Source and Relay Matrices Optimization for Multiuser Multi-Hop MIMO Relay Systems

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***Abstract*—In this paper, we consider multiaccess communication through multi-hop linear non-regenerative relays, where all users, all relay nodes, and the destination node may have multiple antennas. We design the user, relay, and destination matrices that jointly minimize the mean-squared error (MSE) of the signal waveform estimation. It is shown that the optimal amplifying matrix at each relay node can be viewed as a linear minimal MSE filter concatenated with another linear filter. As a consequence, the MSE matrix of the signal waveform estimation at the destination node is decomposed into the sum of the MSE matrices at all relay nodes. We show that at a high signal-to-noise ratio (SNR) environment, this MSE matrix decomposition significantly simplifies the solution to the problem of optimizing the user and relay matrices. Simulation results show that even at the low to medium SNR range, the simplified optimization algorithms have only a marginal performance degradation but a greatly reduced computational complexity and signalling overhead compared with the existing optimal iterative algorithm, and thus are of great interest for practical relay systems.**

I. Introduction

Multiuser multiple-input multiple-output (MIMO) relay communication systems recently have attracted much research interest [1]-[4]. The achievable sum rate of a system with single-antenna users and a multi-antenna relay has been derived in [1]. In [2], the optimal relay amplifying matrix and user precoding matrices were developed to maximize the sum source-destination mutual information of a two-hop multiuser relay system, where the users and the relay node are equipped with multiple antennas. Recently, a minimal mean-squared error (MMSE)-based optimal multiuser MIMO relay system has been proposed [3]. The quality-of-service constraints in a multi-antenna relay broadcast channel were investigated in [4].

In this paper, we focus on multiaccess communication through multi-hop linear non-regenerative relays. In contrast to [1], we consider a relay system where all users, all relay nodes, and the destination node may have multiple antennas. Using a linear MMSE receiver at the destination node, we show that the optimal amplifying matrix at each relay node can be viewed as a linear MMSE filter concatenated with another linear filter. As a consequence, the MSE matrix of the signal waveform estimation at the destination node can be decomposed into the sum of the MSE matrices at all relay nodes. A very useful application of such decomposition is that it greatly simplifies the user precoding matrices and relay amplifying matrices optimization problem at a (moderately) high signal-to-noise ratio (SNR) environment. In particular, it enables the power allocation optimization to be performed locally at each relay node, which has a significant reduction in both the computational complexity and the signalling overhead compared with the iterative algorithm developed in [3]. Simulation results show that even at the low to medium SNR range, the simplified optimization algorithms only slightly increase the MSE of the signal waveform estimation and the system bit-error-rate (BER), but greatly reduce the computational complexity (less than the complexity of carrying out one iteration of the algorithm in [3]). Thus, the simplified optimization algorithms are of great interest for practical relay systems such as multi-hop wireless backhaul networks [5].

We would like to mention that the decomposition of the MSE matrix was first discovered in [6] for a single-user two-hop MIMO relay system. Our paper generalizes [6] from single-user two-hop MIMO relay system to multiuser multi-hop MIMO relay systems with any number of hops and any number of users. Note that due to the introduction of multiusers and multiple relay nodes, a rigorous proof of the MSE matrix decomposition for multi-hop MIMO relay system is much more challenging than that for the two-hop MIMO channel. The generalization from a single-user two-hop MIMO system to multiuser multi-hop MIMO relay systems is significant. Note that although in this paper we focus on uplink multiaccess systems, the downlink broadcast system can be designed by exploiting the uplink-downlink duality for multi-hop linear non-regenerative MIMO relay systems established in [7]. In this paper, for notational convenience, we consider a narrow band single-carrier system. However, our results can be straightforwardly generalized to wide band multi-carrier multi-hop MIMO relay systems as in the case of two-hop MIMO relay system shown in [8].

The rest of this paper is organized as follows. In Section II, we introduce the model of a multi-hop linear non-regenerative multiaccess MIMO relay communication system. The proposed source and relay design algorithms are presented in Section III. In Section IV, we show some numerical examples. Conclusions are drawn in Section V.

II. System Model

We consider a multiaccess (uplink) system where N_u users simultaneously transmit information to a common destination node equipped with a linear receiver as shown in Fig. 1. Due to the long source-destination distance, $L-1$ relay nodes are applied in serial to relay signals from all users to the destination node, where the lth relay node is equipped with N_l antennas, $l = 1, \cdots, L-1$, and the destination node has N_L

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antennas. The ith user transmits M_i independent data streams using M_i antennas, $i = 1, \cdots, N_u$. We denote $N_0 = \sum_{i=1}^{N_u} M_i$ as the total number of independent data streams from all users. For a linear non-regenerative MIMO relay system, there should be $N_0 \leq \min(N_1, \cdots, N_L)$, since otherwise the system can not support N_0 active symbols in each transmission. Such condition is imposed by the inherent physical property of the MIMO channel (which is true also for classical single-hop MIMO communication systems [9]).

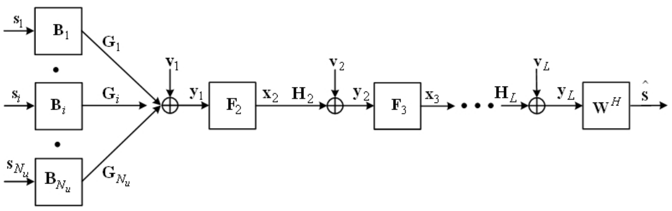

Fig. 1. Block diagram of an N_u-user L-hop linear non-regenerative MIMO relay communication system.

