A ROBUST LINEAR RECEIVER FOR UPLINK MULTI-USER MIMO SYSTEMS BASED ON PROBABILITY-CONSTRAINED OPTIMIZATION AND SECOND-ORDER CONE PROGRAMMING

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ABSTRACT

Traditional receiver algorithms developed for multiple-input multiple-output (MIMO) wireless communication systems are based on the assumption that the channel state information (CSI) is perfectly known at the receiver. However, in real environments the exact CSI is unavailable. In this paper, we address the problem of robustness of multiuser space-time block coded MIMO systems against imperfect CSI and develop a simple linear receiver which guarantees robustness against CSI errors with a certain selected probability. The proposed receiver is formulated via a probability-constrained optimization problem which is further relaxed using Chebyshev inequality to a much simpler second-order cone programming (SOCP) problem. Simulations demonstrate that the proposed receiver has a moderate performance degradation as compared to the exact solution to the original probability-constrained optimization problem, while offering much simpler implementation.

1. INTRODUCTION

In uplink cellular communications with multiple antennas at base stations (BSs) and mobile stations (users), spatial diversity techniques can be employed to combat fading and improve the system performance. Applications of space-time block-coded MIMO wireless systems to multiuser scenarios gained recently a significant interest [1], [2].

Both linear and nonlinear receiver algorithms for multiuser MIMO systems have been recently proposed in the literature [2]-[5]. Although linear techniques such as minimum mean square error (MMSE) or minimum variance (MV) receivers are suboptimal, they recently gained much interest due to their low computational complexity. Unfortunately, such receivers require perfect knowledge of the channel state information (CSI) of at least the user of interest. When the exact CSI is unavailable, the performance of these linear receivers may degrade severely. Therefore, linear receivers robust against *imperfect* CSI are of great demand [5], [6]. In this paper, we design a linear receiver for multi-user space-time block coded MIMO systems that is robust against CSI errors.

Recently, several authors have addressed the problem of robust linear receiver design. For example, the diagonal loading (DL) based MV approach was used in [5] to provide robustness against CSI and data covariance matrix mismatches. However, the selection of the DL factor in [5] is *ad hoc*. In [6], robust modifications of MV multiuser receivers of [5] have been developed based on the idea of worst-case performance optimization which has been earlier used in adaptive beamforming [7], [8] and multiuser detection [9], [10]. Taking into account that the worst-case design may be overly conservative, a probability constrained optimization-based receiver was developed that guarantees the robustness against CSI errors with a certain selected probability [11]. However, the computational cost of the robust receiver in [11] is quite high because it is based on the nonlinear programming (NLP) approach. In this paper, we derive a much simpler probability constrained optimization-based linear receiver by relaxing the problem of [11] to the second-order cone programming (SOCP) form.

2. MULTI-ACCESS MIMO LINEAR RECEIVERS

In this section, we review the point-to-point and multi-access space-time block-coded MIMO wireless system models. The latter model is used to formulate our multi-access MIMO linear receiver design problem.

2.1. Point-to-Point MIMO Model

The point-to-point MIMO model can be written as [12]

$$\boldsymbol{Y} = \boldsymbol{X}\boldsymbol{H} + \boldsymbol{N} \tag{1}$$

where $\boldsymbol{Y}$ is the $T \times M$ complex matrix of the received data, $\boldsymbol{X}$ is the $T \times N$ complex matrix of the transmitted data, $\boldsymbol{H}$ is the $N \times M$ complex matrix of quasi-static Rayleigh flat-fading channel whose coherence time is assumed to be longer than T, $\boldsymbol{N}$ is the $T \times M$ complex additive white

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Gaussian noise (AWGN) at the receiver, N is the number of transmit antennas, M is the number of receive antennas, and T is the data block length.

If the user data $s_1, \ldots, s_K$ are encoded by some space-time block code (STBC), then the matrix $\boldsymbol{X}$ has the following structure

$$\boldsymbol{X} = \sum_{k=1}^{K} (\boldsymbol{C}_k \mathrm{Re}\{s_k\} + \boldsymbol{D}_k \mathrm{Im}\{s_k\}) \tag{2}$$

where $\boldsymbol{C}_k = \boldsymbol{X}(\boldsymbol{e}_k)$, $\boldsymbol{D}_k = \boldsymbol{X}(j\boldsymbol{e}_k)$, $j = \sqrt{-1}$, and $\boldsymbol{e}_k$ is the $K \times 1$ vector having one in the kth position and zeros elsewhere. Inserting (2) into (1) yields the following model [5], [12]

