Investigating Successive Interference Cancellation in MIMO Relay Network

Apriana Toding
Dept. Electrical & Computer Eng.
Curtin University
Bentley, WA 6102, Australia
Email: aprianatoding@postgrad.curtin.edu.au

Yue Rong
Dept. Electrical & Computer Eng.
Curtin University
Bentley, WA 6102, Australia
Email: y.rong@curtin.edu.au

***Abstract*—In this paper, we study the zero-forcing (ZF) and minimum mean-squared error (MMSE) algorithms for a multiple-input multiple-output (MIMO) relay network and compare their performance in terms of bit-error-rate (BER). In particular, we investigate their performance with and without using the successive interference cancellation (SIC) at the receiver. Our results demonstrate that the system performance can be significantly improved by using the SIC technique.**

***Index Terms*—multiple-input multiple-output (MIMO), relay networks, zero-forcing (ZF), minimum mean-squared error (MMSE), successive-interference-cancellation (SIC).**

I. INTRODUCTION

In order to provide a reliable wireless transmission, one needs to compensate for the effects of signal fading due to multi-path propagation and strong shadowing. One way to address these issues is to transmit the signal through one or more relays [1]-[6], which can be accomplished via a wireless network consisting of geographically separated nodes. And then the basic motivation behind the use of cooperative communications lies in the exploitation of spatial diversity provided by the network nodes [7] and [8], as well as the efficient use of power resources [9]-[14] which can be achieved by a scheme that simply receives and forwards a given information, yet designed under certain optimality criterion.

Relay schemes can be broadly categorized into three general groups: amplify-and-forward (AF), decode-and-forward (DF), and compress-and-forward (CF). In the AF scheme, the relay nodes amplify the received signal and rebroadcast the amplified signals toward the destination node [3]-[6]. In the DF scheme, the relay nodes first decode the received signals and then forward the re-encoded signals toward the destination node [7]. In the CF method, the relay nodes compress the received signals by exploiting the statistical dependencies between the signals at the nodes [15]. In this paper we consider the AF strategy which is easier to implement compared with the other two approaches.

When nodes in the relay system are installed with multiple antennas, we call such system multiple-input multiple-output (MIMO) relay communication system. Recently, MIMO relay communication systems have attracted much research interest and provided significant improvement in terms of both spatial efficiency and link reliability [4], [6], and [13]-[15]. In this paper, we investigated the performance of zero-forcing (ZF) and minimum mean-squared error (MMSE) algorithms in a MIMO relay network in terms of bit error rate (BER). Note that the ZF and MMSE algorithms have already been studied with single-hop MIMO [9]-[12] and MIMO relay channels [13] and [14]. In [11] and [12], successive interference cancellation (SIC) has been investigated for single-hop MIMO channel. In this paper, we study the ZF and MMSE algorithms with SIC in MIMO relay channel. Our results show that both algorithms with SIC in MIMO relay system have significant performance improvement over the system without using the SIC technique.

The rest of the paper is organized as follows: the system model is described in Section II; in Section III we study the detection techniques at the receiver in a MIMO relay system; Section IV shows the simulation results which justify the performance gain with SIC detection algorithms under various system scenarios and the conclusion is given in Section V.

II. SYSTEM MODEL

Our proposed MIMO relay system with SIC is illustrated in Fig. 1. For simplicity, we consider a three-node

978-1-4577-0255-6/11/$26.00 ©2011 IEEE

MIMO communication system where the source node transmits information to the destination node with the aid of one relay node. The source, the relay and the destination nodes are equipped with N_s, N_r, and N_d antennas, respectively. All data streams are transmitted through the relay and there is no direct link between the source and the destination. $\mathbf{H}_{s,r}$ and $\mathbf{H}_{r,d}$ are the channel matrices for the source-relay and the relay-destination links with dimensions $N_r \times N_s$ and $N_d \times N_r$, respectively. The communication process between the source and destination nodes is completed in two time slots. In the first slot, the $N_s \times 1$ source signal vector $\mathbf{s}$ is transmitted to the relay. The received signal at the relay node can be written as

$$\mathbf{y}_r = \mathbf{H}_{s,r}\mathbf{s} + \mathbf{v}_r \tag{1}$$

where $\mathbf{y}_r$ and $\mathbf{v}_r$ are the received signal and the additive Gaussian noise vectors at the relay node, respectively.

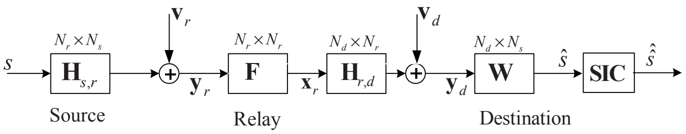

Fig. 1. System model.