At the ith user, the $M_i \times 1$ modulated signal vector $\mathbf{s}_i$ is linearly precoded by the $M_i \times M_i$ user precoding matrix $\mathbf{B}_i$, and the precoded signal vector $\mathbf{u}_i = \mathbf{B}_i\mathbf{s}_i$ is transmitted to the first relay node. The received signal vector at the first relay node is given by

$$\mathbf{y}_1 = \sum_{i=1}^{N_u} \mathbf{G}_i\mathbf{u}_i + \mathbf{v}_1 \triangleq \mathbf{H}_1\mathbf{x}_1 + \mathbf{v}_1 \qquad (1)$$

where $\mathbf{G}_i$, $i = 1, \cdots, N_u$, is the $N_1 \times M_i$ MIMO channel matrix between the first relay node and the ith user, $\mathbf{v}_1$ is the $N_1 \times 1$ independent and identically distributed (i.i.d.) additive white Gaussian noise (AWGN) vector at the first relay node, $\mathbf{x}_1 = \mathbf{F}_1\mathbf{s}$, $\mathbf{s} \triangleq \left[\mathbf{s}_1^T, \cdots, \mathbf{s}_{N_u}^T\right]^T$, and

$$\mathbf{H}_1 \triangleq [\mathbf{G}_1, \cdots, \mathbf{G}_{N_u}], \qquad \mathbf{F}_1 \triangleq \mathrm{bd}(\mathbf{B}_1, \cdots, \mathbf{B}_{N_u}). \quad (2)$$

Here $\mathbf{H}_1$ is the equivalent $N_1 \times N_0$ first-hop MIMO channel, $\mathbf{F}_1$ is the equivalent $N_0 \times N_0$ block diagonal source precoding matrix, $\mathbf{s}$ is an $N_0 \times 1$ vector containing source symbols from all users, $\mathrm{bd}(\cdot)$ stands for a block diagonal matrix, and $(\cdot)^T$ denotes matrix (vector) transpose. We assume that $\mathrm{E}[\mathbf{s}\mathbf{s}^H] = \mathbf{I}_{N_0}$, where $\mathrm{E}[\cdot]$ stands for the statistical expectation, $(\cdot)^H$ denotes the Hermitian transpose, and $\mathbf{I}_n$ is an $n \times n$ identity matrix.

Due to its simplicity, a linear nonregenerative relay matrix is used at each relay as in [1]-[4]. The input-output relationship at the lth relay nodes is

$$\mathbf{x}_{l+1} = \mathbf{F}_{l+1}\mathbf{y}_l, \qquad l = 1, \cdots, L-1 \qquad (3)$$

where $\mathbf{F}_{l+1}$, $l = 1, \cdots, L-1$, is the $N_l \times N_l$ amplifying matrix at the lth relay node, and $\mathbf{y}_l$, $l = 1, \cdots, L-1$, is the $N_l \times 1$ signal vector received at the lth relay node written as

$$\mathbf{y}_l = \mathbf{H}_l\mathbf{x}_l + \mathbf{v}_l, \qquad l = 1, \cdots, L-1 \qquad (4)$$

where $\mathbf{H}_l$, $l = 1, \cdots, L-1$, is the $N_l \times N_{l-1}$ MIMO channel matrix of the lth hop, and $\mathbf{v}_l$ is the i.i.d. AWGN vector at the lth relay node. Finally, at the last hop, the signal vector received at the destination node is given by (4) with $l = L$. We assume that all noises are complex circularly symmetric with zero mean and unit variance. From (1)-(4), we have

$$\mathbf{y}_l = \mathbf{A}_l\mathbf{s} + \bar{\mathbf{v}}_l, \qquad l = 1, \cdots, L$$

where $\mathbf{A}_l$ is the equivalent MIMO channel matrix from the source to the lth hop, and $\bar{\mathbf{v}}_l$ is the equivalent noise vector given by

$$\mathbf{A}_l = \bigotimes_{i=l}^{1}(\mathbf{H}_i\mathbf{F}_i), \qquad l = 1, \cdots, L \qquad (5)$$

$$\bar{\mathbf{v}}_1 = \mathbf{v}_1 \qquad (6)$$

$$\bar{\mathbf{v}}_l = \sum_{j=2}^{l}\Big(\bigotimes_{i=l}^{j}(\mathbf{H}_i\mathbf{F}_i)\mathbf{v}_{j-1}\Big) + \mathbf{v}_l, \quad l = 2, \cdots, L. \quad (7)$$

Here for matrices $\mathbf{X}_i$, $\bigotimes_{i=l}^{k}(\mathbf{X}_i) \triangleq \mathbf{X}_l \cdots \mathbf{X}_k$.

From (6) and (7), the covariance matrix of $\bar{\mathbf{v}}_l$, $\mathbf{C}_l = \mathrm{E}[\bar{\mathbf{v}}_l\bar{\mathbf{v}}_l^H]$, $l = 1, \cdots, L$, is given by

$$\mathbf{C}_1 = \mathbf{I}_{N_1}$$

$$\mathbf{C}_l = \sum_{j=2}^{l}\Big(\bigotimes_{i=l}^{j}(\mathbf{H}_i\mathbf{F}_i)\bigotimes_{i=j}^{l}(\mathbf{F}_i^H\mathbf{H}_i^H)\Big) + \mathbf{I}_{N_l}, \quad l = 2, \cdots, L.$$