$$\underline{\boldsymbol{Y}} = \boldsymbol{A}\underline{\boldsymbol{s}} + \underline{\boldsymbol{N}} \tag{3}$$

where

$$\boldsymbol{A} \triangleq [\underline{\boldsymbol{C}_1 \boldsymbol{H}}, \ldots, \underline{\boldsymbol{C}_K \boldsymbol{H}}, \underline{\boldsymbol{D}_1 \boldsymbol{H}}, \ldots, \underline{\boldsymbol{D}_K \boldsymbol{H}}]$$

and, for any complex matrix $\boldsymbol{P}$, the "underline" operator is defined as

$$\underline{\boldsymbol{P}} \triangleq \begin{bmatrix} \mathrm{vec}(\mathrm{Re}\{\boldsymbol{P}\}) \\ \mathrm{vec}(\mathrm{Im}\{\boldsymbol{P}\}) \end{bmatrix}. \tag{4}$$

2.2. Multi-Access MIMO Model

In the uplink multi-access case, the model (3) can be extended as [5]

$$\begin{aligned} \underline{\boldsymbol{Y}} &= \sum_{p=1}^{P} \boldsymbol{A}_p \underline{\boldsymbol{s}_p} + \underline{\boldsymbol{N}} \\ &= \boldsymbol{M}\boldsymbol{z} + \underline{\boldsymbol{N}} \end{aligned} \tag{5}$$

where

$$\begin{aligned} \boldsymbol{M} &\triangleq [\boldsymbol{A}_1, \ldots, \boldsymbol{A}_P] \\ \boldsymbol{z} &\triangleq [\underline{\boldsymbol{s}_1}^T, \ldots, \underline{\boldsymbol{s}_P}^T]^T \\ \boldsymbol{A}_p &\triangleq [\boldsymbol{a}_{p,1}, \ldots, \boldsymbol{a}_{p,2K}] \\ \boldsymbol{a}_{p,k} &\triangleq \underline{\boldsymbol{F}_k \boldsymbol{H}_p} \\ \boldsymbol{F}_k &\triangleq \begin{cases} \boldsymbol{C}_k, & k = 1, \ldots, K \\ \boldsymbol{D}_{k-K}, & k = K+1, \ldots, 2K \end{cases} \end{aligned}$$

$\boldsymbol{H}_p$ is the channel matrix between the pth user and the BS; and P is the number of users.

2.3. Multi-Access MIMO Linear Receivers

One of conceptually simplest receivers that can be used to decode the user symbols from the data (5) is the standard zero-forcing (ZF) receiver which estimates the vector $\boldsymbol{z}$ as

$$\hat{\boldsymbol{z}} = \boldsymbol{M}^{\dagger} \underline{\boldsymbol{Y}} \tag{6}$$

where $(\cdot)^{\dagger}$ denotes the matrix pseudo-inverse. Unfortunately, the performance of the ZF receiver may be far from the optimal one, especially when the signal-to-noise ratio (SNR) is not sufficiently high.

Let us develop a slightly different framework for the design of linear receivers. We now aim to decode only the symbols of the user-of-interest (desired user) and assume without any loss of generality that the first user is the desired one. Then, the estimate of the symbol vector $\underline{\boldsymbol{s}_1}$ at the output of a linear receiver can be written in the following form [5]

$$\underline{\hat{\boldsymbol{s}}_1} = \boldsymbol{W}^T \underline{\boldsymbol{Y}} \tag{7}$$

where $\boldsymbol{W} = [\boldsymbol{w}_1, \ldots, \boldsymbol{w}_{2K}]$ is the $2MT \times 2K$ real matrix of weight coefficients. Consequently, the kth symbol of the user of interest can be estimated as

$$[\hat{\boldsymbol{s}}_1]_k = [\underline{\hat{\boldsymbol{s}}_1}]_k + j[\underline{\hat{\boldsymbol{s}}_1}]_{k+K} = \boldsymbol{w}_k^T \underline{\boldsymbol{Y}} + j\boldsymbol{w}_{k+K}^T \underline{\boldsymbol{Y}}.$$