In the second slot, the source node remains silent and the relay node multiplies (linearly precodes) the received signal vector by an $N_r \times N_r$ relay amplifying matrix $\mathbf{F}$ and transmits the precoded signal vector $\mathbf{x}_r$ to destination node. Thus

$$\mathbf{x}_r = \mathbf{F}\mathbf{y}_r. \tag{2}$$

The received signal vector at the destination node can be written as

$$\begin{aligned}\mathbf{y}_d &= \mathbf{H}_{r,d}\mathbf{x}_r + \mathbf{v}_d \\ &= \mathbf{H}_{r,d}\mathbf{F}\mathbf{y}_r + \mathbf{v}_d \\ &= \mathbf{H}_{r,d}\mathbf{F}(\mathbf{H}_{s,r}\mathbf{s} + \mathbf{v}_r) + \mathbf{v}_d \\ &= \mathbf{H}_{r,d}\mathbf{F}\mathbf{H}_{s,r}\mathbf{s} + \mathbf{H}_{r,d}\mathbf{F}\mathbf{v}_r + \mathbf{v}_d \end{aligned} \tag{3}$$

where $\mathbf{y}_d$ and $\mathbf{v}_d$ are the received signal and additive Gaussian noise vectors at the destination node, respectively. Thus we see from the above equation that the received signal vector at the destination can be equivalently written as

$$\mathbf{y}_d = \mathbf{H}\mathbf{s} + \mathbf{v} \tag{4}$$

where $\mathbf{H} = \mathbf{H}_{r,d}\mathbf{F}\mathbf{H}_{s,r}$ is the equivalent MIMO channel and $\mathbf{v} = \mathbf{H}_{r,d}\mathbf{F}\mathbf{v}_r + \mathbf{v}_d$ is the equivalent noise.

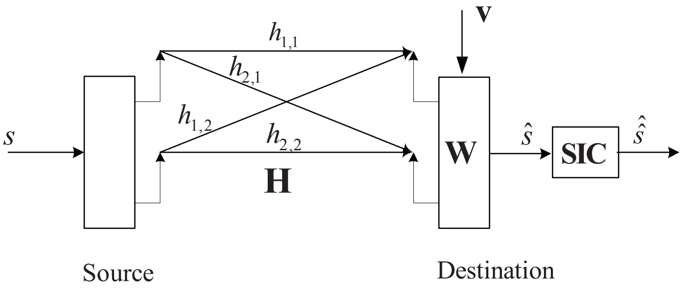

Fig. 2. Equivalent MIMO channel.

In this paper, we try to improve the system BER performance by using SIC in conjunction with ZF and MMSE.

A simple approach to design the relay is to treat it as an all-pass AF unit, which we construct as $\mathbf{F} = \alpha \mathbf{I}_{N_r}$, where α is the amplifying factor of the relay and $\mathbf{I}_{N_r}$ is an identity matrix of dimension N_r. We can find α from $P_r = \alpha^2 tr\{P_s/N_s \mathbf{H}_{s,r}\mathbf{H}_{s,r}^H + \mathbf{I}_{N_r}\}$. Here $P_s > 0$ and $P_r > 0$ are the transmit power available at the source and the relay nodes respectively, $(\cdot)^H$ denotes matrix Hermitian and $tr\{\cdot\}$ indicates trace of a matrix.

III. SIGNAL DETECTION ALGORITHMS FOR MIMO RELAY SYSTEM

We study the following detection algorithms for MIMO relay systems: the ZF and MMSE algorithms with and without SIC technique. If we consider the received signal vector at the destination in (4) then our proposed MIMO relay channel (Fig. 1) reduces to a MIMO channel (Fig. 2) with the equivalent channel matrix of $\mathbf{H} = \mathbf{H}_{r,d}\mathbf{F}\mathbf{H}_{s,r}$, the signals vector of $\mathbf{s}$ and the equivalent noise vector of $\mathbf{v} = \mathbf{H}_{r,d}\mathbf{F}\mathbf{v}_r + \mathbf{v}_d$. Now we can analyze the signal detection at the receiver with the equivalent MIMO channel. For simplicity, we consider here a 2×2 MIMO transmission which essentially means both the transmitter and the receiver are equipped with two antennas. Thus, the received signal on the first receive antenna is

$$\begin{aligned} y_1 &= h_{1,1}\, s_1 + h_{1,2}\, s_2 + v_1 \\ &= \begin{bmatrix} h_{1,1} & h_{1,2} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + v_1 \end{aligned} \tag{5}$$

and that on the second receive antenna is

$$\begin{aligned} y_2 &= h_{2,1}\, s_1 + h_{2,2}\, s_2 + v_2 \\ &= \begin{bmatrix} h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + v_2 \end{aligned} \tag{6}$$

where y_1 and y_2 are the received symbols on the first and the second antennas respectively, $h_{j,i}$ is the channel from ith transmit antenna to jth receive antenna, s_1 and s_2 are the transmitted symbols, and v_1 and v_2 are the noises on first and second receive antennas respectively. For convenience, the above two equations can be combined in matrix notation as