III. Proposed Source and Relay Design Algorithms

With a linear receiver at the destination node, the estimated signal vector is given by $\hat{\mathbf{s}} = \mathbf{W}_L^H\mathbf{y}_L$, where $\mathbf{W}_L$ is the $N_L \times N_0$ weight matrix. The weight matrix of the linear MMSE receiver is $\mathbf{W}_L = (\mathbf{A}_L\mathbf{A}_L^H + \mathbf{C}_L)^{-1}\mathbf{A}_L$, where $(\cdot)^{-1}$ stands for the matrix inversion. Using this MMSE receiver, the MSE matrix $\mathbf{E}_L$ at the destination node is given by

$$\begin{aligned}\mathbf{E}_L &= \left(\mathbf{I}_{N_0} + \mathbf{A}_L^H\mathbf{C}_L^{-1}\mathbf{A}_L\right)^{-1} \\ &= \Bigg[\mathbf{I}_{N_0} + \bigotimes_{i=1}^{L}(\mathbf{F}_i^H\mathbf{H}_i^H)\Bigg(\sum_{l=2}^{L}\Big(\bigotimes_{i=L}^{l}(\mathbf{H}_i\mathbf{F}_i) \\ &\qquad \bigotimes_{i=l}^{L}(\mathbf{F}_i^H\mathbf{H}_i^H)\Big) + \mathbf{I}_{N_L}\Bigg)^{-1}\bigotimes_{i=L}^{1}(\mathbf{H}_i\mathbf{F}_i)\Bigg]^{-1}. \end{aligned} \qquad (8)$$

Let us introduce matrices

$$\mathbf{D}_l \triangleq \mathbf{A}_l\mathbf{A}_l^H + \mathbf{C}_l = \sum_{j=1}^{l}\Big(\bigotimes_{i=l}^{j}(\mathbf{H}_i\mathbf{F}_i)\bigotimes_{i=j}^{l}(\mathbf{F}_i^H\mathbf{H}_i^H)\Big) + \mathbf{I}_{N_l}$$
$$l = 1, \cdots, L-1. \qquad (9)$$

It can be shown from (3) that the transmission power consumed by the lth relay node is

$$\mathrm{tr}\left(\mathrm{E}\big[\mathbf{x}_{l+1}\mathbf{x}_{l+1}^H\big]\right) = \mathrm{tr}\big(\mathbf{F}_{l+1}\mathbf{D}_l\mathbf{F}_{l+1}^H\big), \qquad l = 1, \cdots, L-1 \qquad (10)$$

where $\mathrm{tr}(\cdot)$ denotes matrix trace.

Using (8) and (10), the problem of minimizing the MSE of the signal waveform estimation at the destination node can be written as

$$\min_{\{\mathbf{F}_l\},\{\mathbf{B}_i\}} \quad \mathrm{tr}\Big(\big(\mathbf{I}_{N_0} + \mathbf{A}_L^H \mathbf{C}_L^{-1} \mathbf{A}_L\big)^{-1}\Big) \tag{11}$$

$$\text{s.t.} \quad \mathrm{tr}\big(\mathbf{F}_l \mathbf{D}_{l-1} \mathbf{F}_l^H\big) \le p_l, \quad l = 2, \cdots, L \tag{12}$$

$$\mathrm{tr}\big(\mathbf{B}_i \mathbf{B}_i^H\big) \le q_i, \quad i = 1, \cdots, N_u \tag{13}$$

where (12) and (13) are the transmission power constraint at each relay node and each user, respectively, p_l and q_i are the corresponding power budget, $\{\mathbf{F}_l\} \triangleq [\mathbf{F}_2, \cdots, \mathbf{F}_L]$, and $\{\mathbf{B}_i\} \triangleq [\mathbf{B}_1, \cdots, \mathbf{B}_{N_u}]$. The problem (11)-(13) is non-convex with matrix variables, and a globally optimal solution is very difficult to obtain with a reasonable computational complexity (non-exhaustive searching). In [3], an iterative procedure was developed to obtain (at least) a locally optimal solution of the problem (11)-(13), where in each iteration, the relay amplifying matrices are optimized with fixed user precoding matrices using the results in [10], and then the user precoding matrices are updated with the given relay amplifying matrices through solving an semi-definite programming (SDP) problem. However, the computational complexity and the signalling overhead of the iterative algorithm is quite high for practical relay systems. In the following, we propose simplified algorithms to solve an approximation of the problem (11)-(13). The proposed algorithms have much smaller computational complexity and signalling overhead than the iterative algorithm in [3] as analyzed and shown later.

A. Optimal Structure of Relay Amplifying Matrices

By introducing $N_{l-1} \times N_0$ matrices $\mathbf{T}_l$, $l = 2, \cdots, L$, the following theorem establishes the structure of the optimal relay amplifying matrices, and demonstrates that the MSE matrix at the destination node can be decomposed into the sum of the MSE matrices at all relay nodes.

THEOREM 1: The optimal relay amplifying matrices have the following structure

$$\mathbf{F}_l = \mathbf{T}_l \mathbf{A}_{l-1}^H \mathbf{D}_{l-1}^{-1}, \qquad l = 2, \cdots, L. \tag{14}$$

Using (14), the MSE matrix at the destination node can be equivalently decomposed to

$$\mathbf{E}_L = \big(\mathbf{I}_{N_0} + \mathbf{F}_1^H \mathbf{H}_1^H \mathbf{H}_1 \mathbf{F}_1\big)^{-1} + \sum_{l=2}^{L} \big(\mathbf{R}_l^{-1} + \mathbf{T}_l^H \mathbf{H}_l^H \mathbf{H}_l \mathbf{T}_l\big)^{-1} \tag{15}$$

where

$$\mathbf{R}_l \triangleq \mathbf{A}_{l-1}^H \mathbf{D}_{l-1}^{-1} \mathbf{A}_{l-1}, \quad l = 2, \cdots, L. \tag{16}$$