The problem now is to find $\boldsymbol{W}$ that separates the signals from different users. In the case when the exact CSI is available at the BS, the array processing-type model in (5) opens an avenue for employing techniques similar to that used in adaptive beamforming and multiuser detection [5]. For example, the MMSE receiver can be used. Its coefficient matrix can be computed as

$$\boldsymbol{W}_{\mathrm{MMSE}} = [\boldsymbol{w}_{\mathrm{MMSE},1}, \ldots, \boldsymbol{w}_{\mathrm{MMSE},2K}] \tag{8}$$

where $\boldsymbol{w}_{\mathrm{MMSE},k} = \boldsymbol{R}^{-1}\boldsymbol{r}$; $\boldsymbol{R} = \mathrm{E}\{\underline{\boldsymbol{Y}}\,\underline{\boldsymbol{Y}}^T\}$ is the covariance matrix of the received vectorized (real) data; $\boldsymbol{r} = \mathrm{E}\{\underline{\boldsymbol{Y}} \cdot [\underline{\boldsymbol{s}_1}]_k\}$ is the cross-correlation vector between $[\underline{\boldsymbol{s}_1}]_k$ and $\underline{\boldsymbol{Y}}$; and $\mathrm{E}\{\cdot\}$ is the statistical expectation.

An improved version of the MMSE receiver (8) was developed in [5], where the MV approach was used to reject *multi-access interference* (which is caused by the other users than the user of interest) and, at the same time, to completely eliminate *self-interference* (which is caused by the other entries of $\underline{\boldsymbol{s}_1}$ than the entry of interest) by means of additional zero-forcing constraints. The latter approach is motivated by the fact that zero-forcing of self-interference is a very desirable feature of a linear receiver because, otherwise, the symbol-by-symbol detector becomes non-optimal [5]. The MV receiver of [5] has been shown in that paper to perform better than the conventional MMSE/MV receiver that treats multi-access interference and self-interference in the same way.

Although the MMSE receiver in (8) does not explicitly require any CSI knowledge, it requires the knowledge of second-order statistics (SOSs) of signals, which may not be available at the BS and, therefore, must be estimated using sample data. As a result, the performance of this receiver highly depends on the accuracy of the estimates of $\boldsymbol{R}$ and $\boldsymbol{r}$. In order to obtain accurate estimates of the required SOSs, a large number of data blocks is required. Therefore, the performance of the MMSE receiver using the sample data may degrade substantially due to inaccurate estimates of SOSs.

More accurate estimates of the required SOSs can be obtained if the knowledge of user channel matrices, transmit powers of all users, and the noise power are available at the BS. Then, the estimates of $\boldsymbol{R}$ and $\boldsymbol{r}$ can be obtained not through the received data, but directly, by means of using known values of above-mentioned parameters.

The ZF and MMSE receivers and the modifications of the MV receiver developed in [5] are quite sensitive to the CSI errors. Motivated by this fact, the authors of [6] developed robust modifications of the MV receiver using the worst-case performance optimization approach. However, in practical cases worst-case designs can be overly conservative. Therefore, we develop in the next section a new less conservative robust linear receiver based on the idea of probability-constrained optimization.

3. PROPOSED ROBUST LINEAR RECEIVER

Let us consider the error matrix $\boldsymbol{E}_p = \boldsymbol{H}_p - \hat{\boldsymbol{H}}_p$ between the true channel matrix $\boldsymbol{H}_p$ of the pth user and its estimated value $\hat{\boldsymbol{H}}_p$. Using the notations of model (5), we can write

$$\boldsymbol{e}_{p,q} = \boldsymbol{a}_{p,q} - \hat{\boldsymbol{a}}_{p,q} = \underline{\boldsymbol{F}_q\boldsymbol{H}_p} - \underline{\boldsymbol{F}_q\hat{\boldsymbol{H}}_p} = \underline{\boldsymbol{F}_q\boldsymbol{E}_p} \qquad (9)$$

where $\hat{\boldsymbol{a}}_{p,q}$ denotes the estimated value of $\boldsymbol{a}_{p,q}$ and $\boldsymbol{e}_{p,q}$ is the random mismatch vector between $\boldsymbol{a}_{p,q}$ and $\hat{\boldsymbol{a}}_{p,q}$. Note that the last equality in (9) follows from the linearity of the "underline" operator (4).