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}. \quad (7)$$

A. The ZF Algorithm

The first decoding technique to be described in this paper is ZF. The ZF linear detector meeting the constraint $\mathbf{W}^H\mathbf{H} = \mathbf{I}_{N_s}$ is given by

$$\mathbf{W} = \mathbf{H}(\mathbf{H}^H\mathbf{H})^{-1}. \quad (8)$$

$\mathbf{W}$ is also known as the pseudo-inverse for a general $m \times n$ matrix and $(\cdot)^{-1}$ indicates simple matrix inversion. In order for a pseudo-inverse to exist, N_d must be greater than or equal to N_s. Then the estimate for the transmit signal vector will be

$$\hat{\mathbf{s}} = \mathbf{W}^H\mathbf{y}_d. \quad (9)$$

Using the ZF equalization approach described above, the receiver can obtain an estimate of the two transmitted symbols s_1 and s_2 as

$$\begin{bmatrix} \hat{s}_1 \\ \hat{s}_2 \end{bmatrix} = (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad (10)$$

where $\hat{s}_1$ and $\hat{s}_2$ are the estimates of s_1 and s_2 respectively.

B. The MMSE Algorithm

To solve $\mathbf{s}$ in (4) the MMSE approach tries to find a coefficient matrix $\mathbf{W}$ that minimizes the statistical expectation of the signal estimation error given by

$$tr\{E\{[\hat{\mathbf{s}} - \mathbf{s}][\hat{\mathbf{s}} - \mathbf{s}]^H\}\} \quad (11)$$

where $E\{\cdot\}$ denotes statistical expectation. Substituting (9) into (11), we find that the $\mathbf{W}$ which minimizes (11) can be written as

$$\mathbf{W} = (\mathbf{H}\mathbf{H}^H + \mathbf{C})^{-1}\mathbf{H} \quad (12)$$

where $\mathbf{C} = \mathbf{H}_{r,d}\mathbf{F}\mathbf{F}^H\mathbf{H}_{r,d}^H + \mathbf{I}_{N_d}$ is the noise covariance matrix. Using the MMSE equalization approach described above, the receiver can obtain an estimate of the two transmitted symbols as

$$\begin{bmatrix} \hat{s}_1 \\ \hat{s}_2 \end{bmatrix} = (\mathbf{H}^H\mathbf{H} + \mathbf{C})^{-1}\mathbf{H}^H \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}. \quad (13)$$

C. The ZF and MMSE with SIC Algorithm

In classical SIC, the receiver arbitrarily takes one of the estimated symbols, and subtracts its effect from the received symbols y_1 and y_2 (Fig. 3). However, a better BER performance can be achieved by choosing whether we should subtract the effect of $\hat{\mathbf{s}}_1$ first or $\hat{\mathbf{s}}_2$ first. To make that decision, let us find out the transmit symbol which came at higher power at the receiver. The received power at both antennas corresponding to the transmitted symbol s_1 is

$$P_{s_1} = |\ h_{1,1}|^2 + |\ h_{2,1}|^2 \quad (14)$$

and the received power at both antennas corresponding to the transmitted symbol s_2 is

$$P_{s_2} = |\ h_{1,2}|^2 + |\ h_{2,2}|^2 \quad (15)$$

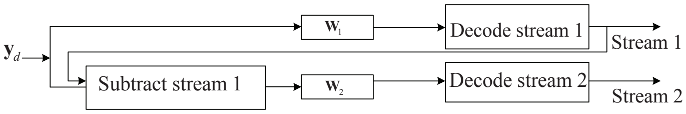

Fig. 3. Successive interference cancellation (SIC) technique.