PROOF: See the journal version of this paper [11]. □

Interestingly, it can be seen from (14) that the optimal relay amplifying matrices $\mathbf{F}_l$, $l = 2, \cdots, L$, can be decomposed into $\mathbf{F}_l = \mathbf{T}_l \mathbf{W}_l^H$, where $\mathbf{W}_l = (\mathbf{A}_{l-1}\mathbf{A}_{l-1}^H + \mathbf{C}_{l-1})^{-1}\mathbf{A}_{l-1}$, $l = 2, \cdots, L$, is the weight matrix of the linear MMSE filter for the received signal vector at the $(l-1)$-th relay node given by $\mathbf{y}_{l-1} = \mathbf{A}_{l-1}\mathbf{s} + \bar{\mathbf{v}}_{l-1}$, and the linear filter $\mathbf{T}_l$ will be designed later. The term $\big(\mathbf{R}_l^{-1} + \mathbf{T}_l^H \mathbf{H}_l^H \mathbf{H}_l \mathbf{T}_l\big)^{-1}$, $l = 2, \cdots, L$, in (15) is the increment of the MSE matrix introduced by the $(l-1)$-th relay node. It is worth noting that $\mathbf{R}_l$ is in fact the covariance matrix of $\mathbf{W}_l^H \mathbf{y}_{l-1}$ as $\mathbf{R}_l = \mathbf{W}_l^H \mathrm{E}[\mathbf{y}_{l-1}\mathbf{y}_{l-1}^H]\mathbf{W}_l$.

By exploiting (14), the transmission power consumed by each relay node can be written as

$$\begin{aligned}\mathrm{tr}\big(\mathbf{F}_l \mathbf{D}_{l-1} \mathbf{F}_l^H\big) &= \mathrm{tr}\big(\mathbf{T}_l \mathbf{A}_{l-1}^H \mathbf{D}_{l-1}^{-1} \mathbf{D}_{l-1} \mathbf{D}_{l-1}^{-1} \mathbf{A}_{l-1} \mathbf{T}_l^H\big) \\ &= \mathrm{tr}\big(\mathbf{T}_l \mathbf{R}_l \mathbf{T}_l^H\big), \quad l = 2, \cdots, L.\end{aligned} \tag{17}$$

From (2) we have

$$\begin{aligned}&\mathrm{tr}\Big(\big(\mathbf{I}_{N_0} + \mathbf{F}_1^H \mathbf{H}_1^H \mathbf{H}_1 \mathbf{F}_1\big)^{-1}\Big) \\ &= \mathrm{tr}\left(\left(\mathbf{I}_{N_1} + \sum_{i=1}^{N_u} \mathbf{G}_i \mathbf{Q}_i \mathbf{G}_i^H\right)^{-1}\right) + N_0 - N_1\end{aligned} \tag{18}$$

where $\mathbf{Q}_i \triangleq \mathrm{E}[\mathbf{u}_i \mathbf{u}_i^H] = \mathbf{B}_i \mathbf{B}_i^H$, $i = 1, \cdots, N_u$, is the covariance matrix of the signal transmitted by the ith user. Now by using (15)-(18), the problem (11)-(13) can be equivalently rewritten as

$$\begin{aligned}\min_{\{\mathbf{Q}_i\},\{\mathbf{T}_l\}} \quad & \mathrm{tr}\left(\left(\mathbf{I}_{N_1} + \sum_{i=1}^{N_u} \mathbf{G}_i \mathbf{Q}_i \mathbf{G}_i^H\right)^{-1}\right) \\ & + \sum_{l=2}^{L} \mathrm{tr}\left(\big(\mathbf{R}_l^{-1} + \mathbf{T}_l^H \mathbf{H}_l^H \mathbf{H}_l \mathbf{T}_l\big)^{-1}\right)\end{aligned} \tag{19}$$

$$\text{s.t.} \quad \mathrm{tr}\big(\mathbf{T}_l \mathbf{R}_l \mathbf{T}_l^H\big) \le p_l, \qquad l = 2, \cdots, L \tag{20}$$

$$\mathrm{tr}(\mathbf{Q}_i) \le q_i, \qquad \mathbf{Q}_i \succeq 0, \quad i = 1, \cdots, N_u \tag{21}$$

where $\{\mathbf{T}_l\} \triangleq [\mathbf{T}_2, \cdots, \mathbf{T}_L]$, $\{\mathbf{Q}_i\} \triangleq [\mathbf{Q}_1, \cdots, \mathbf{Q}_{N_u}]$, and $\succeq$ stands for the matrix positive semi-definiteness.

B. Proposed Algorithm 1

Using the matrix inversion lemma, it can be seen from (16) that for $l = 2, \cdots, L$

$$\mathbf{R}_l = \mathbf{A}_{l-1}^H \mathbf{C}_{l-1}^{-1} \mathbf{A}_{l-1} \big(\mathbf{A}_{l-1}^H \mathbf{C}_{l-1}^{-1} \mathbf{A}_{l-1} + \mathbf{I}_{N_0}\big)^{-1}.$$