The MV linear receiver design problem is to estimate each entry of $\underline{\boldsymbol{s}_1}$ by minimizing the noise and *total* interference power while maintaining the distortionless response for this entry of $\underline{\boldsymbol{s}_1}$. In order to provide the robustness against CSI errors, it is useful to take into account the mismatch vector $\boldsymbol{e}_{p,q}$ in (9), i.e., to consider $\hat{\boldsymbol{a}}_{p,q}+\boldsymbol{e}_{p,q}$ instead of $\hat{\boldsymbol{a}}_{p,q}$ [7]. In [11], it has been proposed to suppress the total interference power and to maintain the distortionless response for the kth entry of $\underline{\boldsymbol{s}_1}$ with a certain (high) probability. It is important to stress that this approach is less conservative that the well-known worst-case robust design approach [6], [7] (which may be overly conservative in some cases). Using such an idea of robust design with probability constraints, the receiver coefficient vector $\boldsymbol{w}_k$ for the kth entry of $\underline{\boldsymbol{s}_1}$ can be found as the solution to the following optimization problem [11]:

$$\begin{aligned} \min_{\boldsymbol{w}_k,\boldsymbol{\delta}} \|\boldsymbol{\delta}\| \quad \text{s.t.} \quad & \mathrm{P}\{\boldsymbol{w}_k^T(\hat{\boldsymbol{a}}_{1,k}+\boldsymbol{e}_{1,k}) \geq 1\} \geq \gamma \\ & \mathrm{P}\{|\boldsymbol{w}_k^T(\hat{\boldsymbol{a}}_{1,l}+\boldsymbol{e}_{1,l})| \leq \delta_{1,l}\} \geq \gamma \\ & \mathrm{P}\{|\boldsymbol{w}_k^T(\hat{\boldsymbol{a}}_{p,q}+\boldsymbol{e}_{p,q})| \leq \delta_{p,q}\} \geq \gamma \quad (10) \\ & l = 1,\cdots,2K;\ l \neq k \\ & p = 2,\cdots,P;\ q = 1,\cdots,2K \end{aligned}$$

where $\boldsymbol{\delta} = [\delta_{1,1},\ldots,\delta_{1,k-1},\delta_{1,k+1},\ldots,\delta_{P,2K}]^T$ is the $(2PK-1)\times 1$ vector whose values limit the contribution of multi-access interference and self-interference, γ is a certain probability value which is selected according to the quality of service (QoS) requirements, $\|\cdot\|$ denotes for Frobenius (Euclidian) norm of a matrix (vector), and $\mathrm{P}\{\cdot\}$ stands for the the probability operator whose analytic form is assumed to be known.

In (10), we minimize the total interference power with a certain probability γ while keeping the probability of the distortionless response to the desired entry of $\underline{\boldsymbol{s}_1}$ larger than γ (see the first constraint of (10)). The second and third constraints of (10) correspond to the self-interference and multi-access interference cancellation, respectively. In (10), we do not consider explicitly noise components because for multi-access scenarios the interference suppression is usually much more important than the noise suppression. Moreover, the effect of noise is implicitly taken into account by introducing CSI errors.

The probability bound γ should be selected from the interval $(0, 1)$.

The problem (10) belongs to the class of *probability-constrained* optimization problems [13], [14]. The nonlinear programming (NLP) approach has been used in [11] to solve (10). Although the NLP approach is able to solve this problem exactly under the assumption that the CSI errors are Gaussian, the complexity of such solution is quite high [11]. Below, we propose a much simpler approach by means of converting the original problem into a SOCP problem using a relaxation of (10) based on Chebyshev inequality.

Assuming that $[\boldsymbol{E}_p]_{n,m} \sim \mathcal{CN}(0,\sigma_e^2)$, we can show that [11]

$$\begin{aligned} \boldsymbol{e}_{p,q} &\sim \mathcal{N}\left(\boldsymbol{0}_{2MT}, \frac{\sigma_e^2}{2}(\boldsymbol{I}_{2M}\otimes \boldsymbol{G}_q\boldsymbol{G}_q^T)\right) \\ \boldsymbol{w}_k^T(\hat{\boldsymbol{a}}_{p,q}+\boldsymbol{e}_{p,q}) &\sim \mathcal{N}\left(\boldsymbol{w}_k^T\hat{\boldsymbol{a}}_{p,q}, \frac{\sigma_e^2}{2}\|(\boldsymbol{I}_{2M}\otimes \boldsymbol{G}_q^T)\boldsymbol{w}_k\|^2\right) \end{aligned} \qquad (11)$$

where $\boldsymbol{G}_q \triangleq \begin{cases} \boldsymbol{C}_q, & q = 1,\ldots,K \\ \mathrm{Im}\{\boldsymbol{D}_{q-K}\}, & q = K+1,\ldots,2K \end{cases}$. Here, $\mathcal{CN}(\cdot,\cdot)$ and $\mathcal{N}(\cdot,\cdot)$ stand for the complex and real Gaussian distributions, respectively, and $\otimes$ denotes the Kronecker product. Therefore, $\boldsymbol{w}_k^T(\hat{\boldsymbol{a}}_{p,q}+\boldsymbol{e}_{p,q})$ ($p = 1,\cdots,P$; $q = 1,\cdots,2K$) are also Gaussian.