Now, if $P_{s_1} > P_{s_2}$ then the receiver decides to remove the effect of $\hat{s}_1$ from the received signals y_1 and y_2 and obtain

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} y_1 - h_{1,1}\hat{s}_1 \\ y_2 - h_{2,1}\hat{s}_1 \end{bmatrix}. \quad (16)$$

If there is no error propagation we can express the above equation in matrix notation as

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} h_{1,2} \\ h_{2,2} \end{bmatrix} s_2 + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad (17)$$

or equivalently as

$$\hat{\mathbf{r}}_2 = \mathbf{h}_1\ s_2 + \mathbf{v}. \quad (18)$$

From (18), we can re-estimate $\hat{s}_2$ as

$$\hat{\hat{s}}_2 = \frac{\mathbf{h}_1^H\hat{\mathbf{r}}_2}{\mathbf{h}_1^H\mathbf{h}_1}. \quad (19)$$

Similarly, if $P_{s_2} > P_{s_1}$ then the receiver decides to remove the effect of $\hat{s}_2$ from the received signals y_1 and y_2 and then re-estimates $\hat{s}_1$ as

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} y_1 - h_{1,2}\hat{s}_2 \\ y_2 - h_{2,2}\hat{s}_2 \end{bmatrix}. \quad (20)$$

If there is no error propagation we can express the above equation in matrix notation as

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} \\ h_{2,1} \end{bmatrix} s_1 + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{21}$$

or equivalently as

$$\hat{\mathbf{r}}_1 = \mathbf{h}_2 \; s_1 + \mathbf{v}. \tag{22}$$

From (22), we can re-estimate $\hat{s}_1$ as

$$\hat{\hat{s}}_1 = \frac{\mathbf{h}_2^H \hat{\mathbf{r}}_1}{\mathbf{h}_2^H \mathbf{h}_2}. \tag{23}$$

IV. SIMULATION RESULTS AND DISCUSSIONS

In the simulations, the transmission signaling is in spatial multiplexing mode (i.e., the source transmits independent data streams from different antennas) with total transmit power uniformly distributed among the transmit antennas. Also, all simulations are conducted in a flat-fading Raleigh environment using the BPSK constellation, and the noise variances are assumed to be the same for all antennas. We transmitted 10^3 randomly generated bits in each channel realization and the BER results are averaged through 200 channel realizations. We plot BER curves versus SNR.

In the first example, we simulate the system BER performance of ZF and MMSE receiver with and without SIC in MIMO relay channel with varying SNR in the source-to-relay link (SNR_s) keeping the relay-to-destination SNR (SNR_r) at 20 dB. Fig. 4 show the BER performance with $N_s = N_r = N_d = 2$. It can been seen that at BER = 10^{-3}, we achieve 5 dB gain from MMSE to MMSE-SIC as well as from ZF to ZF-SIC.

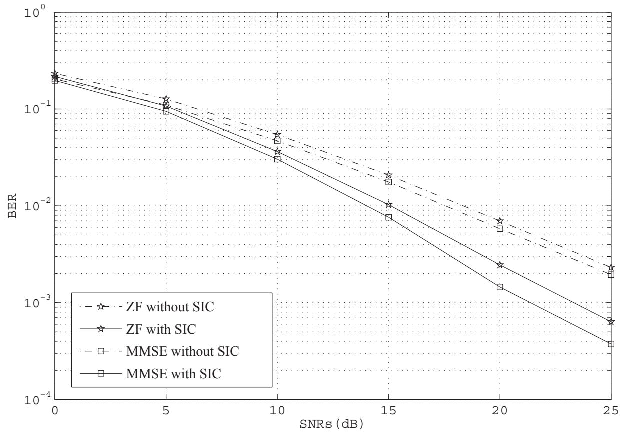

Fig. 4. BER versus SNR_s. $N_s = N_r = N_d = 2$ and $SNR_r = 20$ dB for MIMO relay channel.

In the second example, we simulate the system BER performance of ZF and MMSE receiver with and without SIC in MIMO relay channel with varying SNR in the relay-to-destination SNR (SNR_r) keeping the source-to-relay link (SNR_s) at 20 dB. Fig. 5 show the BER performance with $N_s = N_r = N_d = 2$. Our results demonstrate that ZF-SIC and MMSE-SIC relay algorithms has lower BER compared to the ZF and MMSE relay algorithms.

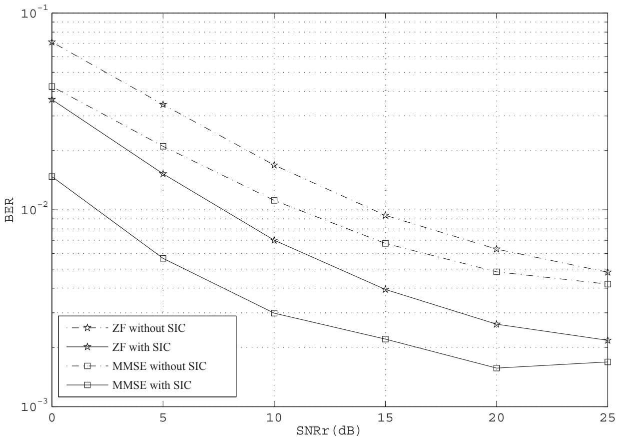

Fig. 5. BER versus SNR_r. $N_s = N_r = N_d = 2$ and $SNR_s = 20$ dB for MIMO relay channel.