In the case of high SNR where $\mathbf{A}_{l-1}^H \mathbf{C}_{l-1}^{-1} \mathbf{A}_{l-1} \gg \mathbf{I}_{N_0}$, we can approximate $\mathbf{R}_l$ as $\mathbf{I}_{N_0}$, $l = 2, \cdots, L$. In other words, in such case, the value of $\mathbf{Q}_i$, $i = 1, \cdots, N_u$, does not affect $\mathbf{R}_l$, $l = 2, \cdots, L$, and $\mathbf{T}_l$ does not affect $\mathbf{R}_j$, $j = l+1, \cdots, L$. This fact implies that the objective function (19) and the constraints in (20) are decoupled with respect to the variables $\{\mathbf{Q}_i\}$ and $\{\mathbf{T}_l\}$. Therefore, the problem (19)-(21) can be approximated and decomposed into the following relay amplifying matrix optimization problem for each $l = 2, \cdots, L$

$$\min_{\mathbf{T}_l} \quad \mathrm{tr}\Big(\big(\mathbf{R}_l^{-1} + \mathbf{T}_l^H \mathbf{H}_l^H \mathbf{H}_l \mathbf{T}_l\big)^{-1}\Big) \tag{22}$$

$$\text{s.t.} \quad \mathrm{tr}\big(\mathbf{T}_l \mathbf{R}_l \mathbf{T}_l^H\big) \le p_l \tag{23}$$

and the user covariance matrices optimization problem

$$\min_{\{\mathbf{Q}_i\}} \quad \mathrm{tr}\left(\left(\mathbf{I}_{N_1} + \sum_{i=1}^{N_u} \mathbf{G}_i \mathbf{Q}_i \mathbf{G}_i^H\right)^{-1}\right) \tag{24}$$

$$\text{s.t.} \quad \mathrm{tr}(\mathbf{Q}_i) \le q_i, \qquad \mathbf{Q}_i \succeq 0, \qquad i = 1, \cdots, N_u. \tag{25}$$

In the following, we show that the problem (22)-(23) has a water-filling solution. Let us introduce the eigenvalue decomposition (EVD) of $\mathbf{H}_l^H\mathbf{H}_l = \mathbf{V}_l\mathbf{\Lambda}_l\mathbf{V}_l^H$ and $\mathbf{R}_l = \mathbf{U}_l\mathbf{\Sigma}_l\mathbf{U}_l^H$, $l = 2, \cdots, L$, where the dimensions of $\mathbf{V}_l$ and $\mathbf{\Lambda}_l$ are $N_{l-1} \times N_{l-1}$, the dimensions of $\mathbf{U}_l$ and $\mathbf{\Sigma}_l$ are $N_0 \times N_0$, and the diagonal elements in $\mathbf{\Lambda}_l$ and $\mathbf{\Sigma}_l$ are sorted in increasing orders. By introducing $\tilde{\mathbf{T}}_l \triangleq \mathbf{T}_l\mathbf{R}^{\frac{1}{2}}$, the problem (22)-(23) can be rewritten as

$$\min_{\tilde{\mathbf{T}}_l} \quad \mathrm{tr}\Big(\mathbf{R}_l^{\frac{1}{2}}\big(\mathbf{I}_{N_0} + \tilde{\mathbf{T}}_l^H\mathbf{H}_l^H\mathbf{H}_l\tilde{\mathbf{T}}_l\big)^{-1}\mathbf{R}_l^{\frac{1}{2}}\Big) \tag{26}$$

$$\text{s.t.} \quad \mathrm{tr}\big(\tilde{\mathbf{T}}_l\tilde{\mathbf{T}}_l^H\big) \le p_l. \tag{27}$$

The solution to the problem (26)-(27) in terms of the singular value decomposition (SVD) of $\tilde{\mathbf{T}}_l$ is given by $\tilde{\mathbf{T}}_l = \mathbf{V}_{l,1}\mathbf{\Omega}_l\mathbf{U}_l^H$, where $\mathbf{V}_{l,1}$ contains the rightmost N_0 columns of $\mathbf{V}_l$. Thus the structure of the optimal linear filter $\mathbf{T}_l$ is given by

$$\mathbf{T}_l = \mathbf{V}_{l,1}\mathbf{\Delta}_l\mathbf{U}_l^H, \qquad \mathbf{\Delta}_l = \mathbf{\Omega}_l\mathbf{\Sigma}_l^{-\frac{1}{2}}, \quad l = 2, \cdots, L \tag{28}$$

where $\mathbf{\Delta}_l$ is an $N_0 \times N_0$ diagonal matrix that remains to be optimized.

Interestingly, it can be seen from (28) that at the $(l-1)$-th relay node, the linear filter $\mathbf{T}_l$ first performs beamforming to the direction of the eigenvectors of $\mathbf{R}_l$, then it allocates power to N_0 streams through $\mathbf{\Delta}_l$, and finally beamforms to the direction of the eigenvectors of $\mathbf{H}_l^H\mathbf{H}_l$. Substituting (28) back into (22)-(23), we find that the matrix-variable optimization problem (22)-(23) is converted to the following optimal power loading problem with scalar variables

$$\min_{\delta_{l,1},\cdots,\delta_{l,N_0}} \quad \sum_{i=1}^{N_0} \frac{1}{\sigma_{l,i}^{-1} + \delta_{l,i}^2\lambda_{l,i}} \tag{29}$$

$$\text{s.t.} \quad \sum_{i=1}^{N_0} \delta_{l,i}^2\sigma_{l,i} \le p_l \tag{30}$$

where $\delta_{l,i}, \sigma_{l,i}, \lambda_{l,i}$, $i = 1, \cdots, N_0$, denote the ith diagonal element of $\mathbf{\Delta}_l$, $\mathbf{\Sigma}_l$, $\mathbf{\Lambda}_l$, respectively. Using the Lagrange multiplier method [12], it can be shown that the problem (29)-(30) has a water-filling solution given by

$$\delta_{l,i}^2 = \frac{1}{\lambda_{l,i}}\left(\sqrt{\frac{\lambda_{l,i}}{\mu_l\,\sigma_{l,i}}} - \frac{1}{\sigma_{l,i}}\right)^+, \qquad i = 1, \cdots, N_0$$

where $(x)^+ \triangleq \max(x, 0)$, and $\mu_l > 0$ is the Lagrangian multiplier and the solution to the nonlinear equation of $\sum_{i=1}^{N_0} \frac{\sigma_{l,i}}{\lambda_{l,i}}\left(\sqrt{\frac{\lambda_{l,i}}{\mu_l\,\sigma_{l,i}}} - \frac{1}{\sigma_{l,i}}\right)^+ = p_l$.