We will use the following theorem which was proven in [11].

Theorem: If $[\boldsymbol{E}_p]_{n,m}$ is Gaussian and $\gamma \in (0.5, 1)$ then the optimization problem (10) is convex.

If $\gamma \in (0.5, 1)$, then using Theorem and (11), we can find that the first constraint of the problem (10) is mathematically equivalent to the following second order-cone (SOC) constraint

$$\sigma_e\|(\boldsymbol{I}_{2M}\otimes \boldsymbol{G}_k^T)\boldsymbol{w}_k\| \leq \frac{\boldsymbol{w}_k^T\hat{\boldsymbol{a}}_{1,k}-1}{\mathrm{erf}^{-1}(2\gamma-1)} \qquad (12)$$

where $\mathrm{erf}^{-1}(\cdot)$ denotes the inverse error function.

However, the second and third constraints of the problem (10) can not be equivalently converted into SOC constrains. To enable such a conversion, we will use Chebyshev inequality which states that for any real random variable ξ and any positive real number α,

$$\mathrm{P}\{|\xi| \geq \alpha\} \leq \mathrm{E}\{\xi^2\}/\alpha^2 . \tag{13}$$

Next, we show that using (13), the problem (10) can be relaxed to a SOCP problem. Since the second and third constraints in (10) share the same structure, we further discuss the second constraint only. Under the assumption that $e_{1,l}$ has Gaussian distribution, we have

$$\mathrm{E}\{|w_k^T(\hat{a}_{1,l} + e_{1,l})|^2\} = {w_k}^T Q_{1,l} w_k \tag{14}$$

where $Q_{1,l} \triangleq \hat{a}_{1,l}\hat{a}_{1,l}^T + \frac{\sigma_e^2}{2}(I_{2M} \otimes G_l G_l^T)$. Applying (13) and (14) to the second constraint of (10), we have

$$w_k^T Q_{1,l} w_k \leq (1-\gamma)\delta_{1,l}^2 . \tag{15}$$

Finally, the constraint (15) can be converted into a SOC constraint in the following way. Let

$$Q_{1,l} = U_{1,l}\Lambda_{1,l}U_{1,l}^T \tag{16}$$

be the eigenvalue decomposition of matrix $Q_{1,l}$, where $U_{1,l}$ and $\Lambda_{1,l}$ are the eigenvector and eigenvalue matrices, respectively. Using (16) and introducing $T_{1,l} \triangleq U_{1,l}\Lambda_{1,l}^{1/2}$, the constraint (15) can be written as

$$\|T_{1,l}^T w_k\| \leq \sqrt{(1-\gamma)}\delta_{1,l}. \tag{17}$$

Using the objective function of (10) along with inequalities (12) and (17), and converting the third constraint of (10) into the SOC form, the optimization problem (10) can be relaxed to the following SOCP problem

$$\begin{aligned}
\min_{w_k,\delta} \|\delta\| \ \text{s.t.} \ & \sigma_e\|(I_{2M} \otimes G_k^T)w_k\| \leq \frac{w_k^T\hat{a}_{1,k} - 1}{\mathrm{erf}^{-1}(2\gamma - 1)} \\
& \|T_{1,l}^T w_k\| \leq \sqrt{(1-\gamma)}\delta_{1,l} \\
& \|T_{p,q}^T w_k\| \leq \sqrt{(1-\gamma)}\delta_{p,q} \\
& l = 1, \cdots, 2K, \quad l \neq k \\
& p = 2, \cdots, P, \quad q = 1, \cdots, 2K.
\end{aligned} \tag{18}$$

This problem can be efficiently solved using standard interior-point algorithms [15], [16]. For example, using the primal-dual potential reduction method the SOCP problem (18) can be solved with the complexity order of $\mathcal{O}(M^3T^3)$ [16]. This is significantly lower than the overall complexity of solving the NLP problem of [11]. It should be stressed here that the NLP problem of [11] can be solved using sequential programming (SQP) technique that is an iterative approach in which each search direction is the solution of a particular quadratic programming (QP) subproblem. The computational complexity of solving each particular QP subproblem using the primal-dual potential reduction method is $\mathcal{O}(M^{4.5}T^{4.5})$ [16]. Therefore, the proposed relaxed approach essentially simplifies the implementation of the robust linear receiver as compared to the approach of [11].

