm{sr}}$ range, since it jointly optimizes the source and relay matrices. In fact, it achieves a higher diversity order than the other algorithms.

V. Conclusion

In this letter, we have proved the optimal beamforming structure of the source precoding matrix for non-regenerative MIMO relay systems with the direct source-destination link. An iterative joint source and relay beamforming algorithm is developed to minimize the MSE of the signal waveform estimation.

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