Substituting (28) back into (14), the structure of the optimal relay amplifying matrices is given by

$$\mathbf{F}_l = \mathbf{V}_{l,1}\mathbf{\Delta}_l\mathbf{U}_l^H\mathbf{A}_{l-1}^H\mathbf{D}_{l-1}^{-1}, \qquad l = 2, \cdots, L. \tag{31}$$

Interestingly, although (31) is derived under a high SNR assumption, this structure is in fact optimal for the whole SNR region as stated by the following theorem.

THEOREM 2: The structure of the relay amplifying matrix given in (31) can be equivalently written as $\mathbf{F}_l = \mathbf{V}_{l,1}\mathbf{\Upsilon}_l\mathbf{U}_{H_{l-1},1}^H$, $l = 2, \cdots, L$, which is optimal for multi-hop MIMO relay systems as proved in [10]. Here $\mathbf{\Upsilon}_l$, $l = 2, \cdots, L$, are $N_0 \times N_0$ diagonal matrices, $\mathbf{H}_1\mathbf{F}_1 = \mathbf{U}_{H_1}\mathbf{\Gamma}_1\mathbf{V}_1^H$, $\mathbf{H}_l = \mathbf{U}_{H_l}\mathbf{\Gamma}_l\mathbf{V}_l^H$, $l = 2, \cdots, L$, are SVDs of $\mathbf{H}_1\mathbf{F}_1$ and $\mathbf{H}_l$ with the diagonal elements of $\mathbf{\Gamma}_l$ sorted in increasing orders, and $\mathbf{U}_{H_l,1}$ contains the rightmost N_0 columns of $\mathbf{U}_{H_l}$.

PROOF: See the journal version of this paper [11]. □

Finally, the user covariance matrices optimization problem (24)-(25) can be solved as follows. By introducing a positive semi-definite (PSD) matrix $\mathbf{X}$ with $\mathbf{X} \succeq \big(\mathbf{I}_{N_1} + \sum_{i=1}^{N_u}\mathbf{G}_i\mathbf{Q}_i\mathbf{G}_i^H\big)^{-1}$ and using the Schur complement [12], the problem (24)-(25) can be converted to the problem of

$$\min_{\{\mathbf{Q}_i\},\mathbf{X}} \quad \mathrm{tr}(\mathbf{X}) \tag{32}$$

$$\text{s.t.} \quad \begin{pmatrix} \mathbf{X} & \mathbf{I}_{N_1} \\ \mathbf{I}_{N_1} & \mathbf{I}_{N_1} + \sum_{i=1}^{N_u}\mathbf{G}_i\mathbf{Q}_i\mathbf{G}_i^H \end{pmatrix} \succeq 0 \tag{33}$$

$$\mathrm{tr}(\mathbf{Q}_i) \le q_i, \qquad \mathbf{Q}_i \succeq 0, \quad i = 1, \cdots, N_u. \tag{34}$$

The problem (32)-(34) is a convex SDP problem which can be efficiently solved by the interior-point method [12].

TABLE I
PROCEDURE OF OPTIMIZING THE SOURCE AND RELAY MATRICES.

1) Solve the SDP problem (32)-(34) to obtain $\{\mathbf{Q}_i\}$.
2) For $l = 2 : L$
   Compute $\mathbf{R}_l$; Solve the problem (22)-(23) to have $\mathbf{T}_l$; Obtain $\mathbf{F}_l$ as in (14).
   End.

The procedure of optimizing all user precoding matrices and relay amplifying matrices is described in Table I. We would like to mention that in each iteration of the algorithm in [3], the complexity of updating the user precoding matrices is similar to that of solving the problem (32)-(34). While at each iteration of [3], an alternating power loading algorithm is applied to update the relay amplifying matrices, which has a higher computational complexity than that of solving the problem (22)-(23). Therefore, the computational complexity of carrying out the procedure in Table I is less than that of each iteration in [3]. Interestingly, the procedure in Table I can be carried out in a distributed manner where each relay node performs the necessary optimization procedure locally. In particular, the first relay node optimizes all user precoding matrices and sends back $\mathbf{Q}_i$ to user i. The first relay node also computes the optimal $\mathbf{F}_2$. Then at the lth relay node, $l = 2, \cdots, L-1$, the optimal $\mathbf{F}_{l+1}$ is computed based on $\mathbf{H}_{l+1}$, $\mathbf{C}_l$, and $\mathbf{A}_l$. The CSI of $\mathbf{H}_{l+1}$ can be first estimated at the $(l+1)$-th relay node through channel training [13], and then fed back to the lth relay node. The knowledge of $\mathbf{C}_l$ and $\mathbf{A}_l$ is forwarded from the $(l-1)$-th relay node. Note that due to its iterative nature, the algorithm in [3] requires centralized processing. Obviously, compared with the centralized method, the distributed approach developed here requires much less information exchange and signalling overhead among different nodes, and thus, is preferred in practical relay systems.