4. SIMULATIONS

In simulations, we assume an uplink cellular communication scenario with multiple users and $N = 2$ antennas per user. The Alamouti code [17] with the QPSK modulation is used. The MIMO channel between the pth user and BS is assumed to be quasi-static Rayleigh flat fading with the channel matrix whose elements are drawn from the complex Gaussian distribution as $[H_p]_{n,m} \sim \mathcal{CN}(0,1)$. The channel mismatch E_p is assumed to be independent of H_p with the distribution $[E_p]_{n,m} \sim \mathcal{CN}(0,\sigma_e^2)$ so that the CSI mismatch level for the channel H_p is characterized by the parameter σ_e. Throughout the simulations, we assume that $\sigma_e = 1/3$, while the interference-to-noise-ratio (INR) is equal to 5 dB.

The following receivers are compared in terms of the symbol error rate (SER): ZF receiver (6), MMSE receiver (8), NLP receiver of [11], and the proposed relaxed robust receiver. All the receivers are computed directly using the mismatched channel state information available. The probability γ for the NLP method of [11] and the proposed method is set to be equal to 0.95. All results are averaged over 300 simulation runs.

In our first example, we model the scenario with $P = 2$ users and $M = 2$ receive antennas at the BS. Figure 1 compares the SERs of the receivers tested versus the SNR.

In our second example, the scenario with $P = 4$ users and $M = 4$ receive antennas at the BS is considered. Figure 2 displays the SERs of the receivers tested versus the SNR.

Both figures clearly demonstrate that the proposed robust receiver consistently enjoys better performance compared to the ZF and MMSE receivers. There is only a moderate performance degradation of the proposed receiver as compared to the NLP receiver of [11] which can be viewed as a price for a substantially reduced computational cost of our technique.

5. CONCLUSIONS

The problem of robustness of multiuser space-time block coded MIMO systems against imperfect CSI has been addressed. A new simple linear receiver has been proposed which guarantees the robustness against CSI errors with a

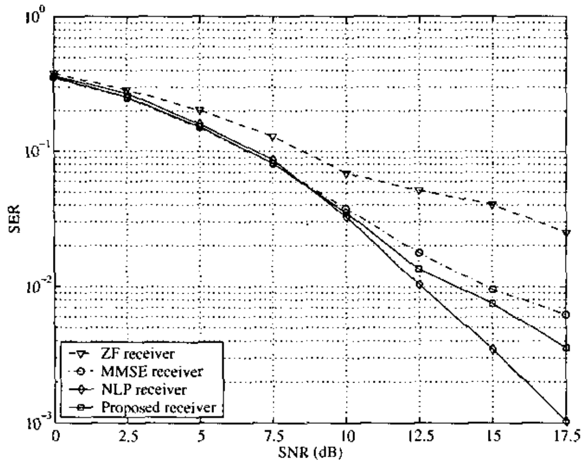

Fig. 1. SERs versus SNR; first example.

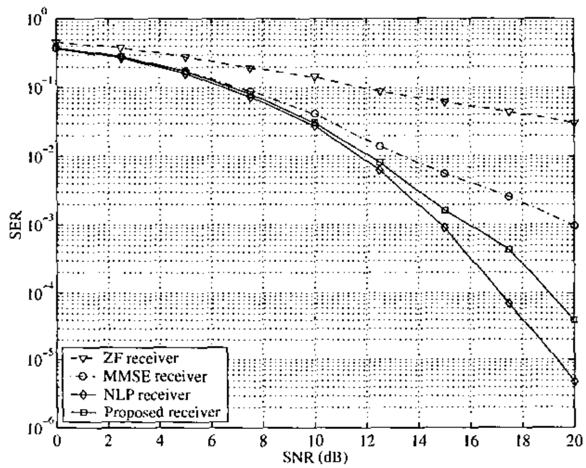

Fig. 2. SERs versus SNR; second example.

certain selected probability. Our receiver is formulated using a probability-constrained optimization problem which is further relaxed to a SOCP problem whose computational complexity is much lower than that of the original problem. Simulations have shown that the proposed receiver has only a moderate performance degradation as compared to the robust NLP receiver.

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