V. CONCLUSIONS

In conclusion, we have demonstrated the advantage of using detection algorithms in combination with SIC in MIMO relay network. We designed relays as all-pass amplify-and-forward (AF) units which are simpler to implement. Our results demonstrate that ZF-SIC and MMSE-SIC relay algorithms outperform the ZF and MMSE relay algorithms. Future works may include investigating the SIC technique to parallel relays for MIMO relay networks and optimizing the source and the relay matrices to allocate power efficiently in a cooperative MIMO relay network.

REFERENCES

[1] H. Bolukbasi, H. Yanikomeroglu, H. Falconer, and S. Periyalwar, “On the capacity of cellular fixed relay networks,” in *Proc. Electrical and Computer Engr., 2004. Canadian Conf.*, May 2004, vol. 4, pp. 2217-2220.

[2] R. Pabst, B. H. Walke, D. C. Schultz, D. C. Herhold, H. Yanikomeroglu, S. Mukherjee, H. Viswanathan, M. Lott, W. Zirwas, M. Dohler, H. Aghvami, D. D. Falconer, and G. P. Fettweis, “Relay-based deployment concepts for wireless and mobile broadband radio,” *IEEE Commun. Mag.*, vol. 42, no. 9, pp. 80-89, Sept. 2004.

[3] H. Hu, H. Yanikomeroglu, D. D. Falconer, and S. Periyalwar, "Range extension without capacity penalty in cellular networks with digital fixed relays," *Proc. Glob. Telecom. Conf.*, Nov. 29 - Dec. 3 2004, vol. 5, pp. 3053-3057

[4] B. Wang, J. Zhang, and A. Host-Madsen, "On the capacity of MIMO relay channels," *IEEE Trans. Inf. Theory*, vol. 51, no. 1, pp. 29-43, Jan. 2005.

[5] I. Kang, W. Sheen, R. Chen, and S. Lin C Hsiao, "Throughput improvement with relay-augmented cellular architecture," *IEEE 802.16mmr-05 008*, http://www.wirelessman.org, Sep. 2005.

[6] T. Tang, C. B. Chae, R. W. Heath. Jr., "On achievable sum rates of a multiuser MIMO relay channel," in *Proc. of IEEE ISIT'06*, Seattle, WA, Jul. 2006.

[7] J. N. Laneman and G. W. Wornell, "Distributed space-time coded protocols for exploiting cooperative diversity in wireless," *Trans. Inf. Theory*, vol. 49, no. 10, pp. 2415-2425, Oct. 2003.

[8] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062-3080, Dec. 2004.

[9] H. Sampath, P. Stoica, and A. Paulraj, "Generalized linear precoder and decoder design for MIMO channels using the weighted MMSE criterion," *IEEE Trans. Commun.*, vol. 49, pp. 2198-2206, Dec. 2001.

[10] S. Serbetli and A. Yener, "Transceiver optimization for multiuser MIMO systems," *IEEE Trans. Signal Processing*, vol. 52, no. 1, pp. 214-226, Jan. 2004.

[11] A. Zanella, M. Chiani, and M. Z. Win, "MMSE reception and successive interference cancellation for MIMO system with high spectral efficiency," *IEEE Trans. Commun.*, vol. 4, no. 3, pp. 1244-1253, May 2005

[12] K. Sayana, S. Nagaraj, and S. B. Gelfand, " A MIMO zero forcing receiver with soft interference cancellation for BICM," *IEEE Workshop on Signal Proc. Commun.*,vol. 4, pp. 837-839, 2005

[13] Y. Rong and Y. Hua, "Optimality of diagonalization of multi-hop MIMO relays," *IEEE Trans. Wireless Commun.*, vol. 8, no. 12, pp. 6068-6077, Dec. 2009.

[14] Y. Rong, X. Tang, and Y. Hua, "A unified framework for optimizing linear non-regenerative multicarrier MIMO relay communication systems," *IEEE Trans. Signal Processing*, vol. 57, no. 12, pp. 4837-4851, Dec. 2009.

[15] S. Simoens, O. Munoz, J. Vidal, and A. D. Coso, "Compress-and-forward cooperative MIMO relaying with full channel state information" in *Proc. IEEE Signal Theory and Commun.*, Nov. 2008.