C. Proposed Algorithm 2

In this algorithm, the user precoding matrices are optimized by solving the problem (32)-(34). However, since $\mathbf{R}_l$ approaches $\mathbf{I}_{N_0}$ as SNR increases, at a high SNR environment, the relay amplifying matrices optimization can be further simplified by substituting $\mathbf{R}_l$ in (22) and (23) with $\mathbf{I}_{N_0}$. Then we have the following optimization problem for each $l = 2, \cdots, L$

$$\min_{\mathbf{T}_l} \quad \operatorname{tr}\Big(\big(\mathbf{I}_{N_0} + \mathbf{T}_l^H \mathbf{H}_l^H \mathbf{H}_l \mathbf{T}_l\big)^{-1}\Big) \tag{35}$$

$$\text{s.t.} \quad \operatorname{tr}\big(\mathbf{T}_l \mathbf{T}_l^H\big) \le p_l. \tag{36}$$

Interestingly, (35) is in fact an upper-bound of (22). The solution to the problem (35)-(36) is given by

$$\mathbf{T}_l = \mathbf{V}_{l,1} \mathbf{\Theta}_l \mathbf{\Pi}, \qquad l = 2, \cdots, L \tag{37}$$

where $\mathbf{\Pi}$ can be any $N_0 \times N_0$ unitary matrix, and $\mathbf{\Theta}_l$ is an $N_0 \times N_0$ diagonal matrix. Substituting (37) back into (35)-(36), we find that the ith diagonal element of $\mathbf{\Theta}_l$ is given by $\theta_{l,i} = \Big[\frac{1}{\lambda_{l,i}}\Big(\sqrt{\frac{\lambda_{l,i}}{\nu_l}} - 1\Big)^+\Big]^{\frac{1}{2}}$, $i = 1, \cdots, N_0$. Here $\nu_l > 0$ is the solution to the nonlinear equation of $\sum_{i=1}^{N_0} \frac{1}{\lambda_{l,i}}\Big(\sqrt{\frac{\lambda_{l,i}}{\nu_l}} - 1\Big)^+ = p_l$.

Compared with the problem (22)-(23), the relay amplifying matrices designed by the problem (35)-(36) has a smaller computational complexity, since the latter algorithm does not need to compute $\mathbf{R}_l$ and its SVD. Comparing (37) with (28), we can choose $\mathbf{\Pi} = \mathbf{U}_l^H = \mathbf{V}_1^H$ which provides an optimal structure of $\mathbf{F}_l$. $l = 2, \cdots, L$, as in Theorem 2. However, in this case, all relay nodes need to know $\mathbf{V}_1$, which increases the signalling overhead. For the reason of simplicity, we choose $\mathbf{\Pi} = \mathbf{I}_{N_0}$. Through numerical simulations in Section IV, we will see that there is only a negligible increase in MSE and BER by using $\mathbf{\Pi} = \mathbf{I}_{N_0}$ instead of $\mathbf{\Pi} = \mathbf{V}_1^H$.

IV. Numerical Examples

In this section, we study the performance of the proposed multiuser multi-hop MIMO relay design algorithms through numerical simulations. We simulate a flat Rayleigh fading environment where all channel matrices have entries with zero mean. In particular, the variance of entries in $\mathbf{G}_i$ is $1/M_i$, $i = 1, \cdots, N_u$, and the variance of entries in $\mathbf{H}_l$ is $1/N_{l-1}$, $l = 2, \cdots, L$. All noises are complex circularly symmetric with zero mean and unit variance. We also assume that $p_l = P$, $l = 2, \cdots, L$, $q_i = Q$, $i = 1, \cdots, N_u$.

All simulation results are averaged over 5000 independent channel realizations. The CVX convex optimization software package [14] is applied to solve the SDP problem (32)-(34). For all examples, we set $Q = 20$dB and compare the performance of the algorithm described in Table I (denoted as Proposed Algorithm 1), the algorithm where the relay amplifying matrices are designed by solving the problem (35)-(36) using $\mathbf{\Pi} = \mathbf{V}_1^H$ (denoted as Proposed Algorithm 2), the algorithm of solving the problem (35)-(36) with $\mathbf{\Pi} = \mathbf{I}_{N_0}$ (denoted as Proposed Algorithm 3), and the optimal iterative algorithm developed in [3] (denoted as Iterative Algorithm).

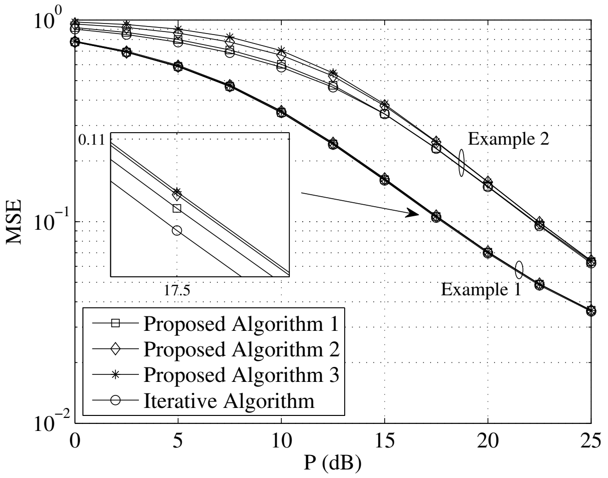

Fig. 2. MSE versus P. Example 1: $L = 2$, $N_u = 2$, $M_1 = 3$, $M_2 = 2$, $N_1 = 8$, and $N_2 = 7$; Example 2: $L = 3$, $N_u = 3$, $M = 2$, and $N = 8$.

In our first example, we simulate a two-hop relay system with $N_u = 2$, $M_1 = 3$, $M_2 = 2$, $N_1 = 8$, and $N_2 = 7$. Fig. 2 shows the MSE performance of all algorithms versus P, and the system BER yielded by all algorithms with QPSK constellations are illustrated in Fig. 3 versus P. Our results clearly demonstrate that the Proposed Algorithms 1-3 only have slightly higher MSE and BER than the Iterative Algorithm. Note that three proposed algorithms have a much smaller computational complexity and signalling overhead than the Iterative Algorithm as analyzed in Subsection III-B.

We simulate multi-hop ($L \ge 3$) multiuser relay systems in the following two examples. Since there are many parameters on the system setup for multi-hop relays, for simplicity, we consider relay systems where all users have the same number of antennas (i.e., $M_i = M$, $i = 1, \cdots, N_u$) and all relay nodes and the destination node have the same number of antennas (i.e., $N_l = N$, $l = 1, \cdots, N_L$). The extension to systems where different nodes have different number of antennas is straight-forward. A three-hop ($L = 3$) MIMO relay system is simulated in the second example with $N_u = 3$, $M = 2$, and $N = 8$. Fig. 2 and Fig. 3 show the MSE and BER comparisons among four algorithms, respectively. It can be seen that due to the approximation from the problem (22)-(23) to the problem (35)-(36), the MSE and BER gaps between the Proposed Algorithm 2, the Proposed Algorithm 3, and the other two algorithm increase at low to medium P. But the performance of the Proposed Algorithm 1 is very close to that of the Iterative Algorithm. It can also be observed from Figs. 2 and 3 that both the MSE and BER values of Example 1 are lower that those of Example 2, indicating that the algorithm yielding a lower MSE indeed guarantees a better BER performance.

In the third example, a five-hop ($L = 5$) relay system is simulated with $N_u = 3$, $M = 2$, and $N = 9$. The MSE and BER comparisons of four algorithms are shown in Fig. 4 and Fig. 5, respectively. It can be clearly seen

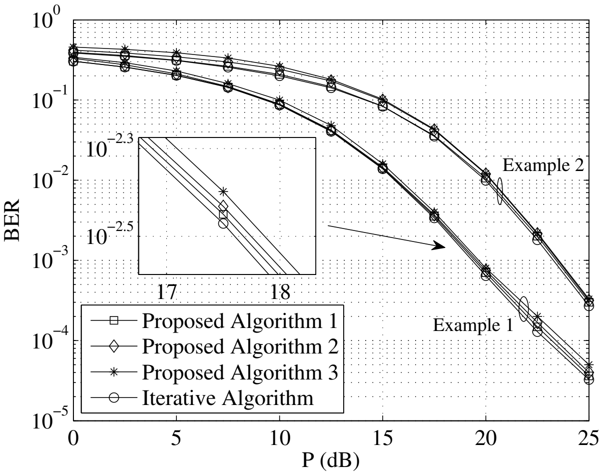

Fig. 3. BER versus P. Example 1: $L = 2$, $N_u = 2$, $M_1 = 3$, $M_2 = 2$, $N_1 = 8$, and $N_2 = 7$; Example 2: $L = 3$, $N_u = 3$, $M = 2$, and $N = 8$.

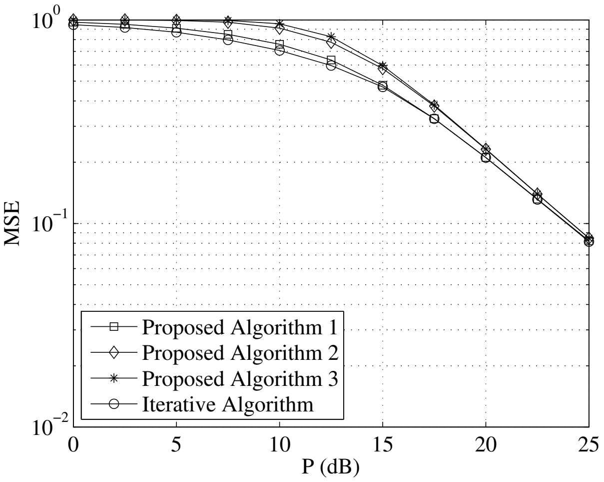

Fig. 4. MSE versus P. Example 3: $L = 5$, $N_u = 3$, $M = 2$, $N = 9$.

that the Proposed Algorithm 1 yields almost the same BER as the Iterative Algorithm. It can also be observed from Figs. 2-5 that there is only a small gap in both the MSE and BER performance between the Proposed Algorithm 2 and the Proposed Algorithm 3. Based on the simulation results and taking into account the complexity-performance tradeoff, the Proposed Algorithm 1 is most suitable for practical multiuser multi-hop MIMO relay systems.

V. Conclusions

We addressed the issue of multiaccess communication through multi-hop non-regenerative MIMO relays. It has been shown that the MSE matrix of the signal waveform estimation at the destination node can be decomposed into the sum of the MSE matrices at all relay nodes. Simplified source and relay optimization algorithms have been proposed which greatly reduce the computational complexity and signalling overhead with only a negligible MSE and BER degradation.

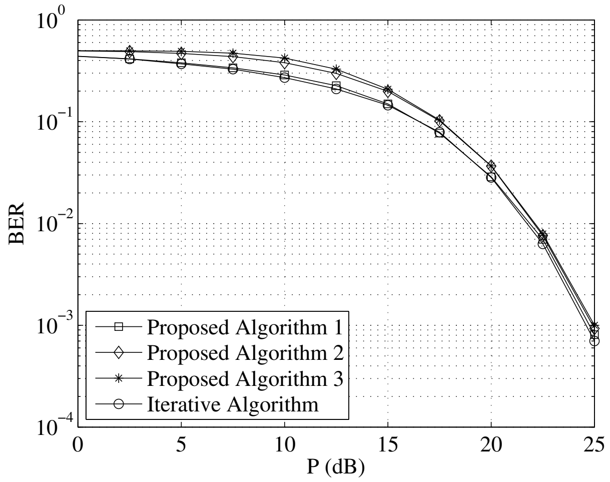

Fig. 5. BER versus P. Example 3: $L = 5$, $N_u = 3$, $M = 2$, $N = 9$.

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